Chapter 2: Polynomials

CONTENTS 2 exercises · 3 questions

2.1	Geometrical Meaning of the Zeroes of a Polynomial	1 Q
2.2	Relationship between Zeroes and Coefficients of a Polynomial	2 Q

Exercise 2.1 — Geometrical Meaning of the Zeroes of a Polynomial

Q1 The graphs of y = p(x) are given below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.
(i) Graph (i)
(ii) Graph (ii)
(iii) Graph (iii)
(iv) Graph (iv)
(v) Graph (v)
(vi) Graph (vi)

SOLUTION

The number of zeroes of p(x) is the number of times the graph of y = p(x) intersects the x-axis.

(i) The graph does not intersect the x-axis.
∴ Number of zeroes = 0

(ii) The graph intersects the x-axis at exactly one point.
∴ Number of zeroes = 1

(iii) The graph intersects the x-axis at exactly three points.
∴ Number of zeroes = 3

(iv) The graph intersects the x-axis at exactly two points.
∴ Number of zeroes = 2

(v) The graph intersects the x-axis at exactly four points.
∴ Number of zeroes = 4

(vi) The graph intersects the x-axis at exactly three points.
∴ Number of zeroes = 3

Answer: (i) 0 (ii) 1 (iii) 3 (iv) 2 (v) 4 (vi) 3

Exercise 2.2 — Relationship between Zeroes and Coefficients of a Polynomial

Q1 Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

(i) $x^2 - 2x - 8$

(ii) $4s^2 - 4s + 1$

(iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$

(v) $t^2 - 15$

(vi) $3x^2 - x - 4$

SOLUTION

(i) $x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4)$

$= (x - 4)(x + 2)$

Zeroes: $x = 4$ and $x = -2$

Verification: $a = 1, b = -2, c = -8$

Sum of zeroes $= 4 + (-2) = 2 = -b/a = -(-2)/1 = 2$ ✓

Product of zeroes $= 4 \times (-2) = -8 = c/a = -8/1 = -8$ ✓

(ii) $4s^2 - 4s + 1$

$= 4s^2 - 2s - 2s + 1$

$= 2s(2s - 1) - 1(2s - 1)$

$= (2s - 1)(2s - 1)$

Zeroes: $s = 1/2$ and $s = 1/2$

Verification: $a = 4, b = -4, c = 1$

Sum of zeroes $= 1/2 + 1/2 = 1 = -b/a = -(-4)/4 = 1$ ✓

Product of zeroes $= 1/2 \times 1/2 = 1/4 = c/a = 1/4$ ✓

(iii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (2x - 3)(3x + 1)$

Zeroes: $x = 3/2$ and $x = -1/3$

Verification: $a = 6, b = -7, c = -3$

Sum of zeroes $= 3/2 + (-1/3) = 9/6 - 2/6 = 7/6 = -b/a = -(-7)/6 = 7/6$ ✓

Product of zeroes $= 3/2 \times (-1/3) = -3/6 = -1/2 = c/a = -3/6 = -1/2$ ✓

(iv) $4u^2 + 8u$
$= 4u(u + 2)$
Zeroes: $u = 0$ and $u = -2$

Verification: $a = 4, b = 8, c = 0$
Sum of zeroes $= 0 + (-2) = -2 = -b/a = -8/4 = -2$ ✓
Product of zeroes $= 0 \times (-2) = 0 = c/a = 0/4 = 0$ ✓

(v) $t^2 - 15$
$= t^2 - (\sqrt{15})^2$
$= (t - \sqrt{15})(t + \sqrt{15})$
Zeroes: $t = \sqrt{15}$ and $t = -\sqrt{15}$

Verification: $a = 1, b = 0, c = -15$
Sum of zeroes $= \sqrt{15} + (-\sqrt{15}) = 0 = -b/a = 0/1 = 0$ ✓
Product of zeroes $= \sqrt{15} \times (-\sqrt{15}) = -15 = c/a = -15/1 = -15$ ✓

(vi) $3x^2 - x - 4$
$= 3x^2 - 4x + 3x - 4$
$= x(3x - 4) + 1(3x - 4)$
$= (3x - 4)(x + 1)$
Zeroes: $x = 4/3$ and $x = -1$

Verification: $a = 3, b = -1, c = -4$
Sum of zeroes $= 4/3 + (-1) = 4/3 - 3/3 = 1/3 = -b/a = -(-1)/3 = 1/3$ ✓
Product of zeroes $= 4/3 \times (-1) = -4/3 = c/a = -4/3$ ✓

Answer: (i) 4, −2 (ii) 1/2, 1/2 (iii) 3/2, −1/3 (iv) 0, −2 (v) √15, −√15 (vi) 4/3, −1

Q2 Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively:
(i) 1/4, −1
(ii) √2, 1/3
(iii) 0, √5
(iv) 1, 1
(v) −1/4, 1/4
(vi) 4, 1

SOLUTION

If α and β are zeroes, the quadratic polynomial is:

$k[x^2 - (\alpha + \beta)x + \alpha\beta]$, where k is any non-zero constant.

(i) Sum = 1/4, Product = −1

$p(x) = x^2 - (1/4)x + (-1)$

$= x^2 - x/4 - 1$

Multiplying by 4: $4x^2 - x - 4$

(ii) Sum = $\sqrt{2}$, Product = 1/3

$p(x) = x^2 - \sqrt{2}\,x + 1/3$

Multiplying by 3: $3x^2 - 3\sqrt{2}\,x + 1$

(iii) Sum = 0, Product = $\sqrt{5}$

$p(x) = x^2 - 0 \cdot x + \sqrt{5}$

$= x^2 + \sqrt{5}$

(iv) Sum = 1, Product = 1

$p(x) = x^2 - x + 1$

(v) Sum = −1/4, Product = 1/4

$p(x) = x^2 - (-1/4)x + 1/4$

$= x^2 + x/4 + 1/4$

Multiplying by 4: $4x^2 + x + 1$

(vi) Sum = 4, Product = 1

$p(x) = x^2 - 4x + 1$

Answer: (i) $4x^2 - x - 4$ (ii) $3x^2 - 3\sqrt{2}\,x + 1$ (iii) $x^2 + \sqrt{5}$ (iv) $x^2 - x + 1$ (v) $4x^2 + x + 1$ (vi) $x^2 - 4x + 1$