Chapter 3: Pair of Linear Equations in Two Variables

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Exercise 3.1 — Graphical Representation and Consistency

Q1 Form the pair of linear equations in the following problems, and find their solutions graphically.
(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.
(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and one pen.

SOLUTION

(i) Let number of boys = x and number of girls = y.
Total students: x + y = 10 ... (1)
Girls are 4 more than boys: y - x = 4 => -x + y = 4 ... (2)

From (1): y = 10 - x
x = 0 => y = 10; x = 4 => y = 6; x = 10 => y = 0

From (2): y = x + 4
x = 0 => y = 4; x = 3 => y = 7; x = 6 => y = 10

Plotting both lines, they intersect at (3, 7).
∴ Number of boys = 3, Number of girls = 7.

Verification: 3 + 7 = 10 ✓ and 7 - 3 = 4 ✓

(ii) Let cost of one pencil = Rs x and cost of one pen = Rs y.
5x + 7y = 50 ... (1)
7x + 5y = 46 ... (2)

From (1): y = (50 - 5x)/7
x = 3 => y = (50 - 15)/7 = 35/7 = 5
x = 10 => y = (50 - 50)/7 = 0
x = -4 => y = (50 + 20)/7 = 70/7 = 10

From (2): $y = (46 - 7x)/5$
$x = 3 \Rightarrow y = (46 - 21)/5 = 25/5 = 5$
$x = 8 \Rightarrow y = (46 - 56)/5 = -10/5 = -2$
$x = -2 \Rightarrow y = (46 + 14)/5 = 60/5 = 12$

Plotting both lines, they intersect at (3, 5).
∴ Cost of one pencil = Rs 3, Cost of one pen = Rs 5.

Verification: $5(3) + 7(5) = 15 + 35 = 50$ ✓
$7(3) + 5(5) = 21 + 25 = 46$ ✓

Answer: (i) Boys = 3, Girls = 7 (ii) Pencil = Rs 3, Pen = Rs 5

Q2 On comparing the ratios a_1/a_2, b_1/b_2 and c_1/c_2, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:
(i) $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$
(ii) $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$
(iii) $6x - 3y + 10 = 0$, $2x - y + 9 = 0$

SOLUTION

(i) $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$
$a_1/a_2 = 5/7$, $b_1/b_2 = -4/6 = -2/3$
Since $a_1/a_2 \neq b_1/b_2$ $(5/7 \neq -2/3)$
∴ The lines intersect at a point. (Consistent)

(ii) $9x + 3y + 12 = 0$ and $18x + 6y + 24 = 0$
$a_1/a_2 = 9/18 = 1/2$, $b_1/b_2 = 3/6 = 1/2$, $c_1/c_2 = 12/24 = 1/2$
Since $a_1/a_2 = b_1/b_2 = c_1/c_2 = 1/2$
∴ The lines are coincident. (Dependent, consistent — infinitely many solutions)

(iii) $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$
$a_1/a_2 = 6/2 = 3$, $b_1/b_2 = -3/-1 = 3$, $c_1/c_2 = 10/9$
Since $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ $(3 = 3 \neq 10/9)$
∴ The lines are parallel. (Inconsistent — no solution)

Answer: (i) Intersecting (ii) Coincident (iii) Parallel

Q3 On comparing the ratios a_1/a_2, b_1/b_2 and c_1/c_2, find out whether the following pair of linear equations are consistent, or inconsistent.
(i) 3x + 2y = 5; 2x - 3y = 7
(ii) 2x - 3y = 8; 4x - 6y = 9
(iii) (3/2)x + (5/3)y = 7; 9x - 10y = 14
(iv) 5x - 3y = 11; -10x + 6y = -22

SOLUTION

(i) 3x + 2y - 5 = 0 and 2x - 3y - 7 = 0
$a_1/a_2 = 3/2$, $b_1/b_2 = 2/(-3) = -2/3$
Since $a_1/a_2 \neq b_1/b_2$ $(3/2 \neq -2/3)$
∴ Consistent (unique solution). Lines intersect.

Graphical solution:
From 3x + 2y = 5: y = (5 - 3x)/2
x = 1 => y = 1; x = 3 => y = -2; x = -1 => y = 4
From 2x - 3y = 7: y = (2x - 7)/3
x = 2 => y = -1; x = 5 => y = 1; x = -1 => y = -3

The lines intersect at approximately (29/13, -11/13).
Check: Solving algebraically: multiply eq(1) by 3 and eq(2) by 2:
9x + 6y = 15 and 4x - 6y = 14
Adding: 13x = 29 => x = 29/13
From (1): 2y = 5 - 3(29/13) = 5 - 87/13 = (65 - 87)/13 = -22/13
y = -11/13
∴ Solution is x = 29/13, y = -11/13

(ii) 2x - 3y - 8 = 0 and 4x - 6y - 9 = 0
$a_1/a_2 = 2/4 = 1/2$, $b_1/b_2 = -3/(-6) = 1/2$, $c_1/c_2 = -8/(-9) = 8/9$
Since $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ $(1/2 = 1/2 \neq 8/9)$
∴ Inconsistent (no solution). Lines are parallel.

(iii) (3/2)x + (5/3)y - 7 = 0 and 9x - 10y - 14 = 0
$a_1/a_2 = (3/2)/9 = 3/18 = 1/6$
$b_1/b_2 = (5/3)/(-10) = 5/(-30) = -1/6$
Since $a_1/a_2 \neq b_1/b_2$ $(1/6 \neq -1/6)$
∴ Consistent (unique solution). Lines intersect.

Graphical solution:
From (3/2)x + (5/3)y = 7 => 9x + 10y = 42 (multiply by 6)
From 9x - 10y = 14

Adding: 18x = 56 => x = 56/18 = 28/9
From 9(28/9) - 10y = 14 => 28 - 10y = 14 => 10y = 14 => y = 14/10 = 7/5
∴ Solution is x = 28/9, y = 7/5

(iv) 5x - 3y - 11 = 0 and -10x + 6y + 22 = 0
$a_1/a_2 = 5/(-10) = -1/2$, $b_1/b_2 = -3/6 = -1/2$, $c_1/c_2 = -11/22 = -1/2$
Since $a_1/a_2 = b_1/b_2 = c_1/c_2 = -1/2$
∴ Consistent (dependent — infinitely many solutions). Lines are coincident.

The second equation is just -2 times the first equation.
Every point on 5x - 3y = 11 is a solution.

Answer: (i) Consistent, x = 29/13, y = -11/13 (ii) Inconsistent (iii) Consistent, x = 28/9, y = 7/5 (iv) Consistent (dependent, infinite solutions)

Exercise 3.2 — Solving by Substitution Method

Q1 Solve the following pair of linear equations by the substitution method.

(i) x + y = 14, x - y = 4

(ii) s - t = 3, s/3 + t/2 = 6

(iii) 3x - y = 3, 9x - 3y = 9

SOLUTION

(i) x + y = 14 ... (1) and x - y = 4 ... (2)

From (2): x = 4 + y

Substituting in (1): (4 + y) + y = 14

4 + 2y = 14

2y = 10 => y = 5

x = 4 + 5 = 9

∴ x = 9, y = 5

Verification: 9 + 5 = 14 ✓ and 9 - 5 = 4 ✓

(ii) s - t = 3 ... (1) and s/3 + t/2 = 6 ... (2)

From (1): s = 3 + t

Substituting in (2): (3 + t)/3 + t/2 = 6

Multiply through by 6: 2(3 + t) + 3t = 36

6 + 2t + 3t = 36

5t = 30 => t = 6

s = 3 + 6 = 9

∴ s = 9, t = 6

Verification: 9 - 6 = 3 ✓ and 9/3 + 6/2 = 3 + 3 = 6 ✓

(iii) 3x - y = 3 ... (1) and 9x - 3y = 9 ... (2)

From (1): y = 3x - 3

Substituting in (2): 9x - 3(3x - 3) = 9

9x - 9x + 9 = 9

9 = 9 (always true)

This is a dependent system — equation (2) is just 3 times equation (1).

∴ Infinitely many solutions of the form y = 3x - 3.

For example: x = 1, y = 0; x = 2, y = 3; x = 0, y = -3

Answer: (i) x = 9, y = 5 (ii) s = 9, t = 6 (iii) Infinitely many solutions, y = 3x - 3

Q2 Solve 2x + 3y = 11 and 2x - 4y = -24 and hence find the value of 'm' for which y = mx + 3.

SOLUTION

2x + 3y = 11 ... (1)
2x - 4y = -24 ... (2)

From (1): 2x = 11 - 3y => x = (11 - 3y)/2
Substituting in (2): (11 - 3y) - 4y = -24
11 - 3y - 4y = -24
11 - 7y = -24
-7y = -35 => y = 5

From (1): 2x + 3(5) = 11
2x + 15 = 11
2x = -4 => x = -2
∴ x = -2, y = 5

Verification: 2(-2) + 3(5) = -4 + 15 = 11 ✓
2(-2) - 4(5) = -4 - 20 = -24 ✓

Now finding m: y = mx + 3
5 = m(-2) + 3
5 = -2m + 3
-2m = 2 => m = -1

Answer: x = -2, y = 5, m = -1

Q3 Form the pair of linear equations for the following problem and find their solution by substitution method.
The difference between two numbers is 26 and one number is three times the other. Find them.

SOLUTION

Let the two numbers be x and y, where x > y.
x - y = 26 ... (1)
x = 3y ... (2)

Substituting (2) in (1):
3y - y = 26
2y = 26 => y = 13

x = 3(13) = 39

∴ The two numbers are 39 and 13.

Verification: 39 - 13 = 26 ✓ and 39 = 3 × 13 ✓

Answer: The numbers are 39 and 13

Q4 Solve the following pair of linear equations by the substitution method:

0.2x + 0.3y = 1.3

0.4x + 0.5y = 2.3

SOLUTION

0.2x + 0.3y = 1.3 ... (1)

0.4x + 0.5y = 2.3 ... (2)

Multiply both equations by 10 to remove decimals:

2x + 3y = 13 ... (3)

4x + 5y = 23 ... (4)

From (3): 2x = 13 - 3y => x = (13 - 3y)/2

Substituting in (4): 4 × (13 - 3y)/2 + 5y = 23

2(13 - 3y) + 5y = 23

26 - 6y + 5y = 23

26 - y = 23

-y = -3 => y = 3

x = (13 - 3(3))/2 = (13 - 9)/2 = 4/2 = 2

∴ x = 2, y = 3

Verification: 0.2(2) + 0.3(3) = 0.4 + 0.9 = 1.3 ✓

0.4(2) + 0.5(3) = 0.8 + 1.5 = 2.3 ✓

Answer: x = 2, y = 3

Q5 Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden by substitution method.

SOLUTION

Let length = x m and width = y m.

Half perimeter: x + y = 36 ... (1)

Length is 4m more than width: x = y + 4 ... (2)

Substituting (2) in (1):
(y + 4) + y = 36
2y + 4 = 36
2y = 32 => y = 16
x = 16 + 4 = 20
∴ Length = 20 m, Width = 16 m

Verification: Half perimeter = 20 + 16 = 36 ✓
20 - 16 = 4 ✓

Answer: Length = 20 m, Width = 16 m

Q6 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and one pen.

SOLUTION

Let cost of one pencil = Rs x and cost of one pen = Rs y.
5x + 7y = 50 ... (1)
7x + 5y = 46 ... (2)

From (1): 5x = 50 - 7y => x = (50 - 7y)/5
Substituting in (2):
7 × (50 - 7y)/5 + 5y = 46
Multiply through by 5:
7(50 - 7y) + 25y = 230
350 - 49y + 25y = 230
350 - 24y = 230
-24y = -120 => y = 5

x = (50 - 7(5))/5 = (50 - 35)/5 = 15/5 = 3
∴ Cost of one pencil = Rs 3, Cost of one pen = Rs 5

Verification: 5(3) + 7(5) = 15 + 35 = 50 ✓
7(3) + 5(5) = 21 + 25 = 46 ✓

Answer: Pencil = Rs 3, Pen = Rs 5

Q7 The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

SOLUTION

Let the larger angle = x° and the smaller angle = y°.
Supplementary angles: x + y = 180 ... (1)
Larger exceeds smaller by 18: x - y = 18 ... (2)

From (2): x = y + 18
Substituting in (1):
(y + 18) + y = 180
2y + 18 = 180
2y = 162 => y = 81
x = 81 + 18 = 99
∴ The two angles are 99° and 81°.

Verification: 99 + 81 = 180 ✓ and 99 - 81 = 18 ✓

Answer: 99° and 81°

Exercise 3.3 — Solving by Elimination and Cross-Multiplication

Q1 Solve the following pair of linear equations by the elimination method and the substitution method:
(i) x + y = 5 and 2x - 3y = 4
(ii) 3x + 4y = 10 and 2x - 2y = 2
(iii) 3x - 5y - 4 = 0 and 9x = 2y + 7

SOLUTION

(i) x + y = 5 ... (1) and 2x - 3y = 4 ... (2)

Elimination method:
Multiply (1) by 2: 2x + 2y = 10 ... (3)
Subtract (2) from (3): (2x + 2y) - (2x - 3y) = 10 - 4
2y + 3y = 6 => 5y = 6 => y = 6/5
From (1): x = 5 - 6/5 = 25/5 - 6/5 = 19/5

Substitution method:
From (1): x = 5 - y
Substituting in (2): 2(5 - y) - 3y = 4
10 - 2y - 3y = 4 => -5y = -6 => y = 6/5
x = 5 - 6/5 = 19/5
∴ x = 19/5, y = 6/5

Verification: 19/5 + 6/5 = 25/5 = 5 ✓
2(19/5) - 3(6/5) = 38/5 - 18/5 = 20/5 = 4 ✓

(ii) 3x + 4y = 10 ... (1) and 2x - 2y = 2 ... (2)

Elimination method:
Multiply (2) by 2: 4x - 4y = 4 ... (3)
Add (1) and (3): (3x + 4y) + (4x - 4y) = 10 + 4
7x = 14 => x = 2
From (2): 2(2) - 2y = 2 => 4 - 2y = 2 => 2y = 2 => y = 1

Substitution method:
From (2): 2x = 2 + 2y => x = 1 + y
Substituting in (1): 3(1 + y) + 4y = 10
3 + 3y + 4y = 10 => 7y = 7 => y = 1
x = 1 + 1 = 2
∴ x = 2, y = 1

Verification: 3(2) + 4(1) = 6 + 4 = 10 ✓
2(2) - 2(1) = 4 - 2 = 2 ✓

(iii) 3x - 5y - 4 = 0 => 3x - 5y = 4 ... (1)
9x = 2y + 7 => 9x - 2y = 7 ... (2)

Elimination method:
Multiply (1) by 3: 9x - 15y = 12 ... (3)
Subtract (2) from (3): (9x - 15y) - (9x - 2y) = 12 - 7
-15y + 2y = 5 => -13y = 5 => y = -5/13
From (1): 3x = 4 + 5(-5/13) = 4 - 25/13 = (52 - 25)/13 = 27/13
x = 27/39 = 9/13

Substitution method:
From (1): 3x = 4 + 5y => x = (4 + 5y)/3
Substituting in (2): 9 × (4 + 5y)/3 - 2y = 7
3(4 + 5y) - 2y = 7
12 + 15y - 2y = 7 => 13y = -5 => y = -5/13
x = (4 + 5(-5/13))/3 = (4 - 25/13)/3 = (52/13 - 25/13)/3 = (27/13)/3 = 9/13
∴ x = 9/13, y = -5/13

Verification: 3(9/13) - 5(-5/13) = 27/13 + 25/13 = 52/13 = 4 ✓
9(9/13) = 81/13 and 2(-5/13) + 7 = -10/13 + 91/13 = 81/13 ✓

Answer: (i) x = 19/5, y = 6/5 (ii) x = 2, y = 1 (iii) x = 9/13, y = -5/13

Q2 Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

SOLUTION

Let length = x m and width = y m.
Half perimeter: x + y = 36 ... (1)
Length is 4m more than width: x - y = 4 ... (2)

Using elimination:
Adding (1) and (2): 2x = 40 => x = 20
From (1): 20 + y = 36 => y = 16
∴ Length = 20 m, Width = 16 m

Verification: 20 + 16 = 36 ✓ and 20 - 16 = 4 ✓

Answer: Length = 20 m, Width = 16 m

Q3 Form the pair of linear equations for the following problem and find their solution by elimination method.
The sum of the digits of a two-digit number is 9. The number obtained by reversing the digits exceeds the original number by 27. Find the number.

SOLUTION

Let the tens digit = x and the units digit = y.
Original number = 10x + y, Reversed number = 10y + x.

Sum of digits: x + y = 9 ... (1)
Reversed number exceeds original by 27:
(10y + x) - (10x + y) = 27
9y - 9x = 27
-x + y = 3 ... (2)

Using elimination:
Adding (1) and (2): (x + y) + (-x + y) = 9 + 3
2y = 12 => y = 6
From (1): x + 6 = 9 => x = 3

∴ The number = 10(3) + 6 = 36

Verification: Sum of digits = 3 + 6 = 9 ✓
Reversed number = 63, and 63 - 36 = 27 ✓

Answer: The number is 36

Q4 Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes of Rs 50 and Rs 100 she received.

SOLUTION

Let number of Rs 50 notes = x and Rs 100 notes = y.
Total notes: x + y = 25 ... (1)
Total amount: 50x + 100y = 2000 ... (2)

Simplify (2) by dividing by 50: x + 2y = 40 ... (3)

Using elimination:
Subtract (1) from (3): (x + 2y) - (x + y) = 40 - 25
y = 15
From (1): x + 15 = 25 => x = 10

∴ Rs 50 notes = 10, Rs 100 notes = 15

Verification: 10 + 15 = 25 ✓
50(10) + 100(15) = 500 + 1500 = 2000 ✓

Answer: 10 notes of Rs 50, 15 notes of Rs 100

Q5 A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for a book kept for five days. Find the fixed charge and the charge for each extra day.

SOLUTION

Let the fixed charge for first 3 days = Rs x
Let the charge per extra day = Rs y

Saritha kept for 7 days = 3 days (fixed) + 4 extra days:
x + 4y = 27 ... (1)

Susy kept for 5 days = 3 days (fixed) + 2 extra days:
x + 2y = 21 ... (2)

Using elimination:
Subtract (2) from (1): (x + 4y) - (x + 2y) = 27 - 21
2y = 6 => y = 3
From (2): x + 2(3) = 21 => x + 6 = 21 => x = 15
∴ Fixed charge = Rs 15, Charge per extra day = Rs 3

Verification: 15 + 4(3) = 15 + 12 = 27 ✓
15 + 2(3) = 15 + 6 = 21 ✓

Answer: Fixed charge = Rs 15, Charge per extra day = Rs 3

Q6 Solve the following pairs of equations by the cross-multiplication method:
(i) x - 3y - 3 = 0, 3x - 9y - 2 = 0
(ii) 2x + y = 5, 3x + 2y = 8
(iii) 3x - 5y = 20, 6x - 10y = 40
(iv) x - 3y - 7 = 0, 3x - 3y - 15 = 0

SOLUTION

(i) x - 3y - 3 = 0 and 3x - 9y - 2 = 0

$a_1 = 1, b_1 = -3, c_1 = -3$
$a_2 = 3, b_2 = -9, c_2 = -2$

Check: $a_1/a_2 = 1/3$, $b_1/b_2 = -3/-9 = 1/3$, $c_1/c_2 = -3/-2 = 3/2$
Since $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, the lines are parallel.
$\therefore$ No solution (inconsistent).

(ii) $2x + y - 5 = 0$ and $3x + 2y - 8 = 0$
$a_1 = 2, b_1 = 1, c_1 = -5$
$a_2 = 3, b_2 = 2, c_2 = -8$

By cross-multiplication:
$x / (b_1c_2 - b_2c_1) = y / (c_1a_2 - c_2a_1) = 1 / (a_1b_2 - a_2b_1)$
$x / (1\times(-8) - 2\times(-5)) = y / ((-5)\times3 - (-8)\times2) = 1 / (2\times2 - 3\times1)$
$x / (-8 + 10) = y / (-15 + 16) = 1 / (4 - 3)$
$x / 2 = y / 1 = 1 / 1$
$\therefore x = 2, y = 1$

Verification: $2(2) + 1 = 5$ ✓ and $3(2) + 2(1) = 8$ ✓

(iii) $3x - 5y - 20 = 0$ and $6x - 10y - 40 = 0$
$a_1/a_2 = 3/6 = 1/2$, $b_1/b_2 = -5/-10 = 1/2$, $c_1/c_2 = -20/-40 = 1/2$
Since $a_1/a_2 = b_1/b_2 = c_1/c_2 = 1/2$
The lines are coincident (equation 2 is just 2 times equation 1).
$\therefore$ Infinitely many solutions.

(iv) $x - 3y - 7 = 0$ and $3x - 3y - 15 = 0$
$a_1 = 1, b_1 = -3, c_1 = -7$
$a_2 = 3, b_2 = -3, c_2 = -15$

By cross-multiplication:
$x / (b_1c_2 - b_2c_1) = y / (c_1a_2 - c_2a_1) = 1 / (a_1b_2 - a_2b_1)$
$x / ((-3)(-15) - (-3)(-7)) = y / ((-7)(3) - (-15)(1)) = 1 / ((1)(-3) - (3)(-3))$
$x / (45 - 21) = y / (-21 + 15) = 1 / (-3 + 9)$
$x / 24 = y / (-6) = 1 / 6$
$x = 24/6 = 4$
$y = -6/6 = -1$
$\therefore x = 4, y = -1$

Verification: $4 - 3(-1) - 7 = 4 + 3 - 7 = 0$ ✓
$3(4) - 3(-1) - 15 = 12 + 3 - 15 = 0$ ✓

Answer: (i) No solution (ii) x = 2, y = 1 (iii) Infinitely many solutions (iv) x = 4, y = -1

Q7 (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?
2x + 3y = 7
(a - b)x + (a + b)y = 3a + b - 2
(ii) For which value of k will the following pair of linear equations have no solution?
3x + y = 1
(2k - 1)x + (k - 1)y = 2k + 1

SOLUTION

(i) 2x + 3y = 7 and (a-b)x + (a+b)y = 3a+b-2
For infinitely many solutions: $a_1/a_2 = b_1/b_2 = c_1/c_2$
2/(a-b) = 3/(a+b) = 7/(3a+b-2)

From 2/(a-b) = 3/(a+b):
2(a+b) = 3(a-b)
2a + 2b = 3a - 3b
5b = a => a = 5b ... (A)

From 3/(a+b) = 7/(3a+b-2):
3(3a+b-2) = 7(a+b)
9a + 3b - 6 = 7a + 7b
2a - 4b = 6
a - 2b = 3 ... (B)

Substituting (A) in (B): 5b - 2b = 3 => 3b = 3 => b = 1
a = 5(1) = 5
∴ a = 5, b = 1

Verification: a-b = 4, a+b = 6, 3a+b-2 = 14
2/4 = 1/2, 3/6 = 1/2, 7/14 = 1/2 ✓

(ii) 3x + y = 1 and (2k-1)x + (k-1)y = 2k+1
For no solution: $a_1/a_2 = b_1/b_2 \neq c_1/c_2$
3/(2k-1) = 1/(k-1)
3(k-1) = 2k-1
3k - 3 = 2k - 1
k = 2

Check $c_1/c_2 \neq a_1/a_2$ when k = 2:

$a_1/a_2 = 3/(2(2)-1) = 3/3 = 1$

$c_1/c_2 = 1/(2(2)+1) = 1/5$

Since $1 \neq 1/5$ ✓

$\therefore k = 2$

Answer: (i) a = 5, b = 1 (ii) k = 2

Q8 Solve the following pair of equations:

(i) $a/x - b/y = 0$; $ab^2/x + a^2b/y = a^2 + b^2$, where $x \neq 0, y \neq 0$

(ii) $2(ax - by) + (a + 4b) = 0$; $2(bx + ay) + (b - 4a) = 0$

SOLUTION

(i) Let $1/x = u$ and $1/y = v$.

$au - bv = 0$ ... (1)

$ab^2u + a^2bv = a^2 + b^2$ ... (2)

From (1): $au = bv \Rightarrow u = bv/a$

Substituting in (2):

$ab^2(bv/a) + a^2bv = a^2 + b^2$

$b^3v + a^2bv = a^2 + b^2$

$bv(b^2 + a^2) = a^2 + b^2$

$bv = 1 \Rightarrow v = 1/b$

From (1): $au = b(1/b) = 1 \Rightarrow u = 1/a$

Since $u = 1/x$ and $v = 1/y$:

$1/x = 1/a \Rightarrow x = a$

$1/y = 1/b \Rightarrow y = b$

$\therefore x = a, y = b$

Verification: $a/a - b/b = 1 - 1 = 0$ ✓

$ab^2/a + a^2b/b = b^2 + a^2 = a^2 + b^2$ ✓

(ii) $2(ax - by) + (a + 4b) = 0$

$\Rightarrow 2ax - 2by + a + 4b = 0$

$\Rightarrow 2ax - 2by = -(a + 4b)$ ... (1)

$2(bx + ay) + (b - 4a) = 0$

$\Rightarrow 2bx + 2ay + b - 4a = 0$

$\Rightarrow 2bx + 2ay = -(b - 4a) = 4a - b$ ... (2)

From (1): $2ax - 2by = -a - 4b$ ... (1)

From (2): $2bx + 2ay = 4a - b$ ... (2)

Multiply (1) by a: $2a^2x - 2aby = -a^2 - 4ab$ ... (3)
Multiply (2) by b: $2b^2x + 2aby = 4ab - b^2$ ... (4)

Adding (3) and (4):
$2a^2x + 2b^2x = -a^2 - 4ab + 4ab - b^2$
$2x(a^2 + b^2) = -(a^2 + b^2)$
$2x = -1 \Rightarrow x = -1/2$

Substituting $x = -1/2$ in (1):
$2a(-1/2) - 2by = -a - 4b$
$-a - 2by = -a - 4b$
$-2by = -4b$
$y = 2$ (assuming $b \neq 0$)
$\therefore x = -1/2, y = 2$

Verification:
$2(a(-1/2) - b(2)) + (a + 4b) = 2(-a/2 - 2b) + a + 4b = -a - 4b + a + 4b = 0$ ✓
$2(b(-1/2) + a(2)) + (b - 4a) = 2(-b/2 + 2a) + b - 4a = -b + 4a + b - 4a = 0$ ✓

Answer: (i) $x = a, y = b$ (ii) $x = -1/2, y = 2$