Chapter 1: Real Numbers

CONTENTS 2 exercises · 10 questions

1.1	Fundamental Theorem of Arithmetic — HCF & LCM	7 Q
1.2	Irrational Numbers — proofs	3 Q

Exercise 1.1 — Fundamental Theorem of Arithmetic — HCF & LCM

Q1 Express each number as a product of its prime factors: (i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

SOLUTION

(i) 140
140 ÷ 2 = 70
70 ÷ 2 = 35
35 ÷ 5 = 7
7 is prime.
=> 140 = $2^2 \times 5 \times 7$

(ii) 156
156 ÷ 2 = 78
78 ÷ 2 = 39
39 ÷ 3 = 13
13 is prime.
=> 156 = $2^2 \times 3 \times 13$

(iii) 3825
3825 ÷ 3 = 1275
1275 ÷ 3 = 425
425 ÷ 5 = 85
85 ÷ 5 = 17
17 is prime.
=> 3825 = $3^2 \times 5^2 \times 17$

(iv) 5005
5005 ÷ 5 = 1001
1001 ÷ 7 = 143
143 ÷ 11 = 13
13 is prime.

=> 5005 = 5 × 7 × 11 × 13

(v) 7429
7429 ÷ 17 = 437
437 ÷ 19 = 23
23 is prime.
=> 7429 = 17 × 19 × 23

Answer: 140 = 2^2 × 5 × 7; 156 = 2^2 × 3 × 13; 3825 = 3^2 × 5^2 × 17; 5005 = 5 × 7 × 11 × 13; 7429 = 17 × 19 × 23

Q2 Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers. (i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

SOLUTION

(i) 26 and 91
Prime factorisation:
26 = 2 × 13
91 = 7 × 13
HCF = 13 (only common prime, lowest power = 13^1)
LCM = 2 × 7 × 13 = 182 (highest powers of all primes)
Verification: LCM × HCF = 182 × 13 = 2366; 26 × 91 = 2366 ✓

(ii) 510 and 92
Prime factorisation:
510 = 2 × 3 × 5 × 17
92 = 2^2 × 23
HCF = 2 (only common prime, lowest power = 2^1)
LCM = 2^2 × 3 × 5 × 17 × 23
= 4 × 3 × 5 × 17 × 23 = 12 × 5 × 17 × 23 = 60 × 17 × 23 = 1020 × 23 = 23460
Verification: LCM × HCF = 23460 × 2 = 46920; 510 × 92 = 46920 ✓

(iii) 336 and 54
Prime factorisation:
336 = 2^4 × 3 × 7
54 = 2 × 3^3
HCF = 2^1 × 3^1 = 6 (common primes at their lowest powers)
LCM = 2^4 × 3^3 × 7 = 16 × 27 × 7 = 432 × 7 = 3024
Verification: LCM × HCF = 3024 × 6 = 18144; 336 × 54 = 18144 ✓

Answer: (i) HCF = 13, LCM = 182; (ii) HCF = 2, LCM = 23460; (iii) HCF = 6, LCM = 3024

Q3 Find the LCM and HCF of the following integers by applying the prime factorisation method. (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

SOLUTION

(i) 12, 15 and 21
Prime factorisation:
$12 = 2^2 \times 3$
$15 = 3 \times 5$
$21 = 3 \times 7$
HCF = 3 (only prime common to all three, at its lowest power 3^1)
LCM = $2^2 \times 3 \times 5 \times 7 = 4 \times 3 \times 5 \times 7 = 420$

(ii) 17, 23 and 29
17, 23, and 29 are all prime numbers.
No prime is common to all three, so HCF = 1.
LCM = $17 \times 23 \times 29 = 17 \times 667 = 11339$

(iii) 8, 9 and 25
Prime factorisation:
$8 = 2^3$
$9 = 3^2$
$25 = 5^2$
All three have different prime factors, so HCF = 1.
LCM = $2^3 \times 3^2 \times 5^2 = 8 \times 9 \times 25 = 1800$

Answer: (i) HCF = 3, LCM = 420; (ii) HCF = 1, LCM = 11339; (iii) HCF = 1, LCM = 1800

Q4 Given that HCF (306, 657) = 9, find LCM (306, 657).

SOLUTION

Using the relation: LCM × HCF = Product of the two numbers
LCM × 9 = 306 × 657
306 × 657 = 306 × 600 + 306 × 57 = 183600 + 17442 = 201042
LCM = 201042 ÷ 9 = 22338

Answer: LCM (306, 657) = 22338

Q5 Check whether 6^n can end with the digit 0 for any natural number n.

SOLUTION

A number ends with digit 0 if and only if 10 is a factor of it, i.e., it has both 2 and 5 as prime factors.

Prime factorisation of 6:
$6 = 2 \times 3$
$\Rightarrow 6^n = 2^n \times 3^n$

The prime factorisation of 6^n contains only the primes 2 and 3 — it does not contain 5.

By the Fundamental Theorem of Arithmetic, the prime factorisation of every positive integer is unique.
Since 5 never appears as a prime factor of 6^n, 5 can never divide 6^n.
Therefore 6^n cannot end with the digit 0 for any natural number n.

Answer: 6^n cannot end with the digit 0 for any natural number n, because 5 is not a prime factor of $6^n = 2^n \times 3^n$.

Q6 Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

SOLUTION

A composite number has factors other than 1 and itself.

Expression 1: $7 \times 11 \times 13 + 13$
$= 13 \times (7 \times 11) + 13 \times 1$
$= 13 \times (77 + 1)$
$= 13 \times 78$
This is a product of 13 and 78, both greater than 1.
Therefore it is a composite number.
(Verify: $7 \times 11 \times 13 = 1001$; $1001 + 13 = 1014 = 13 \times 78$ ✓)

Expression 2: $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$
$= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1) + 5 \times 1$
$= 5 \times (1008 + 1)$
$= 5 \times 1009$

This is a product of 5 and 1009, both greater than 1.
Therefore it is a composite number.
(Verify: 7! = 5040; 5040 ÷ 5 = 1008; so 5040 + 5 = 5 × 1009 = 5045 ✓)

Answer: 7 × 11 × 13 + 13 = 13 × 78 (composite); 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 = 5 × 1009 (composite)

Q7 There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

SOLUTION

Sonia completes one round every 18 minutes.
Ravi completes one round every 12 minutes.

They will meet at the starting point again after a time that is a multiple of both 18 and 12 — i.e., after LCM(18, 12) minutes.

Prime factorisation:
$18 = 2 \times 3^2$
$12 = 2^2 \times 3$
$LCM = 2^2 \times 3^2 = 4 \times 9 = 36$

They meet again at the starting point after 36 minutes.
(Check: Sonia completes 36 ÷ 18 = 2 rounds; Ravi completes 36 ÷ 12 = 3 rounds — both back at start ✓)

Answer: They will meet again at the starting point after 36 minutes.

Exercise 1.2 — Irrational Numbers — proofs

Q1 Prove that √5 is irrational.

SOLUTION

Assume, for contradiction, that √5 is rational.
Then √5 = a/b, where a and b are integers, b ≠ 0, and HCF(a, b) = 1 (i.e., a/b is in its lowest terms).

Squaring both sides:
5 = a²/b²
=> a² = 5b² ... (1)

Since 5 divides a², and 5 is prime, by the property of primes, 5 must divide a.
So let a = 5c for some integer c.

Substituting in (1):
(5c)² = 5b²
25c² = 5b²
=> b² = 5c² ... (2)

Since 5 divides b², and 5 is prime, 5 must divide b.

So 5 divides both a and b, which means HCF(a, b) >= 5.
This contradicts our assumption that HCF(a, b) = 1.

The contradiction arose from assuming √5 is rational.
Therefore, √5 is irrational.

Answer: √5 is irrational (proved by contradiction).

Q2 Prove that 3 + 2√5 is irrational.

SOLUTION

Assume, for contradiction, that 3 + 2√5 is rational.
Then 3 + 2√5 = r for some rational number r.

Rearranging:
2√5 = r - 3
√5 = (r - 3) / 2

Now, r is rational (by assumption) and 3 is rational, so r - 3 is rational.
Also, 2 is a non-zero rational, so (r - 3)/2 is rational.
This would make √5 rational.

But we proved in Q1 that √5 is irrational. This is a contradiction.

Therefore, our assumption is false, and 3 + 2√5 is irrational.

Answer: 3 + 2√5 is irrational (proved by contradiction, reducing to irrationality of √5).

Q3 Prove that the following are irrationals: (i) 1/√2 (ii) 7√5 (iii) 6 + √2

SOLUTION

(i) Prove 1/√2 is irrational.
Note: √2 is irrational (proved by contradiction — same method as Q1 for √5).

Assume, for contradiction, that 1/√2 is rational.
Then 1/√2 = a/b for integers a, b with b ≠ 0.
=> √2 = b/a
Since b and a are integers (a ≠ 0), b/a is rational.
So √2 would be rational — a contradiction.
Therefore 1/√2 is irrational.

(ii) Prove 7√5 is irrational.
Assume, for contradiction, that 7√5 is rational.
Then 7√5 = r for some rational r.
=> √5 = r/7
Since r is rational and 7 is a non-zero rational, r/7 is rational.
So √5 would be rational — but √5 is irrational (proved in Q1). Contradiction.
Therefore 7√5 is irrational.

(iii) Prove 6 + √2 is irrational.
Assume, for contradiction, that 6 + √2 is rational.
Then 6 + √2 = r for some rational r.
=> √2 = r - 6
Since r is rational and 6 is rational, r - 6 is rational.
So √2 would be rational — but √2 is irrational. Contradiction.
Therefore 6 + √2 is irrational.

Answer: (i) 1/√2 is irrational; (ii) 7√5 is irrational; (iii) 6 + √2 is irrational. All proved by contradiction.