Chapter 5: Arithmetic Progressions

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Exercise 5.1 — Introduction to Arithmetic Progressions

Q1 In which of the following situations, does the list of numbers involved make an Arithmetic Progression, and why?
(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.
(ii) The amount of air present in a cylinder when a vacuum pump removes 1/4 of the air remaining in the cylinder at a time.
(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.
(iv) The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

SOLUTION

(i) Taxi fare:
Fare for 1 km = Rs 15
Fare for 2 km = 15 + 8 = Rs 23
Fare for 3 km = 23 + 8 = Rs 31
Fare for 4 km = 31 + 8 = Rs 39
List: 15, 23, 31, 39, ...
Difference: 23 - 15 = 8, 31 - 23 = 8, 39 - 31 = 8
Common difference is constant (d = 8).
∴ Yes, this forms an AP with d = 8. Next three terms: 47, 55, 63.

(ii) Air in cylinder:
Let initial volume = V
After 1st pump: V - V/4 = 3V/4
After 2nd pump: 3V/4 - (3V/4)/4 = 3V/4 × 3/4 = 9V/16

After 3rd pump: 9V/16 × 3/4 = 27V/64
List: V, 3V/4, 9V/16, 27V/64, ...
Differences: 3V/4 - V = -V/4, 9V/16 - 3V/4 = -3V/16
-V/4 ≠ -3V/16
∴ No, this does not form an AP (it is a GP with ratio 3/4).

(iii) Cost of digging well:
Cost of 1st metre = Rs 150
Cost of 2nd metre = 150 + 50 = Rs 200
Cost of 3rd metre = 200 + 50 = Rs 250
Cost of 4th metre = 250 + 50 = Rs 300
List: 150, 200, 250, 300, ...
Difference: 200 - 150 = 50, 250 - 200 = 50, 300 - 250 = 50
Common difference is constant (d = 50).
∴ Yes, this forms an AP with d = 50. Next three terms: 350, 400, 450.

(iv) Compound interest account:
Amount after 1 year = 10000(1 + 8/100) = 10000 × 1.08 = Rs 10800
Amount after 2 years = 10800 × 1.08 = Rs 11664
Amount after 3 years = 11664 × 1.08 = Rs 12597.12
List: 10000, 10800, 11664, 12597.12, ...
Differences: 10800 - 10000 = 800, 11664 - 10800 = 864
800 ≠ 864
∴ No, this does not form an AP (compound interest gives a GP).

Answer: (i) Yes, AP with d = 8 (ii) No (iii) Yes, AP with d = 50 (iv) No

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Q2 Write first four terms of the AP when the first term a and common difference d are given as follows:
(i) a = 10, d = 10
(ii) a = -2, d = 0
(iii) a = 4, d = -3
(iv) a = -1, d = 1/2
(v) a = -1.25, d = -0.25

SOLUTION

Terms of an AP: a, a + d, a + 2d, a + 3d, ...

(i) a = 10, d = 10:

10, 10 + 10, 10 + 20, 10 + 30 = 10, 20, 30, 40

(ii) a = -2, d = 0:
-2, -2 + 0, -2 + 0, -2 + 0 = -2, -2, -2, -2

(iii) a = 4, d = -3:
4, 4 + (-3), 4 + (-6), 4 + (-9) = 4, 1, -2, -5

(iv) a = -1, d = 1/2:
-1, -1 + 1/2, -1 + 1, -1 + 3/2 = -1, -1/2, 0, 1/2

(v) a = -1.25, d = -0.25:
-1.25, -1.25 + (-0.25), -1.25 + (-0.50), -1.25 + (-0.75)
= -1.25, -1.50, -1.75, -2.00

Answer: (i) 10, 20, 30, 40 (ii) -2, -2, -2, -2 (iii) 4, 1, -2, -5 (iv) -1, -1/2, 0, 1/2 (v) -1.25, -1.50, -1.75, -2.00

Q3 For the following APs, write the first term a and the common difference d:
(i) 3, 1, -1, -3, ...
(ii) -5, -1, 3, 7, ...
(iii) 1/3, 5/3, 9/3, 13/3, ...
(iv) 0.6, 1.7, 2.8, 3.9, ...

SOLUTION

(i) 3, 1, -1, -3, ...
a = 3, d = 1 - 3 = -2

(ii) -5, -1, 3, 7, ...
a = -5, d = -1 - (-5) = -1 + 5 = 4

(iii) 1/3, 5/3, 9/3, 13/3, ...
a = 1/3, d = 5/3 - 1/3 = 4/3

(iv) 0.6, 1.7, 2.8, 3.9, ...
a = 0.6, d = 1.7 - 0.6 = 1.1

Answer: (i) a = 3, d = -2 (ii) a = -5, d = 4 (iii) a = 1/3, d = 4/3 (iv) a = 0.6, d = 1.1

Q4 Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(i) 2, 4, 8, 16, ...

(ii) 2, 5/2, 3, 7/2, ...

(iii) -1.2, -3.2, -5.2, -7.2, ...

(iv) -10, -6, -2, 2, ...

(v) 3, 3 + √2, 3 + 2√2, 3 + 3√2, ...

(vi) 0.2, 0.22, 0.222, 0.2222, ...

(vii) 0, -4, -8, -12, ...

(viii) -1/2, -1/2, -1/2, -1/2, ...

(ix) 1, 3, 9, 27, ...

(x) a, 2a, 3a, 4a, ...

(xi) a, a^2, a^3, a^4, ...

(xii) √2, √8, √18, √32, ...

(xiii) √3, √6, √9, √12, ...

(xiv) 1^2, 5^2, 7^2, 73, ...

SOLUTION

(i) 2, 4, 8, 16, ...

4 - 2 = 2, 8 - 4 = 4, 16 - 8 = 8

Differences are not equal. Not an AP.

(ii) 2, 5/2, 3, 7/2, ...

5/2 - 2 = 1/2, 3 - 5/2 = 1/2, 7/2 - 3 = 1/2

AP with d = 1/2. Next three: 4, 9/2, 5

(iii) -1.2, -3.2, -5.2, -7.2, ...

-3.2 - (-1.2) = -2, -5.2 - (-3.2) = -2, -7.2 - (-5.2) = -2

AP with d = -2. Next three: -9.2, -11.2, -13.2

(iv) -10, -6, -2, 2, ...

-6 - (-10) = 4, -2 - (-6) = 4, 2 - (-2) = 4

AP with d = 4. Next three: 6, 10, 14

(v) 3, 3 + √2, 3 + 2√2, 3 + 3√2, ...

(3 + √2) - 3 = √2, (3 + 2√2) - (3 + √2) = √2, (3 + 3√2) - (3 + 2√2) = √2

AP with d = √2. Next three: 3 + 4√2, 3 + 5√2, 3 + 6√2

(vi) 0.2, 0.22, 0.222, 0.2222, ...

0.22 - 0.2 = 0.02, 0.222 - 0.22 = 0.002

0.02 ≠ 0.002. Not an AP.

(vii) 0, -4, -8, -12, ...
-4 - 0 = -4, -8 - (-4) = -4, -12 - (-8) = -4
AP with d = -4. Next three: -16, -20, -24

(viii) -1/2, -1/2, -1/2, -1/2, ...
-1/2 - (-1/2) = 0, -1/2 - (-1/2) = 0
AP with d = 0. Next three: -1/2, -1/2, -1/2

(ix) 1, 3, 9, 27, ...
3 - 1 = 2, 9 - 3 = 6, 27 - 9 = 18
Differences are not equal. Not an AP.

(x) a, 2a, 3a, 4a, ...
2a - a = a, 3a - 2a = a, 4a - 3a = a
AP with d = a. Next three: 5a, 6a, 7a

(xi) a, a^2, a^3, a^4, ...
$a^2 - a = a(a - 1)$, $a^3 - a^2 = a^2(a - 1)$
$a(a - 1) \neq a^2(a - 1)$ in general (unless a = 0 or a = 1).
Not an AP.

(xii) $\sqrt{2}$, $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$, ...
$\sqrt{8} = 2\sqrt{2}$, $\sqrt{18} = 3\sqrt{2}$, $\sqrt{32} = 4\sqrt{2}$
So the sequence is: $\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$, ...
$2\sqrt{2} - \sqrt{2} = \sqrt{2}$, $3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$, $4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$
AP with d = $\sqrt{2}$. Next three: $5\sqrt{2}$, $6\sqrt{2}$, $7\sqrt{2}$

(xiii) $\sqrt{3}$, $\sqrt{6}$, $\sqrt{9}$, $\sqrt{12}$, ...
$\sqrt{6} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1) \approx 0.717$
$\sqrt{9} - \sqrt{6} = 3 - \sqrt{6} \approx 3 - 2.449 = 0.551$
$0.717 \neq 0.551$. Not an AP.

(xiv) 1^2, 5^2, 7^2, 73, ...
= 1, 25, 49, 73
25 - 1 = 24, 49 - 25 = 24, 73 - 49 = 24
AP with d = 24. Next three: 97, 121, 145

Answer: (ii) d = 1/2 (iii) d = -2 (iv) d = 4 (v) d = $\sqrt{2}$ (vii) d = -4 (viii) d = 0 (x) d = a (xii) d = $\sqrt{2}$ (xiv) d = 24. Not APs: (i), (vi), (ix), (xi), (xiii)

Exercise 5.2 — nth Term of an AP

Q1 Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the nth term of the AP:

(i) a = 7, d = 3, n = 8, a_n = ?

(ii) a = -18, d = ?, n = 10, a_n = 0

(iii) a = ?, d = -3, n = 18, a_n = -5

(iv) a = -18.9, d = 2.5, n = ?, a_n = 3.6

(v) a = 3.5, d = 0, n = 105, a_n = ?

SOLUTION

Using $a_n = a + (n - 1)d$

(i) a = 7, d = 3, n = 8:

$a_8 = 7 + (8 - 1) \times 3 = 7 + 21 = 28$

(ii) a = -18, n = 10, a_n = 0:

$0 = -18 + (10 - 1)d$

$0 = -18 + 9d$

$9d = 18$

$d = 2$

(iii) d = -3, n = 18, a_n = -5:

$-5 = a + (18 - 1)(-3)$

$-5 = a - 51$

$a = -5 + 51 = 46$

(iv) a = -18.9, d = 2.5, a_n = 3.6:

$3.6 = -18.9 + (n - 1)(2.5)$

$3.6 + 18.9 = (n - 1)(2.5)$

$22.5 = (n - 1)(2.5)$

$n - 1 = 9$

$n = 10$

(v) a = 3.5, d = 0, n = 105:

$a_{105} = 3.5 + (105 - 1)(0) = 3.5 + 0 = 3.5$

Answer: (i) a_n = 28 (ii) d = 2 (iii) a = 46 (iv) n = 10 (v) a_n = 3.5

Q2 Choose the correct choice in the following and justify:
(i) 30th term of the AP: 10, 7, 4, ... is
(A) 97 (B) 77 (C) -77 (D) -87

SOLUTION

AP: 10, 7, 4, ...
a = 10, d = 7 - 10 = -3
$a_n = a + (n - 1)d$
$a_{30} = 10 + (30 - 1)(-3)$
$= 10 + 29 \times (-3)$
$= 10 - 87$
$= -77$
∴ Answer is (C) -77

Answer: (C) -77

Q3 In the following APs, find the missing terms in the boxes:
(i) 2, __, 26
(ii) __, 13, __, 3
(iii) 5, __, __, 9½
(iv) -4, __, __, __, __, 6
(v) __, 38, __, __, __, -22

SOLUTION

(i) 2, __, 26
a = 2, $a_3 = 26$
$a_3 = a + 2d \Rightarrow 26 = 2 + 2d \Rightarrow 2d = 24 \Rightarrow d = 12$
$a_2 = 2 + 12 = 14$

(ii) __, 13, __, 3
$a_2 = 13$, $a_4 = 3$
$a_4 = a_2 + 2d \Rightarrow 3 = 13 + 2d \Rightarrow 2d = -10 \Rightarrow d = -5$
$a_1 = a_2 - d = 13 - (-5) = 18$
$a_3 = a_2 + d = 13 + (-5) = 8$

(iii) 5, __, __, 9½
a = 5, $a_4 = 9.5$
$a_4 = a + 3d \Rightarrow 9.5 = 5 + 3d \Rightarrow 3d = 4.5 \Rightarrow d = 1.5$
$a_2 = 5 + 1.5 = 6.5$

$a_3 = 5 + 3 = 8$

(iv) -4, __, __, __, __, 6

$a = -4$, $a_6 = 6$

$a_6 = a + 5d \Rightarrow 6 = -4 + 5d \Rightarrow 5d = 10 \Rightarrow d = 2$

$a_2 = -4 + 2 = -2$

$a_3 = -4 + 4 = 0$

$a_4 = -4 + 6 = 2$

$a_5 = -4 + 8 = 4$

(v) __, 38, __, __, __, -22

$a_2 = 38$, $a_6 = -22$

$a_6 = a_2 + 4d \Rightarrow -22 = 38 + 4d \Rightarrow 4d = -60 \Rightarrow d = -15$

$a_1 = a_2 - d = 38 - (-15) = 53$

$a_3 = 38 + (-15) = 23$

$a_4 = 23 + (-15) = 8$

$a_5 = 8 + (-15) = -7$

Answer: (i) 14 (ii) 18, 8 (iii) 6.5, 8 (iv) -2, 0, 2, 4 (v) 53, 23, 8, -7

Q4 Which term of the AP: 3, 8, 13, 18, ... is 78?

SOLUTION

$a = 3$, $d = 8 - 3 = 5$

Let $a_n = 78$

$a_n = a + (n - 1)d$

$78 = 3 + (n - 1)(5)$

$78 - 3 = 5(n - 1)$

$75 = 5(n - 1)$

$n - 1 = 15$

$n = 16$

$\therefore$ 78 is the 16th term.

Answer: 16th term

Q5 Find the number of terms in each of the following APs:
(i) 7, 13, 19, ..., 205
(ii) 18, 15½, 13, ..., -47

SOLUTION

(i) a = 7, d = 13 - 7 = 6, a_n = 205
205 = 7 + (n - 1)(6)
198 = 6(n - 1)
n - 1 = 33
n = 34

(ii) a = 18, d = 15.5 - 18 = -2.5, a_n = -47
-47 = 18 + (n - 1)(-2.5)
-47 - 18 = -2.5(n - 1)
-65 = -2.5(n - 1)
n - 1 = 26
n = 27

Answer: (i) 34 terms (ii) 27 terms

Q6 Check whether -150 is a term of the AP: 11, 8, 5, 2, ...

SOLUTION

a = 11, d = 8 - 11 = -3
Let a_n = -150
-150 = 11 + (n - 1)(-3)
-150 - 11 = -3(n - 1)
-161 = -3(n - 1)
n - 1 = 161/3 = 53.67 (not a whole number)

Since n must be a positive integer, -150 is not a term of this AP.

Answer: No, -150 is not a term of this AP

Q7 Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

SOLUTION

$a_{11} = a + 10d = 38$ ... (1)
$a_{16} = a + 15d = 73$ ... (2)

Subtracting (1) from (2):

5d = 35
d = 7

From (1): a + 10(7) = 38
a + 70 = 38
a = -32

a_{31} = a + 30d = -32 + 30(7) = -32 + 210 = 178

Answer: 178

Q8 An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

SOLUTION

n = 50, a_3 = 12, a_{50} = 106
a_3 = a + 2d = 12 ... (1)
a_{50} = a + 49d = 106 ... (2)

Subtracting (1) from (2):
47d = 94
d = 2

From (1): a + 2(2) = 12 => a = 8

a_{29} = a + 28d = 8 + 28(2) = 8 + 56 = 64

Answer: 64

Q9 If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?

SOLUTION

a_3 = a + 2d = 4 ... (1)
a_9 = a + 8d = -8 ... (2)

Subtracting (1) from (2):
6d = -12
d = -2

From (1): a + 2(-2) = 4 => a = 8

Let a_n = 0

0 = 8 + (n - 1)(-2)
0 = 8 - 2n + 2
0 = 10 - 2n
2n = 10
n = 5
∴ The 5th term is zero.

Answer: 5th term

Q10 The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

SOLUTION

$a_{17} - a_{10} = 7$
(a + 16d) - (a + 9d) = 7
7d = 7
d = 1

Answer: d = 1

Q11 Which term of the AP: 3, 15, 27, 39, ... will be 132 more than its 54th term?

SOLUTION

a = 3, d = 15 - 3 = 12
a54 = 3 + (54 - 1)(12) = 3 + 53 × 12 = 3 + 636 = 639

We need a_n = a54 + 132 = 639 + 132 = 771
771 = 3 + (n - 1)(12)
768 = 12(n - 1)
n - 1 = 64
n = 65
∴ The 65th term is 132 more than the 54th term.

Answer: 65th term

Q12 Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

SOLUTION

Let the two APs have first terms a_1 and a_2, common difference d.
100th term of 1st AP = $a_1 + 99d$
100th term of 2nd AP = $a_2 + 99d$

Difference = $(a_1 + 99d) - (a_2 + 99d) = a_1 - a_2 = 100$

1000th term of 1st AP = $a_1 + 999d$
1000th term of 2nd AP = $a_2 + 999d$
Difference = $(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2 = 100$

∴ The difference between their 1000th terms is also 100.

Answer: 100

Q13 How many three-digit numbers are divisible by 7?

SOLUTION

First three-digit number divisible by 7:
100 ÷ 7 = 14 remainder 2, so first number = 7 × 15 = 105

Last three-digit number divisible by 7:
999 ÷ 7 = 142 remainder 5, so last number = 7 × 142 = 994

AP: 105, 112, 119, ..., 994
a = 105, d = 7, a_n = 994
994 = 105 + (n - 1)(7)
889 = 7(n - 1)
n - 1 = 127
n = 128

Answer: 128

Q14 How many multiples of 4 lie between 10 and 250?

SOLUTION

First multiple of 4 after 10: 12
Last multiple of 4 before 250: 248

AP: 12, 16, 20, ..., 248

a = 12, d = 4, a_n = 248
248 = 12 + (n - 1)(4)
236 = 4(n - 1)
n - 1 = 59
n = 60

Answer: 60

Q15 For what value of n, are the nth terms of two APs: 63, 65, 67, ... and 3, 10, 17, ... equal?

SOLUTION

First AP: $a_1 = 63$, $d_1 = 2$
nth term = 63 + (n - 1)(2) = 61 + 2n

Second AP: $a_2 = 3$, $d_2 = 7$
nth term = 3 + (n - 1)(7) = -4 + 7n

Setting them equal:
61 + 2n = -4 + 7n
61 + 4 = 7n - 2n
65 = 5n
n = 13

Answer: n = 13

Q16 Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

SOLUTION

$a_3 = a + 2d = 16$ ... (1)

$a_7 - a_5 = 12$
(a + 6d) - (a + 4d) = 12
2d = 12
d = 6

From (1): a + 2(6) = 16 => a = 16 - 12 = 4

AP: 4, 10, 16, 22, 28, ...

Answer: AP: 4, 10, 16, 22, ...

Q17 Find the 20th term from the last term of the AP: 3, 8, 13, ..., 253.

SOLUTION

AP: 3, 8, 13, ..., 253
a = 3, d = 5

To find the 20th term from the last, we reverse the AP:
Last term l = 253, common difference = -d = -5

20th term from last = l + (20 - 1)(-d)
= 253 + 19 × (-5)
= 253 - 95
= 158

Answer: 158

Q18 The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

SOLUTION

$a_4 + a_8 = 24$
(a + 3d) + (a + 7d) = 24
2a + 10d = 24 ... (1)

$a_6 + a_{10} = 44$
(a + 5d) + (a + 9d) = 44
2a + 14d = 44 ... (2)

Subtracting (1) from (2):
4d = 20
d = 5

From (1): 2a + 10(5) = 24
2a + 50 = 24
2a = -26
a = -13

First three terms: -13, -13 + 5, -13 + 10 = -13, -8, -3

Answer: -13, -8, -3

Q19 Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

SOLUTION

Salary in 1995 = Rs 5000 (first term, a = 5000)
Annual increment d = Rs 200

Salary in nth year: $a_n = 5000 + (n - 1)(200)$

When salary = Rs 7000:
$7000 = 5000 + (n - 1)(200)$
$2000 = 200(n - 1)$
$n - 1 = 10$
$n = 11$

∴ His salary reached Rs 7000 in the 11th year, i.e., 1995 + 10 = 2005.

Answer: In the year 2005 (11th year of service)

Q20 Ramkali saved Rs 5 in the first week of a year and then increased her weekly savings by Rs 1.75. If in the nth week, her weekly savings become Rs 20.75, find n.

SOLUTION

$a = 5, d = 1.75, a_n = 20.75$
$a_n = a + (n - 1)d$
$20.75 = 5 + (n - 1)(1.75)$
$15.75 = 1.75(n - 1)$
$n - 1 = 15.75 / 1.75 = 9$
$n = 10$

Answer: n = 10

Exercise 5.3 — Sum of First n Terms of an AP

Q1 Find the sum of the following APs:
(i) 2, 7, 12, ..., to 10 terms
(ii) -37, -33, -29, ..., to 12 terms
(iii) 0.6, 1.7, 2.8, ..., to 100 terms
(iv) 1/15, 1/12, 1/10, ..., to 11 terms

SOLUTION

Using $S_n = n/2 [2a + (n - 1)d]$

(i) a = 2, d = 5, n = 10
$S_{10} = 10/2 [2(2) + 9(5)] = 5[4 + 45] = 5 \times 49 = 245$

(ii) a = -37, d = -33 - (-37) = 4, n = 12
$S_{12} = 12/2 [2(-37) + 11(4)] = 6[-74 + 44] = 6 \times (-30) = -180$

(iii) a = 0.6, d = 1.1, n = 100
$S_{100} = 100/2 [2(0.6) + 99(1.1)] = 50[1.2 + 108.9] = 50 \times 110.1 = 5505$

(iv) a = 1/15, d = 1/12 - 1/15 = (5 - 4)/60 = 1/60, n = 11
$S_{11} = 11/2 [2(1/15) + 10(1/60)]$
$= 11/2 [2/15 + 10/60]$
$= 11/2 [2/15 + 1/6]$
$= 11/2 [(4 + 5)/30]$
$= 11/2 \times 9/30$
$= 11/2 \times 3/10$
$= 33/20$

Answer: (i) 245 (ii) -180 (iii) 5505 (iv) 33/20

Q2 Find the sums given below:
(i) 7 + 10½ + 14 + ... + 84
(ii) 34 + 32 + 30 + ... + 10
(iii) -5 + (-8) + (-11) + ... + (-230)

SOLUTION

(i) a = 7, d = 10.5 - 7 = 3.5, l = 84
First find n: 84 = 7 + (n - 1)(3.5)
77 = 3.5(n - 1)
n - 1 = 22, n = 23

$S_{23} = 23/2\ (7 + 84) = 23/2 \times 91 = 2093/2 = 1046.5$

(ii) a = 34, d = -2, l = 10
10 = 34 + (n - 1)(-2)
-24 = -2(n - 1)
n - 1 = 12, n = 13
$S_{13} = 13/2\ (34 + 10) = 13/2 \times 44 = 13 \times 22 = 286$

(iii) a = -5, d = -3, l = -230
-230 = -5 + (n - 1)(-3)
-225 = -3(n - 1)
n - 1 = 75, n = 76
$S_{76} = 76/2\ (-5 + (-230)) = 38 \times (-235) = -8930$

Answer: (i) 1046.5 (ii) 286 (iii) -8930

Q3 In an AP:
(i) Given a = 5, d = 3, a_n = 50, find n and S_n.
(ii) Given a = 7, $a_{13} = 35$, find d and S_{13}.
(iii) Given $a_{12} = 37$, d = 3, find a and S_{12}.
(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.
(v) Given d = 5, $S_9 = 75$, find a and a_9.

SOLUTION

(i) a = 5, d = 3, a_n = 50
50 = 5 + (n - 1)(3) => 45 = 3(n - 1) => n - 1 = 15 => n = 16
$S_{16} = 16/2\ (5 + 50) = 8 \times 55 = 440$

(ii) a = 7, $a_{13} = 35$
35 = 7 + 12d => 12d = 28 => d = 7/3
$S_{13} = 13/2\ (7 + 35) = 13/2 \times 42 = 13 \times 21 = 273$

(iii) $a_{12} = 37$, d = 3
37 = a + 11(3) => a = 37 - 33 = 4
$S_{12} = 12/2\ (4 + 37) = 6 \times 41 = 246$

(iv) $a_3 = 15$, $S_{10} = 125$
$a_3 = a + 2d = 15$ ... (1)
$S_{10} = 10/2\ [2a + 9d] = 5(2a + 9d) = 125$
=> 2a + 9d = 25 ... (2)
From (1): a = 15 - 2d. Substituting in (2):

2(15 - 2d) + 9d = 25
30 - 4d + 9d = 25
5d = -5 => d = -1
a = 15 - 2(-1) = 17
a_{10} = 17 + 9(-1) = 8

(v) d = 5, S_9 = 75
S_9 = 9/2 [2a + 8(5)] = 9/2 [2a + 40] = 75
9(2a + 40) = 150
2a + 40 = 150/9 = 50/3
2a = 50/3 - 40 = 50/3 - 120/3 = -70/3
a = -35/3
a_9 = a + 8d = -35/3 + 40 = -35/3 + 120/3 = 85/3

Answer: (i) n = 16, S_n = 440 (ii) d = 7/3, S_{13} = 273 (iii) a = 4, S_{12} = 246 (iv) d = -1, a_{10} = 8 (v) a = -35/3, a_9 = 85/3

Q4 How many terms of the AP: 9, 17, 25, ... must be taken to give a sum of 636?

SOLUTION

a = 9, d = 8, S_n = 636
S_n = n/2 [2a + (n - 1)d]
636 = n/2 [2(9) + (n - 1)(8)]
636 = n/2 [18 + 8n - 8]
636 = n/2 [10 + 8n]
1272 = n(10 + 8n)
1272 = 10n + $8n^2$
$8n^2$ + 10n - 1272 = 0
$4n^2$ + 5n - 636 = 0

Using the quadratic formula:
n = (-5 ± √(25 + 4 × 4 × 636)) / (2 × 4)
= (-5 ± √(25 + 10176)) / 8
= (-5 ± √10201) / 8
= (-5 ± 101) / 8

n = (-5 + 101)/8 = 96/8 = 12
or n = (-5 - 101)/8 = -106/8 (rejected, n must be positive)

∴ 12 terms must be taken.

Answer: 12 terms

Q5 The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

SOLUTION

$a = 5, l = 45, S_n = 400$

$S_n = n/2 (a + l)$

$400 = n/2 (5 + 45)$

$400 = n/2 \times 50$

$400 = 25n$

$n = 16$

$l = a + (n - 1)d$

$45 = 5 + (16 - 1)d$

$40 = 15d$

$d = 8/3$

Answer: n = 16, d = 8/3

Q6 The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

SOLUTION

$a = 17, l = 350, d = 9$

$l = a + (n - 1)d$

$350 = 17 + (n - 1)(9)$

$333 = 9(n - 1)$

$n - 1 = 37$

$n = 38$

$S_n = n/2 (a + l) = 38/2 (17 + 350) = 19 \times 367 = 6973$

Answer: n = 38, S = 6973

Q7 Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

SOLUTION

$d = 7, a_{22} = 149$

$a_{22} = a + 21d \Rightarrow 149 = a + 21(7) \Rightarrow 149 = a + 147 \Rightarrow a = 2$

$S_{22} = 22/2\ (a + a_{22}) = 11(2 + 149) = 11 \times 151 = 1661$

Answer: 1661

Q8 Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

SOLUTION

$a_2 = 14, a_3 = 18$

$d = a_3 - a_2 = 18 - 14 = 4$

$a = a_2 - d = 14 - 4 = 10$

$S_{51} = 51/2\ [2(10) + 50(4)]$

$= 51/2\ [20 + 200]$

$= 51/2 \times 220$

$= 51 \times 110$

$= 5610$

Answer: 5610

Q9 If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

SOLUTION

$S_7 = 49$: $7/2\ [2a + 6d] = 49 \Rightarrow 2a + 6d = 14$ ... (1)

$S_{17} = 289$: $17/2\ [2a + 16d] = 289 \Rightarrow 2a + 16d = 34$ ... (2)

Subtracting (1) from (2):

$10d = 20 \Rightarrow d = 2$

From (1): $2a + 12 = 14 \Rightarrow 2a = 2 \Rightarrow a = 1$

$S_n = n/2\ [2(1) + (n - 1)(2)]$

$= n/2\ [2 + 2n - 2]$

$= n/2 \times 2n$

$= n^2$

Answer: $S_n = n^2$

Q10 Show that $a_1, a_2, ..., a_n, ...$ form an AP where a_n is defined as below:
(i) $a_n = 3 + 4n$
(ii) $a_n = 9 - 5n$
Also find the sum of the first 15 terms in each case.

SOLUTION

(i) $a_n = 3 + 4n$
$a_1 = 3 + 4 = 7$, $a_2 = 3 + 8 = 11$, $a_3 = 3 + 12 = 15$
$a_{n+1} - a_n = [3 + 4(n + 1)] - [3 + 4n] = 4$ (constant)
$\therefore$ It forms an AP with $a = 7$, $d = 4$.

$S_{15} = 15/2\ [2(7) + 14(4)] = 15/2\ [14 + 56] = 15/2 \times 70 = 525$

(ii) $a_n = 9 - 5n$
$a_1 = 9 - 5 = 4$, $a_2 = 9 - 10 = -1$, $a_3 = 9 - 15 = -6$
$a_{n+1} - a_n = [9 - 5(n + 1)] - [9 - 5n] = -5$ (constant)
$\therefore$ It forms an AP with $a = 4$, $d = -5$.

$S_{15} = 15/2\ [2(4) + 14(-5)] = 15/2\ [8 - 70] = 15/2 \times (-62) = -465$

Answer: (i) $S_{15} = 525$ (ii) $S_{15} = -465$

Q11 If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

SOLUTION

$S_n = 4n - n^2$

First term: $S_1 = 4(1) - 1^2 = 4 - 1 = 3$
$a_1 = 3$

$S_2 = 4(2) - 2^2 = 8 - 4 = 4$
$a_2 = S_2 - S_1 = 4 - 3 = 1$

$S_3 = 4(3) - 3^2 = 12 - 9 = 3$
$a_3 = S_3 - S_2 = 3 - 4 = -1$

For $n \geq 2$: $a_n = S_n - S_{n-1}$
$= (4n - n^2) - [4(n - 1) - (n - 1)^2]$

$= 4n - n^2 - 4n + 4 + n^2 - 2n + 1$
$= 5 - 2n$

Check: $a_1 = 5 - 2(1) = 3$ ✓

$a_{10} = 5 - 2(10) = 5 - 20 = -15$
$a_n = 5 - 2n$

Answer: $a_1 = 3$, $S_2 = 4$, $a_2 = 1$, $a_3 = -1$, $a_{10} = -15$, $a_n = 5 - 2n$

Q12 Find the sum of the first 40 positive integers divisible by 6.

SOLUTION

Positive integers divisible by 6: 6, 12, 18, 24, ...
$a = 6$, $d = 6$, $n = 40$

$S_{40} = 40/2 [2(6) + 39(6)]$
$= 20 [12 + 234]$
$= 20 \times 246$
$= 4920$

Answer: 4920

Q13 Find the sum of the first 15 multiples of 8.

SOLUTION

Multiples of 8: 8, 16, 24, 32, ...
$a = 8$, $d = 8$, $n = 15$

$S_{15} = 15/2 [2(8) + 14(8)]$
$= 15/2 [16 + 112]$
$= 15/2 \times 128$
$= 15 \times 64$
$= 960$

Answer: 960

Q14 Find the sum of the odd numbers between 0 and 50.

SOLUTION

Odd numbers between 0 and 50: 1, 3, 5, 7, ..., 49
a = 1, d = 2, l = 49

Number of terms: 49 = 1 + (n - 1)(2) => 48 = 2(n - 1) => n = 25

S_{25} = 25/2 (1 + 49) = 25/2 × 50 = 625

Answer: 625

Q15 A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

SOLUTION

Penalty: 200, 250, 300, ... (AP with a = 200, d = 50)
n = 30 days

S_{30} = 30/2 [2(200) + 29(50)]
= 15 [400 + 1450]
= 15 × 1850
= 27750

∴ Total penalty = Rs 27,750

Answer: Rs 27,750

Q16 A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

SOLUTION

Let the first (highest) prize = a
d = -20 (each prize is Rs 20 less)
n = 7, S_7 = 700

S_7 = 7/2 [2a + 6(-20)] = 700
7/2 [2a - 120] = 700

7(2a - 120) = 1400
2a - 120 = 200
2a = 320
a = 160

The prizes are: 160, 140, 120, 100, 80, 60, 40
Verification: 160 + 140 + 120 + 100 + 80 + 60 + 40 = 700 ✓

Answer: Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, Rs 40

Q17 In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

SOLUTION

Each section plants trees = class number.
3 sections per class, so trees per class = 3 × class number.

Total trees = 3(1 + 2 + 3 + ... + 12)

Sum 1 + 2 + 3 + ... + 12:
This is an AP with a = 1, d = 1, n = 12
S_{12} = 12/2 (1 + 12) = 6 × 13 = 78

Total trees = 3 × 78 = 234

Answer: 234 trees

Q18 A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take π = 22/7)

SOLUTION

Radii: 0.5, 1.0, 1.5, 2.0, ..., (13 terms)
Length of each semicircle = πr

Total length = π(0.5 + 1.0 + 1.5 + 2.0 + ... + 6.5)

The radii form an AP: a = 0.5, d = 0.5, n = 13
13th term = 0.5 + 12(0.5) = 0.5 + 6 = 6.5 ✓

Sum of radii = 13/2 (0.5 + 6.5) = 13/2 × 7 = 91/2 = 45.5

Total length = π × 45.5 = 22/7 × 45.5
= 22 × 45.5 / 7
= 1001/7
= 143 cm

Answer: 143 cm

Q19 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row?

SOLUTION

AP: 20, 19, 18, ... (a = 20, d = -1)
S_n = 200

n/2 [2(20) + (n - 1)(-1)] = 200
n/2 [40 - n + 1] = 200
n/2 [41 - n] = 200
n(41 - n) = 400
41n - n^2 = 400
n^2 - 41n + 400 = 0

Using the quadratic formula:
n = (41 ± √(1681 - 1600)) / 2
= (41 ± √81) / 2
= (41 ± 9) / 2

n = (41 + 9)/2 = 25 or n = (41 - 9)/2 = 16

If n = 25: a_{25} = 20 + 24(-1) = -4 (negative, not possible)
∴ n = 16

Logs in top row: a_{16} = 20 + 15(-1) = 5

Verify: S_{16} = 16/2 (20 + 5) = 8 × 25 = 200 ✓

Answer: 16 rows, 5 logs in the top row

Q20 In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see figure). A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

SOLUTION

Distance to 1st potato = 5 m, to 2nd = 5 + 3 = 8 m,
to 3rd = 8 + 3 = 11 m, ..., to nth = 5 + (n - 1)(3) m

For each potato, the competitor runs to it and back.
Distance for 1st potato = 2 × 5 = 10 m
Distance for 2nd potato = 2 × 8 = 16 m
Distance for 3rd potato = 2 × 11 = 22 m
...
Distance for nth potato = 2[5 + (n - 1)(3)] = 10 + 6(n - 1)

This is an AP: a = 10, d = 6, n = 10

Total distance = S_{10} = 10/2 [2(10) + 9(6)]
= 5 [20 + 54]
= 5 × 74
= 370 m

Answer: 370 m

Exercise 5.4 — Optional Exercise

Q1 Which term of the AP: 121, 117, 113, ..., is its first negative term?

SOLUTION

$a = 121, d = 117 - 121 = -4$

$a_n = 121 + (n - 1)(-4) = 121 - 4n + 4 = 125 - 4n$

For first negative term: $a_n < 0$

$125 - 4n < 0$

$125 < 4n$

$n > 31.25$

Since n must be a positive integer, the first negative term is when $n = 32$.

$a_{32} = 125 - 4(32) = 125 - 128 = -3$

Check: $a_{31} = 125 - 124 = 1$ (still positive) ✓

Answer: 32nd term ($a_{32} = -3$)

Q2 The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

SOLUTION

$a_3 + a_7 = 6$

$(a + 2d) + (a + 6d) = 6$

$2a + 8d = 6$

$a + 4d = 3$

$a = 3 - 4d$ ... (1)

$a_3 \times a_7 = 8$

$(a + 2d)(a + 6d) = 8$

Substituting $a = 3 - 4d$:

$(3 - 4d + 2d)(3 - 4d + 6d) = 8$

$(3 - 2d)(3 + 2d) = 8$

$9 - 4d^2 = 8$

$4d^2 = 1$

$d^2 = 1/4$

$d = 1/2$ or $d = -1/2$

Case 1: $d = 1/2$, $a = 3 - 4(1/2) = 1$

S_{16} = 16/2 [2(1) + 15(1/2)] = 8[2 + 7.5] = 8 × 9.5 = 76

Case 2: d = -1/2, a = 3 - 4(-1/2) = 5
S_{16} = 16/2 [2(5) + 15(-1/2)] = 8[10 - 7.5] = 8 × 2.5 = 20

∴ Sum of first 16 terms = 76 or 20.

Answer: 76 or 20

Q3 A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are 2½ m apart, how much length of wood is required for the rungs?

SOLUTION

Distance between rungs = 25 cm
Total height = 2.5 m = 250 cm
Number of rungs = 250/25 + 1 = 10 + 1 = 11

Bottom rung = 45 cm, Top rung = 25 cm
Rungs decrease uniformly, so lengths form an AP:
a = 45, l = 25, n = 11

Total wood for rungs = S_{11} = 11/2 (45 + 25)
= 11/2 × 70
= 385 cm

∴ Total length of wood required = 385 cm

Answer: 385 cm

Q4 The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x equals the sum of the numbers of the houses following it. Find this value of x.

SOLUTION

Houses: 1, 2, 3, ..., 49
Sum of houses before x: S = 1 + 2 + ... + (x - 1) = x(x - 1)/2
Sum of houses after x: S' = (x + 1) + (x + 2) + ... + 49

Total sum = 49 × 50/2 = 1225
S' = 1225 - x - S = 1225 - x - x(x - 1)/2

Given: S = S'

$x(x - 1)/2 = 1225 - x - x(x - 1)/2$

$x(x - 1)/2 + x(x - 1)/2 = 1225 - x$

$x(x - 1) = 1225 - x$

$x^2 - x = 1225 - x$

$x^2 = 1225$

$x = 35$ (taking positive value)

Verify:

Sum before 35: $1 + 2 + ... + 34 = 34 \times 35/2 = 595$

Sum after 35: $36 + 37 + ... + 49 = S_{49} - S_{35} = 1225 - 35 - 595 = 595$ ✓

Answer: $x = 35$

Q5 A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of 1/4 m and a tread of 1/2 m (see figure). Calculate the total volume of concrete required to build the terrace.

SOLUTION

Each step is 50 m long.

Rise (height) of each step = 1/4 m

Tread (depth) of each step = 1/2 m

Step 1: height = 1/4 m, depth = 1/2 m, length = 50 m

Volume = $50 \times 1/2 \times 1/4 = 50/8$ m³

Step 2: height = 2/4 m (built on step 1), depth = 1/2 m

Volume = $50 \times 1/2 \times 2/4 = 100/8$ m³

Step 3: height = 3/4 m, depth = 1/2 m

Volume = $50 \times 1/2 \times 3/4 = 150/8$ m³

Step n: Volume = $50 \times 1/2 \times n/4 = 50n/8$ m³

Total volume = $50/8 (1 + 2 + 3 + ... + 15)$

$= 50/8 \times 15 \times 16/2$

$= 50/8 \times 120$

$= 6000/8$

$= 750$ m³

∴ Total volume of concrete = 750 m³

Answer: 750 m³