Chapter 4: Quadratic Equations

CONTENTS 3 exercises · 19 questions

4.1	Checking and Forming Quadratic Equations	2 Q
4.2	Solution of a Quadratic Equation by Factorisation	6 Q
4.3	Nature of Roots	11 Q

Exercise 4.1 — Checking and Forming Quadratic Equations

Q1 Check whether the following are quadratic equations:

(i) $(x + 1)^2 = 2(x - 3)$

(ii) $x^2 - 2x = (-2)(3 - x)$

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iv) $(x - 3)(2x + 1) = x(x + 5)$

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

(vi) $x^2 + 3x + 1 = (x - 2)^2$

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

SOLUTION

(i) $(x + 1)^2 = 2(x - 3)$

$x^2 + 2x + 1 = 2x - 6$

$x^2 + 2x + 1 - 2x + 6 = 0$

$x^2 + 7 = 0$

This is of the form $ax^2 + bx + c = 0$ with $a = 1$, $b = 0$, $c = 7$.

$\therefore$ Yes, it is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3 - x)$

$x^2 - 2x = -6 + 2x$

$x^2 - 2x + 6 - 2x = 0$

$x^2 - 4x + 6 = 0$

This is of the form $ax^2 + bx + c = 0$ with $a = 1$, $b = -4$, $c = 6$.

$\therefore$ Yes, it is a quadratic equation.

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

LHS: $x^2 + x - 2x - 2 = x^2 - x - 2$

RHS: $x^2 + 3x - x - 3 = x^2 + 2x - 3$

$x^2 - x - 2 = x^2 + 2x - 3$

$x^2 - x - 2 - x^2 - 2x + 3 = 0$

$-3x + 1 = 0$

This is a linear equation (no x^2 term), not quadratic.

$\therefore$ No, it is not a quadratic equation.

(iv) $(x - 3)(2x + 1) = x(x + 5)$

LHS: $2x^2 + x - 6x - 3 = 2x^2 - 5x - 3$

RHS: $x^2 + 5x$

$2x^2 - 5x - 3 = x^2 + 5x$

$2x^2 - 5x - 3 - x^2 - 5x = 0$

$x^2 - 10x - 3 = 0$

This is of the form $ax^2 + bx + c = 0$ with $a = 1$, $b = -10$, $c = -3$.

$\therefore$ Yes, it is a quadratic equation.

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

LHS: $2x^2 - 6x - x + 3 = 2x^2 - 7x + 3$

RHS: $x^2 - x + 5x - 5 = x^2 + 4x - 5$

$2x^2 - 7x + 3 = x^2 + 4x - 5$

$2x^2 - 7x + 3 - x^2 - 4x + 5 = 0$

$x^2 - 11x + 8 = 0$

This is of the form $ax^2 + bx + c = 0$ with $a = 1$, $b = -11$, $c = 8$.

$\therefore$ Yes, it is a quadratic equation.

(vi) $x^2 + 3x + 1 = (x - 2)^2$

$x^2 + 3x + 1 = x^2 - 4x + 4$

$x^2 + 3x + 1 - x^2 + 4x - 4 = 0$

$7x - 3 = 0$

This is a linear equation (no x^2 term), not quadratic.

$\therefore$ No, it is not a quadratic equation.

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

LHS: $x^3 + 3(x^2)(2) + 3(x)(4) + 8 = x^3 + 6x^2 + 12x + 8$

RHS: $2x^3 - 2x$

$x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$

$x^3 + 6x^2 + 12x + 8 - 2x^3 + 2x = 0$

$-x^3 + 6x^2 + 14x + 8 = 0$

This is a cubic equation (highest power is 3), not quadratic.

$\therefore$ No, it is not a quadratic equation.

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

RHS: $(x-2)^3 = x^3 - 3(x^2)(2) + 3(x)(4) - 8 = x^3 - 6x^2 + 12x - 8$

$x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$

$x^3 - 4x^2 - x + 1 - x^3 + 6x^2 - 12x + 8 = 0$

$2x^2 - 13x + 9 = 0$

This is of the form $ax^2 + bx + c = 0$ with $a = 2$, $b = -13$, $c = 9$.

$\therefore$ Yes, it is a quadratic equation.

Answer: (i) Yes (ii) Yes (iii) No (iv) Yes (v) Yes (vi) No (vii) No (viii) Yes

---

Q2 Represent the following situations in the form of quadratic equations:
(i) The area of a rectangular plot is 528 m². The length of the plot (in metres) is one more than twice its breadth. We need to find the dimensions of the plot.
(ii) The product of two consecutive positive integers is 306. We need to find the integers.
(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.
(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

SOLUTION

(i) Let breadth = x metres.

Then length = $(2x + 1)$ metres.

Area = length × breadth

$528 = (2x + 1) \times x$

$528 = 2x^2 + x$

$2x^2 + x - 528 = 0$

$\therefore$ The quadratic equation is $2x^2 + x - 528 = 0$.

(ii) Let the two consecutive positive integers be x and $(x + 1)$.

Product = $x(x + 1) = 306$

$x^2 + x = 306$

$x^2 + x - 306 = 0$

$\therefore$ The quadratic equation is $x^2 + x - 306 = 0$.

(iii) Let Rohan's present age = x years.

His mother's present age = $(x + 26)$ years.

After 3 years: Rohan's age = $(x + 3)$, Mother's age = $(x + 29)$.

Product of their ages after 3 years = 360

$(x + 3)(x + 29) = 360$

$x^2 + 29x + 3x + 87 = 360$

$x^2 + 32x + 87 - 360 = 0$

$x^2 + 32x - 273 = 0$

$\therefore$ The quadratic equation is $x^2 + 32x - 273 = 0$.

(iv) Let the speed of the train = x km/h.

Time taken at speed x = 480/x hours.

Time taken at speed $(x - 8) = 480/(x - 8)$ hours.

According to the condition:

$480/(x - 8) - 480/x = 3$

$480[1/(x - 8) - 1/x] = 3$

$480[x - (x - 8)] / [x(x - 8)] = 3$

$480 \times 8 / [x(x - 8)] = 3$

$3840 = 3x(x - 8)$

$3840 = 3x^2 - 24x$

$3x^2 - 24x - 3840 = 0$

Dividing by 3: $x^2 - 8x - 1280 = 0$

$\therefore$ The quadratic equation is $x^2 - 8x - 1280 = 0$.

Answer: (i) $2x^2 + x - 528 = 0$ (ii) $x^2 + x - 306 = 0$ (iii) $x^2 + 32x - 273 = 0$ (iv) $x^2 - 8x - 1280 = 0$

Exercise 4.2 — Solution of a Quadratic Equation by Factorisation

Q1 Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + 1/8 = 0$

(v) $100x^2 - 20x + 1 = 0$

SOLUTION

(i) $x^2 - 3x - 10 = 0$

We need two numbers whose product = −10 and sum = −3.

Those numbers are −5 and 2.

$x^2 - 5x + 2x - 10 = 0$

$x(x - 5) + 2(x - 5) = 0$

$(x - 5)(x + 2) = 0$

$x = 5$ or $x = -2$

(ii) $2x^2 + x - 6 = 0$

We need two numbers whose product = $2 \times (-6) = -12$ and sum = 1.

Those numbers are 4 and −3.

$2x^2 + 4x - 3x - 6 = 0$

$2x(x + 2) - 3(x + 2) = 0$

$(x + 2)(2x - 3) = 0$

$x = -2$ or $x = 3/2$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Product of $a \times c = \sqrt{2} \times 5\sqrt{2} = 5 \times 2 = 10$. Sum = 7.

Two numbers: 5 and 2.

$\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$\sqrt{2}x + 5 = 0 \Rightarrow x = -5/\sqrt{2} = -5\sqrt{2}/2$

$x + \sqrt{2} = 0 \Rightarrow x = -\sqrt{2}$

(iv) $2x^2 - x + 1/8 = 0$

Multiplying by 8: $16x^2 - 8x + 1 = 0$

We need two numbers whose product = $16 \times 1 = 16$ and sum = −8.

Those numbers are −4 and −4.

$16x^2 - 4x - 4x + 1 = 0$
$4x(4x - 1) - 1(4x - 1) = 0$
$(4x - 1)(4x - 1) = 0$
$(4x - 1)^2 = 0$
$x = 1/4$ (repeated root)

(v) $100x^2 - 20x + 1 = 0$
We need two numbers whose product = $100 \times 1 = 100$ and sum = -20.
Those numbers are -10 and -10.
$100x^2 - 10x - 10x + 1 = 0$
$10x(10x - 1) - 1(10x - 1) = 0$
$(10x - 1)(10x - 1) = 0$
$(10x - 1)^2 = 0$
$x = 1/10$ (repeated root)

Answer: (i) $x = 5, -2$ (ii) $x = -2, 3/2$ (iii) $x = -5\sqrt{2}/2, -\sqrt{2}$ (iv) $x = 1/4, 1/4$ (v) $x = 1/10, 1/10$

Q2 Solve the problems given:
(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find how many marbles they had to start with.
(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find the number of toys produced on that day.

SOLUTION

(i) Let John have x marbles.
Then Jivanti has $(45 - x)$ marbles.
After losing 5 each: John has $(x - 5)$, Jivanti has $(45 - x - 5) = (40 - x)$.
Product = $(x - 5)(40 - x) = 124$
$40x - x^2 - 200 + 5x = 124$
$-x^2 + 45x - 200 = 124$
$-x^2 + 45x - 324 = 0$
$x^2 - 45x + 324 = 0$

We need two numbers whose product = 324 and sum = 45.
Those numbers are 36 and 9.

$x^2 - 36x - 9x + 324 = 0$
$x(x - 36) - 9(x - 36) = 0$
$(x - 36)(x - 9) = 0$
$x = 36$ or $x = 9$

If $x = 36$: John = 36, Jivanti = $45 - 36 = 9$.
If $x = 9$: John = 9, Jivanti = $45 - 9 = 36$.
∴ One had 36 marbles and the other had 9 marbles.

(ii) Let the number of toys produced in a day = x.
Cost of each toy = Rs $(55 - x)$.
Total cost = $x(55 - x) = 750$
$55x - x^2 = 750$
$x^2 - 55x + 750 = 0$

We need two numbers whose product = 750 and sum = 55.
Those numbers are 25 and 30.
$x^2 - 25x - 30x + 750 = 0$
$x(x - 25) - 30(x - 25) = 0$
$(x - 25)(x - 30) = 0$
$x = 25$ or $x = 30$

If $x = 25$: cost per toy = $55 - 25$ = Rs 30.
If $x = 30$: cost per toy = $55 - 30$ = Rs 25.
∴ Number of toys = 25 or 30.

Answer: (i) John: 36, Jivanti: 9 (or vice versa) (ii) 25 or 30 toys

---

Q3 Find two numbers whose sum is 27 and product is 182.

SOLUTION

Let one number = x.
Then the other number = $27 - x$.
Product = $x(27 - x) = 182$
$27x - x^2 = 182$
$x^2 - 27x + 182 = 0$

We need two numbers whose product = 182 and sum = 27.
Those numbers are 13 and 14.
$x^2 - 13x - 14x + 182 = 0$
$x(x - 13) - 14(x - 13) = 0$

$(x - 13)(x - 14) = 0$

$x = 13$ or $x = 14$

If $x = 13$, other number $= 27 - 13 = 14$.

If $x = 14$, other number $= 27 - 14 = 13$.

Verification: $13 + 14 = 27$ ✓ and $13 \times 14 = 182$ ✓

∴ The two numbers are 13 and 14.

Answer: 13 and 14

Q4 Find two consecutive positive integers, sum of whose squares is 365.

SOLUTION

Let the two consecutive positive integers be x and $(x + 1)$.

$x^2 + (x + 1)^2 = 365$

$x^2 + x^2 + 2x + 1 = 365$

$2x^2 + 2x + 1 = 365$

$2x^2 + 2x - 364 = 0$

$x^2 + x - 182 = 0$

We need two numbers whose product $= -182$ and sum $= 1$.

Those numbers are 14 and −13.

$x^2 + 14x - 13x - 182 = 0$

$x(x + 14) - 13(x + 14) = 0$

$(x + 14)(x - 13) = 0$

$x = -14$ or $x = 13$

Since we need positive integers, $x = 13$.

The consecutive integers are 13 and 14.

Verification: $13^2 + 14^2 = 169 + 196 = 365$ ✓

∴ The two consecutive positive integers are 13 and 14.

Answer: 13 and 14

Q5 The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

SOLUTION

Let the base = x cm.
Then altitude = (x − 7) cm.
Hypotenuse = 13 cm.

By Pythagoras theorem:
base² + altitude² = hypotenuse²
$x^2 + (x - 7)^2 = 13^2$
$x^2 + x^2 - 14x + 49 = 169$
$2x^2 - 14x + 49 - 169 = 0$
$2x^2 - 14x - 120 = 0$
$x^2 - 7x - 60 = 0$

We need two numbers whose product = −60 and sum = −7.
Those numbers are −12 and 5.
$x^2 - 12x + 5x - 60 = 0$
$x(x - 12) + 5(x - 12) = 0$
$(x - 12)(x + 5) = 0$
$x = 12$ or $x = -5$

Since length cannot be negative, x = 12.
Base = 12 cm, Altitude = 12 − 7 = 5 cm.

Verification: $12^2 + 5^2 = 144 + 25 = 169 = 13^2$ ✓
∴ The other two sides are 12 cm and 5 cm.

Answer: Base = 12 cm, Altitude = 5 cm

Q6 A cottage industry produces a certain number of pottery articles in a day. It was observed that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

SOLUTION

Let the number of articles produced = x.
Cost of each article = Rs (2x + 3).
Total cost = x(2x + 3) = 90

$2x^2 + 3x = 90$

$2x^2 + 3x - 90 = 0$

We need two numbers whose product = $2 \times (-90) = -180$ and sum = 3.

Those numbers are 15 and −12.

$2x^2 + 15x - 12x - 90 = 0$

$x(2x + 15) - 6(2x + 15) = 0$

$(2x + 15)(x - 6) = 0$

$x = -15/2$ or $x = 6$

Since number of articles cannot be negative, $x = 6$.

Cost of each article = $2(6) + 3$ = Rs 15.

Verification: $6 \times 15 = 90$ ✓

∴ Number of articles = 6, Cost of each article = Rs 15.

Answer: 6 articles, Rs 15 each

Exercise 4.3 — Nature of Roots

Q1 Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:
(i) $2x^2 - 3x + 5 = 0$
(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$
(iii) $2x^2 - 6x + 3 = 0$

SOLUTION

(i) $2x^2 - 3x + 5 = 0$
Here $a = 2, b = -3, c = 5$.
Discriminant $D = b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31$
Since $D < 0$, the equation has no real roots.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$
Here $a = 3, b = -4\sqrt{3}, c = 4$.
$D = b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$
Since $D = 0$, the equation has two equal real roots.
$x = -b/(2a) = 4\sqrt{3}/(2 \times 3) = 4\sqrt{3}/6 = 2\sqrt{3}/3$
$\therefore$ The roots are $x = 2\sqrt{3}/3$ and $x = 2\sqrt{3}/3$.

(iii) $2x^2 - 6x + 3 = 0$
Here $a = 2, b = -6, c = 3$.
$D = b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12$
Since $D > 0$, the equation has two distinct real roots.
$x = [-b \pm \sqrt{D}]/(2a) = [6 \pm \sqrt{12}]/(2 \times 2) = [6 \pm 2\sqrt{3}]/4$
$x = (6 + 2\sqrt{3})/4 = (3 + \sqrt{3})/2$
or $x = (6 - 2\sqrt{3})/4 = (3 - \sqrt{3})/2$
$\therefore$ The roots are $x = (3 + \sqrt{3})/2$ and $x = (3 - \sqrt{3})/2$.

Answer: (i) No real roots ($D = -31$) (ii) Equal roots: $x = 2\sqrt{3}/3$ (iii) Distinct roots: $x = (3 \pm \sqrt{3})/2$

Q2 Find the values of k for each of the following quadratic equations, so that they have two equal roots:
(i) $2x^2 + kx + 3 = 0$
(ii) $kx(x - 2) + 6 = 0$

SOLUTION

(i) $2x^2 + kx + 3 = 0$

Here a = 2, b = k, c = 3.
For equal roots, D = 0.
$b^2 - 4ac = 0$
$k^2 - 4(2)(3) = 0$
$k^2 - 24 = 0$
$k^2 = 24$
$k = \pm\sqrt{24} = \pm 2\sqrt{6}$
$\therefore k = 2\sqrt{6}$ or $k = -2\sqrt{6}$.

(ii) $kx(x - 2) + 6 = 0$
$kx^2 - 2kx + 6 = 0$
Here a = k, b = −2k, c = 6.
For equal roots, D = 0.
$b^2 - 4ac = 0$
$(-2k)^2 - 4(k)(6) = 0$
$4k^2 - 24k = 0$
$4k(k - 6) = 0$
k = 0 or k = 6

But if k = 0, the equation becomes 6 = 0, which is not a quadratic equation.
$\therefore k = 6$.

Answer: (i) $k = \pm 2\sqrt{6}$ (ii) k = 6

Q3 Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m²? If so, find its length and breadth.

SOLUTION

Let breadth = x metres.
Then length = 2x metres.
Area = length × breadth = $2x \times x = 2x^2$

Given: $2x^2 = 800$
$x^2 = 400$
$x = \pm 20$

Since breadth cannot be negative, x = 20.
Breadth = 20 m, Length = 2 × 20 = 40 m.

This equation ($x^2 = 400$) has $D = 0^2 - 4(1)(-400) = 1600 > 0$.
So real roots exist, and it is possible to design such a grove.

∴ Length = 40 m, Breadth = 20 m.

Answer: Yes. Length = 40 m, Breadth = 20 m

Q4 Is it possible to design a rectangular park of perimeter 80 m and area 400 m²? If so, find its length and breadth.

SOLUTION

Let length = x metres and breadth = y metres.
Perimeter = 2(x + y) = 80 => x + y = 40 => y = 40 − x
Area = x × y = 400
x(40 − x) = 400
40x − x² = 400
x² − 40x + 400 = 0

D = b² − 4ac = (−40)² − 4(1)(400) = 1600 − 1600 = 0

Since D = 0, the equation has two equal real roots.
x = −b/(2a) = 40/(2 × 1) = 20

Length = 20 m, Breadth = 40 − 20 = 20 m.
∴ Yes, it is possible. The park is a square with side 20 m.

Answer: Yes. Length = 20 m, Breadth = 20 m (square)

Q5 Find the roots of the following quadratic equations, if they exist, using the quadratic formula:
(i) $2x^2 - 7x + 3 = 0$
(ii) $2x^2 + x - 4 = 0$
(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$
(iv) $2x^2 + x + 4 = 0$

SOLUTION

(i) $2x^2 - 7x + 3 = 0$
Here a = 2, b = −7, c = 3.
D = b² − 4ac = (−7)² − 4(2)(3) = 49 − 24 = 25
Since D > 0, two distinct real roots exist.
x = [−b ± √D]/(2a) = [7 ± √25]/(2 × 2) = [7 ± 5]/4
x = (7 + 5)/4 = 12/4 = 3
or x = (7 − 5)/4 = 2/4 = 1/2

∴ x = 3 or x = 1/2.

(ii) $2x^2 + x - 4 = 0$
Here a = 2, b = 1, c = −4.
$D = b^2 - 4ac = (1)^2 - 4(2)(-4) = 1 + 32 = 33$
Since D > 0, two distinct real roots exist.
$x = [-b \pm \sqrt{D}]/(2a) = [-1 \pm \sqrt{33}]/(2 \times 2) = [-1 \pm \sqrt{33}]/4$
∴ $x = (-1 + \sqrt{33})/4$ or $x = (-1 - \sqrt{33})/4$.

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$
Here $a = 4, b = 4\sqrt{3}, c = 3$.
$D = b^2 - 4ac = (4\sqrt{3})^2 - 4(4)(3) = 48 - 48 = 0$
Since D = 0, two equal real roots exist.
$x = -b/(2a) = -4\sqrt{3}/(2 \times 4) = -4\sqrt{3}/8 = -\sqrt{3}/2$
∴ $x = -\sqrt{3}/2$ (repeated root).

(iv) $2x^2 + x + 4 = 0$
Here a = 2, b = 1, c = 4.
$D = b^2 - 4ac = (1)^2 - 4(2)(4) = 1 - 32 = -31$
Since D < 0, no real roots exist.
∴ The equation has no real roots.

Answer: (i) x = 3, 1/2 (ii) $x = (-1 \pm \sqrt{33})/4$ (iii) $x = -\sqrt{3}/2$ (repeated) (iv) No real roots

---

Q6 The sum of the areas of two squares is 468 m². If the difference of their perimeters is 24 m, find the sides of the two squares.

SOLUTION

Let side of the first square = x m and side of the second square = y m.
Perimeter of first square = 4x, Perimeter of second square = 4y.

Difference of perimeters: 4x − 4y = 24 => x − y = 6 => x = y + 6
Sum of areas: $x^2 + y^2 = 468$

Substituting x = y + 6:
$(y + 6)^2 + y^2 = 468$
$y^2 + 12y + 36 + y^2 = 468$
$2y^2 + 12y + 36 = 468$
$2y^2 + 12y - 432 = 0$
$y^2 + 6y - 216 = 0$

We need two numbers whose product = −216 and sum = 6.
Those numbers are 18 and −12.
$y^2 + 18y - 12y - 216 = 0$
$y(y + 18) - 12(y + 18) = 0$
$(y + 18)(y - 12) = 0$
$y = -18$ or $y = 12$

Since side cannot be negative, $y = 12$.
$x = 12 + 6 = 18$.

Verification: $18^2 + 12^2 = 324 + 144 = 468$ ✓
Perimeter difference: $4(18) - 4(12) = 72 - 48 = 24$ ✓
∴ The sides of the two squares are 18 m and 12 m.

Answer: 18 m and 12 m

Q7 The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48. Is this situation possible? If so, determine their present ages.

SOLUTION

Let the present age of one friend = x years.
Then the present age of the other friend = (20 − x) years.

Four years ago:
Age of first friend = (x − 4) years.
Age of second friend = (20 − x − 4) = (16 − x) years.

Product = $(x - 4)(16 - x) = 48$
$16x - x^2 - 64 + 4x = 48$
$-x^2 + 20x - 64 = 48$
$-x^2 + 20x - 64 - 48 = 0$
$-x^2 + 20x - 112 = 0$
$x^2 - 20x + 112 = 0$

Checking the discriminant:
$D = b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48$

Since $D < 0$, the equation has no real roots.
∴ This situation is not possible.

Answer: Not possible (D < 0, no real roots)

Q8 The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is 2 16/21, find the fraction.

SOLUTION

Let the numerator = x.
Then denominator = 2x + 1.
The fraction = x/(2x + 1).

Sum of fraction and its reciprocal = 2 16/21 = 58/21
$x/(2x + 1) + (2x + 1)/x = 58/21$

$[x^2 + (2x + 1)^2] / [x(2x + 1)] = 58/21$
$[x^2 + 4x^2 + 4x + 1] / [x(2x + 1)] = 58/21$
$[5x^2 + 4x + 1] / [x(2x + 1)] = 58/21$

Cross-multiplying:
$21(5x^2 + 4x + 1) = 58 \times x(2x + 1)$
$105x^2 + 84x + 21 = 116x^2 + 58x$
$105x^2 + 84x + 21 - 116x^2 - 58x = 0$
$-11x^2 + 26x + 21 = 0$
$11x^2 - 26x - 21 = 0$

We need two numbers whose product = $11 \times (-21) = -231$ and sum = -26.
Those numbers are -33 and 7 (since $-33 + 7 = -26$ and $-33 \times 7 = -231$).
$11x^2 - 33x + 7x - 21 = 0$
$11x(x - 3) + 7(x - 3) = 0$
$(x - 3)(11x + 7) = 0$
$x = 3$ or $x = -7/11$

Since the numerator should be a positive integer, x = 3.
Denominator = 2(3) + 1 = 7.
The fraction = 3/7.

Verification: 3/7 + 7/3 = 9/21 + 49/21 = 58/21 = 2 16/21 ✓
∴ The fraction is 3/7.

Answer: 3/7

Q9 An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore. If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

SOLUTION

Let the speed of the passenger train = x km/h.
Then speed of the express train = (x + 11) km/h.

Time taken by passenger train = 132/x hours.
Time taken by express train = 132/(x + 11) hours.

The express train takes 1 hour less:
$132/x - 132/(x + 11) = 1$
$132[(x + 11) - x] / [x(x + 11)] = 1$
$132 \times 11 / [x(x + 11)] = 1$
$1452 = x(x + 11)$
$1452 = x^2 + 11x$
$x^2 + 11x - 1452 = 0$

We need two numbers whose product = −1452 and sum = 11.
Since $1452 = 4 \times 363 = 4 \times 3 \times 121 = 12 \times 121$
Try 44 and −33: $44 \times (-33) = -1452$ and $44 + (-33) = 11$ ✓
$x^2 + 44x - 33x - 1452 = 0$
$x(x + 44) - 33(x + 44) = 0$
$(x + 44)(x - 33) = 0$
$x = -44$ or $x = 33$

Since speed cannot be negative, x = 33.
Speed of passenger train = 33 km/h.
Speed of express train = 33 + 11 = 44 km/h.

Verification: 132/33 = 4 hours, 132/44 = 3 hours. Difference = 1 hour ✓
∴ Speed of passenger train = 33 km/h, Speed of express train = 44 km/h.

Answer: Passenger train: 33 km/h, Express train: 44 km/h

Q10 Sum of the areas of two squares is 468 m². If the difference of their perimeters is 24 m, find the sides of the two squares.

SOLUTION

Let side of the smaller square = x m.
Difference of perimeters = 24 m => 4(larger side) − 4x = 24
Larger side − x = 6 => Larger side = (x + 6) m.

Sum of areas: $x^2 + (x + 6)^2 = 468$
$x^2 + x^2 + 12x + 36 = 468$
$2x^2 + 12x + 36 = 468$
$2x^2 + 12x - 432 = 0$
$x^2 + 6x - 216 = 0$

We need two numbers whose product = −216 and sum = 6.
Those numbers are 18 and −12.
$x^2 + 18x - 12x - 216 = 0$
$x(x + 18) - 12(x + 18) = 0$
$(x + 18)(x - 12) = 0$
$x = -18$ or $x = 12$

Since side cannot be negative, x = 12.
Side of smaller square = 12 m.
Side of larger square = 12 + 6 = 18 m.

Verification: $12^2 + 18^2 = 144 + 324 = 468$ ✓
Perimeter difference: 4(18) − 4(12) = 72 − 48 = 24 ✓
∴ The sides of the two squares are 12 m and 18 m.

Answer: 12 m and 18 m

Q11 A rectangular field is 20 m longer than it is broad. If the area of the field is 8000 m², find the cost of fencing it at Rs 25.50 per metre.

SOLUTION

Let the breadth of the field = x m.
Then length = (x + 20) m.

Area = x(x + 20) = 8000
$x^2 + 20x = 8000$
$x^2 + 20x - 8000 = 0$

Using the quadratic formula:
$D = (20)^2 - 4(1)(-8000) = 400 + 32000 = 32400$
$\sqrt{D} = \sqrt{32400} = 180$

$x = (-20 \pm 180)/2$
$x = (-20 + 180)/2 = 160/2 = 80$
or $x = (-20 - 180)/2 = -200/2 = -100$

Since breadth cannot be negative, $x = 80$.
Breadth = 80 m, Length = 80 + 20 = 100 m.

Verification: $80 \times 100 = 8000$ ✓

Perimeter = 2(length + breadth) = $2(100 + 80) = 2 \times 180 = 360$ m.
Cost of fencing = 360×25.50 = Rs 9180.
∴ The cost of fencing is Rs 9180.

Answer: Rs 9180