Chapter 6: Triangles

CONTENTS 3 exercises · 29 questions

6.1	Similar Figures	3 Q
6.2	Basic Proportionality Theorem (Thales Theorem)	10 Q
6.3	Criteria for Similarity of Triangles	16 Q

Exercise 6.1 — Similar Figures

Q1 Fill in the blanks using the correct word given in brackets: (i) All circles are ___. (congruent, similar) (ii) All squares are ___. (similar, congruent) (iii) All ___ triangles are similar. (isosceles, equilateral) (iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are ___ and (b) their corresponding sides are ___. (equal, proportional)

SOLUTION

(i) All circles are similar.
Reason: All circles have the same shape (every circle is a scaled version of every other circle). Their corresponding angles (360°) are equal and their radii are in a constant ratio, so all circles are similar but not necessarily congruent.

(ii) All squares are similar.
Reason: All squares have all angles equal to 90° and all sides in the same ratio (side : side = 1 : 1 proportionally after scaling). Hence all squares are similar.

(iii) All equilateral triangles are similar.
Reason: In every equilateral triangle each angle is 60°, so corresponding angles are always equal. Their sides are also in the same ratio. Hence all equilateral triangles are similar.

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are equal and (b) their corresponding sides are proportional.

Answer: (i) similar, (ii) similar, (iii) equilateral, (iv)(a) equal, (b) proportional

Q2 Give two different examples of pair of (i) similar figures. (ii) non-similar figures.

SOLUTION

(i) Two examples of similar figures:

1. Any two circles (e.g., a circle of radius 2 cm and a circle of radius 5 cm) — same shape, different sizes.
2. Any two equilateral triangles (e.g., with sides 3 cm and 6 cm) — all angles are 60°, sides are proportional.

(ii) Two examples of non-similar figures:

1. A circle and a square — different shapes altogether; no consistent angle or side-ratio relationship.
2. A right-angled triangle and an equilateral triangle — angles are not equal, so they cannot be similar.

Answer: Similar: (1) two circles, (2) two equilateral triangles. Non-similar: (1) a circle and a square, (2) a right triangle and an equilateral triangle.

Q3 State whether the two quadrilaterals in Fig. 6.8 are similar or not.

SOLUTION

From Fig. 6.8 the two quadrilaterals are: square ABCD with every side 3 cm and every angle 90°, and rhombus PQRS with every side 1.5 cm.

Corresponding sides are proportional: 1.5/3 = 1/2 for each pair of sides.

But the corresponding angles are NOT equal — the square ABCD has all angles 90°, while the rhombus PQRS is slanted, so its angles are not 90°.

Two polygons are similar only if BOTH corresponding sides are proportional AND corresponding angles are equal. Here only the sides are proportional; the angles differ.

Therefore the two quadrilaterals are NOT similar.

Answer: Not similar — corresponding sides are proportional (ratio 1 : 2) but corresponding angles are not equal (the square has 90° angles, the rhombus does not).

Exercise 6.2 — Basic Proportionality Theorem (Thales Theorem)

Q1 In Fig. 6.17, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii). [Fig 6.17(i): AD = 1.5 cm, DB = 3 cm, AE = 1 cm. Fig 6.17(ii): DB = 7.2 cm, AE = 1.8 cm, EC = 5.4 cm.]

SOLUTION

Since DE || BC in both figures, by the Basic Proportionality Theorem (BPT / Theorem 6.1):
AD/DB = AE/EC

(i) Given: AD = 1.5 cm, DB = 3 cm, AE = 1 cm. Find EC.
AD/DB = AE/EC
1.5/3 = 1/EC
EC = 1 × 3/1.5
EC = 3/1.5 = 2 cm

(ii) Given: DB = 7.2 cm, AE = 1.8 cm, EC = 5.4 cm. Find AD.
AD/DB = AE/EC
AD/7.2 = 1.8/5.4
AD/7.2 = 1/3
AD = 7.2/3 = 2.4 cm

Answer: (i) EC = 2 cm; (ii) AD = 2.4 cm

Q2 E and F are points on the sides PQ and PR respectively of a triangle PQR. For each of the following cases, state whether EF || QR: (i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm (ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm (iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

SOLUTION

By the converse of BPT, EF || QR if and only if PE/EQ = PF/FR.

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm
PE/EQ = 3.9/3 = 1.3
PF/FR = 3.6/2.4 = 1.5
Since 1.3 ≠ 1.5, EF is NOT parallel to QR.

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm
PE/QE = 4/4.5 = 8/9
PF/RF = 8/9

Since PE/QE = PF/RF = 8/9, EF IS parallel to QR.

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm
EQ = PQ - PE = 1.28 - 0.18 = 1.10 cm
FR = PR - PF = 2.56 - 0.36 = 2.20 cm
PE/EQ = 0.18/1.10 = 18/110 = 9/55
PF/FR = 0.36/2.20 = 36/220 = 9/55
Since PE/EQ = PF/FR = 9/55, EF IS parallel to QR.

Answer: (i) EF is not parallel to QR; (ii) EF || QR; (iii) EF || QR

Q3 In Fig. 6.18, if LM || CB and LN || CD, prove that AM/AB = AN/AD.

SOLUTION

Given: In triangle ABC, LM || CB. In triangle ACD, LN || CD.

In triangle ABC, LM || CB (given).
By BPT (Theorem 6.1): AL/LB = AM/MC
Adding 1 to both sides: (AL + LB)/LB = (AM + MC)/MC
=> AB/LB = AC/MC
Taking reciprocals and subtracting from 1: AL/AB = AM/AC ... (1)

In triangle ACD, LN || CD (given).
By the same reasoning: AL/AD = AN/AC ... (2)

Dividing (1) by (2):
(AL/AB) / (AL/AD) = (AM/AC) / (AN/AC)
=> AD/AB = AM/AN
=> AM/AB = AN/AD (cross-multiplying)

Hence proved: AM/AB = AN/AD.

Answer: Proved: AM/AB = AN/AD (using BPT in triangles ABC and ACD separately).

Q4 In Fig. 6.19, DE || AC and DF || AE. Prove that BF/FE = BE/EC.

SOLUTION

Given: In triangle ABC, DE || AC. In triangle ABE, DF || AE.

In triangle ABC, DE || AC (given).
By BPT: BD/DA = BE/EC ... (1)

In triangle ABE, DF || AE (given).
By BPT: BD/DA = BF/FE ... (2)

From (1) and (2):
BF/FE = BE/EC

Hence proved.

Answer: Proved: BF/FE = BE/EC (applying BPT twice in triangles ABC and ABE).

Q5 In Fig. 6.20, DE || OQ and DF || OR. Show that EF || QR.

SOLUTION

Given: In triangle POQ, DE || OQ. In triangle POR, DF || OR.

In triangle POQ, DE || OQ (given).
By BPT: PD/DO = PE/EQ ... (1)

In triangle POR, DF || OR (given).
By BPT: PD/DO = PF/FR ... (2)

From (1) and (2):
PE/EQ = PF/FR

In triangle PQR, since PE/EQ = PF/FR, by the converse of BPT (Theorem 6.2):
EF || QR

Hence proved.

Answer: Proved: EF || QR (by applying BPT in triangles POQ and POR, then the converse of BPT in triangle PQR).

Q6 In Fig. 6.21, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

SOLUTION

Given: In triangle OPQ, AB || PQ. In triangle OPR, AC || PR.

In triangle OPQ, AB || PQ (given).
By BPT: OA/AP = OB/BQ ... (1)

In triangle OPR, AC || PR (given).
By BPT: OA/AP = OC/CR ... (2)

From (1) and (2):

OB/BQ = OC/CR

In triangle OQR, since OB/BQ = OC/CR, by the converse of BPT (Theorem 6.2):
BC || QR

Hence proved.

Answer: Proved: BC || QR (using BPT in triangles OPQ and OPR, then converse of BPT in triangle OQR).

Q7 Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

SOLUTION

Let triangle ABC be given. Let D be the mid-point of AB (so AD = DB).
Let DE be drawn through D parallel to BC, meeting AC at E.
We need to prove that E is the mid-point of AC (i.e., AE = EC).

Since DE || BC in triangle ABC, by Theorem 6.1 (BPT):
AD/DB = AE/EC

Since D is the mid-point of AB: AD = DB, so AD/DB = 1.

Therefore: AE/EC = 1
=> AE = EC

Hence E is the mid-point of AC, i.e., the line through the mid-point of AB parallel to BC bisects AC.

Answer: Proved: E is the mid-point of AC (using BPT: AD/DB = AE/EC, and AD = DB => AE = EC).

Q8 Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.

SOLUTION

Let triangle ABC be given. Let D and E be the mid-points of AB and AC respectively.
So AD = DB and AE = EC.
We need to prove that DE || BC.

Since D is the mid-point of AB: AD/DB = 1.
Since E is the mid-point of AC: AE/EC = 1.

Therefore: AD/DB = AE/EC (both equal 1).

By Theorem 6.2 (converse of BPT), since AD/DB = AE/EC:
DE || BC

Hence proved.

Answer: Proved: DE || BC (since AD/DB = AE/EC = 1, by converse of BPT).

Q9 ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that AO/BO = CO/DO.

SOLUTION

Given: Trapezium ABCD with AB || DC. Diagonals AC and BD intersect at O.
To prove: AO/BO = CO/DO (equivalently AO/CO = BO/DO).

In triangles OAB and OCD:

angle OAB = angle OCD (alternate interior angles, AB		DC, transversal AC)
angle OBA = angle ODC (alternate interior angles, AB		DC, transversal BD)

By AA similarity: triangle OAB ~ triangle OCD.

Therefore corresponding sides are proportional:
OA/OC = OB/OD
Cross-multiplying (OA·OD = OC·OB) and rearranging:
AO/BO = CO/DO

Hence proved.

Answer: Proved: AO/BO = CO/DO (triangles OAB and OCD are similar by AA, since AB || DC gives alternate angles).

Q10 The diagonals of a quadrilateral ABCD intersect each other at the point O such that AO/CO = BO/DO. Show that ABCD is a trapezium.

SOLUTION

Given: Diagonals AC and BD of quadrilateral ABCD intersect at O, and AO/CO = BO/DO.
To prove: ABCD is a trapezium (i.e., AB || DC).

AO/CO = BO/DO (given)

=> AO/BO = CO/DO ... (1) (rearranging)

In triangles AOB and COD:
AO/CO = BO/DO (from given, rewritten as AO/CO = BO/DO)
angle AOB = angle COD (vertically opposite angles)

By SAS similarity criterion: triangle AOB ~ triangle COD.

Therefore: angle OAB = angle OCD (corresponding angles of similar triangles)

These are alternate interior angles formed by transversal AC with lines AB and DC.
Since alternate interior angles are equal, AB || DC.

Since one pair of opposite sides (AB and DC) is parallel, ABCD is a trapezium.

Hence proved.

Answer: Proved: ABCD is a trapezium (triangle AOB ~ triangle COD by SAS, giving alternate angles equal, so AB || DC).

Exercise 6.3 — Criteria for Similarity of Triangles

Q1 State which pairs of triangles in Fig. 6.34 are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form.

SOLUTION

(i) In triangle ABC: angle A = 60°, angle B = 80°, angle C = 40°. In triangle PQR: angle P = 60°, angle Q = 80°, angle R = 40°.
All three pairs of corresponding angles are equal.
By the AAA similarity criterion: triangle ABC ~ triangle PQR.

(ii) In triangle ABC: AB = 2, BC = 2.5, CA = 3. In triangle PQR: QR = 4, RP = 5, PQ = 6.
Correspondence A<->Q, B<->R, C<->P: AB/QR = 2/4 = 1/2, BC/RP = 2.5/5 = 1/2, CA/PQ = 3/6 = 1/2.
All three pairs of corresponding sides are in the same ratio 1/2.
By the SSS similarity criterion: triangle ABC ~ triangle QRP.

(iii) In triangle LMP: LM = 2.7, MP = 2, PL = 3. In triangle DEF: DE = 4, EF = 5, FD = 6.
All six sides are given. Comparing ratios: 2/4 = 1/2 and 3/6 = 1/2, but 2.7/5 = 0.54, which is not 1/2.
The three pairs of corresponding sides are NOT in the same ratio, so no SSS correspondence exists.
The two triangles are NOT similar.

(iv) In triangle MNL: MN = 2.5, ML = 5, included angle M = 70°. In triangle QPR: QP = 5, QR = 10, included angle Q = 70°.
MN/QP = 2.5/5 = 1/2 and ML/QR = 5/10 = 1/2, and the included angles are equal (angle M = angle Q = 70°).
By the SAS similarity criterion: triangle MNL ~ triangle QPR.

(v) In triangle ABC: AB = 2.5, BC = 3, angle A = 80°. In triangle DEF: DF = 5, EF = 6, angle D = 80°.
The equal 80° angle is at the apex A (resp. D) — it is NOT the angle included between the two given sides. A single pair of sides with a non-included angle satisfies no criterion.
The two triangles are NOT similar.

(vi) In triangle DEF: angle D = 70°, angle E = 80°, so angle F = 180° - 70° - 80° =

30°. In triangle PQR: angle Q = 80°, angle R = 30°, so angle P = 180° - 80° - 30° = 70°.

Two pairs of corresponding angles are equal (angle E = angle Q = 80° and angle F = angle R = 30°).

By the AA similarity criterion: triangle DEF ~ triangle PQR.

Summary:

(i) triangle ABC ~ triangle PQR (AAA)

(ii) triangle ABC ~ triangle QRP (SSS)

(iii) not similar

(iv) triangle MNL ~ triangle QPR (SAS)

(v) not similar

(vi) triangle DEF ~ triangle PQR (AA)

Answer: (i) ABC ~ PQR (AAA); (ii) ABC ~ QRP (SSS); (iii) not similar; (iv) MNL ~ QPR (SAS); (v) not similar; (vi) DEF ~ PQR (AA)

Q2 In Fig. 6.35, triangle ODC ~ triangle OBA, angle BOC = 125° and angle CDO = 70°. Find angle DOC, angle DCO and angle OAB.

SOLUTION

Given: triangle ODC ~ triangle OBA, angle BOC = 125°, angle CDO = 70°.

Step 1: Find angle DOC.

Points D, O, B are on a straight line (B, O, C are on straight lines forming the figure — O is the intersection of diagonals).

angle DOC + angle BOC = 180° (DOB is a straight line, since D-O-B are collinear)

angle DOC = 180° - 125° = 55°

Step 2: Find angle DCO.

In triangle DOC:

angle CDO + angle DOC + angle DCO = 180°

70° + 55° + angle DCO = 180°

angle DCO = 180° - 125° = 55°

Step 3: Find angle OAB.

Since triangle ODC ~ triangle OBA, the correspondence is O<->O, D<->B, C<->A.

So angle C (= angle DCO = 55°) corresponds to angle A (= angle OAB).

Therefore angle OAB = angle DCO = 55°.

Answer: angle DOC = 55°, angle DCO = 55°, angle OAB = 55°

Q3 Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that OA/OC = OB/OD.

SOLUTION

Given: Trapezium ABCD with AB || DC. Diagonals AC and BD intersect at O.
To prove: OA/OC = OB/OD.

In triangles OAB and OCD:

angle OAB = angle OCD (alternate interior angles, since AB		DC and AC is a transversal)
angle OBA = angle ODC (alternate interior angles, since AB		DC and BD is a transversal)

By AA similarity criterion:
triangle OAB ~ triangle OCD

Therefore, corresponding sides are proportional:
OA/OC = OB/OD = AB/CD

Hence proved: OA/OC = OB/OD.

Answer: Proved: triangle OAB ~ triangle OCD (AA), so OA/OC = OB/OD.

Q4 In Fig. 6.36, QR/QS = QT/PR and angle 1 = angle 2. Show that triangle PQS ~ triangle TQR.

SOLUTION

Given: In triangle PQR, QR/QS = QT/PR and angle 1 = angle 2 (angle 1 = angle PQR, angle 2 = angle PRQ).

In triangle PQR, angle 1 = angle 2 (given).
Sides opposite equal angles are equal, so PQ = PR (triangle PQR is isosceles).

Given QR/QS = QT/PR. Substituting PR = PQ:
QR/QS = QT/PQ

Rearranging: PQ/QT = QS/QR ... (i)

In triangles PQS and TQR:
angle PQS = angle TQR (both are angle 1, the angle at Q)
PQ/QT = QS/QR (from (i) — the sides about this angle are proportional)

By SAS similarity: triangle PQS ~ triangle TQR.

Hence proved.

Answer: Proved: triangle PQS ~ triangle TQR by SAS (angle 1 = angle 2 => PQ = PR, giving PQ/QT = QS/QR, with included angle PQS = angle TQR).

Q5 S and T are points on sides PR and QR of triangle PQR such that angle P = angle RTS. Show that triangle RPQ ~ triangle RTS.

SOLUTION

Given: In triangle PQR, S is on PR, T is on QR, and angle P = angle RTS.
To prove: triangle RPQ ~ triangle RTS.

In triangles RPQ and RTS:
angle R = angle R (common angle)
angle RPQ = angle RTS (given, angle P = angle RTS)

By AA similarity criterion:
triangle RPQ ~ triangle RTS

Hence proved.

Answer: Proved: triangle RPQ ~ triangle RTS (AA: angle R common, angle RPQ = angle RTS given).

Q6 In Fig. 6.37, if triangle ABE is congruent to triangle ACD, show that triangle ADE ~ triangle ABC.

SOLUTION

Given: triangle ABE ≅ triangle ACD.
To prove: triangle ADE ~ triangle ABC.

Since triangle ABE ≅ triangle ACD (given), corresponding parts are equal:
AB = AC ... (1)
AD = AE ... (2) (D in ACD corresponds to E in ABE)

From (1) and (2):

AD/AB = AE/AC ... (3) (dividing: both ratios equal AE/AB = AD/AC...)

More precisely: AB = AC and AD = AE, so:
AD/AB = AE/AC (both ratios are equal since AB = AC and AD = AE).

In triangles ADE and ABC:
AD/AB = AE/AC (proved above)
angle DAE = angle BAC (common angle A)

By SAS similarity criterion:
triangle ADE ~ triangle ABC

Hence proved.

Answer: Proved: triangle ADE ~ triangle ABC by SAS (from congruence: AB = AC and AD = AE, giving AD/AB = AE/AC, with common angle A).

Q7 In Fig. 6.38, altitudes AD and CE of triangle ABC intersect each other at the point P. Show that: (i) triangle AEP ~ triangle CDP (ii) triangle ABD ~ triangle CBE (iii) triangle AEP ~ triangle ADB (iv) triangle PDC ~ triangle BEC.

SOLUTION

Given: AD and CE are altitudes of triangle ABC, intersecting at P.
So angle ADB = angle AEC = angle CDA = 90° (altitude means perpendicular to opposite side).
More precisely: AD ⊥ BC so angle ADB = angle ADC = 90°; CE ⊥ AB so angle CEA = angle CEB = 90°.

(i) triangle AEP ~ triangle CDP:
In triangles AEP and CDP:
angle AEP = angle CDP = 90° (CE ⊥ AB => angle AEP = 90°; AD ⊥ BC => angle CDP = 90°)
angle APE = angle CPD (vertically opposite angles at P)
By AA: triangle AEP ~ triangle CDP.

(ii) triangle ABD ~ triangle CBE:
In triangles ABD and CBE:
angle ADB = 90° = angle CEB (altitudes)
angle ABD = angle CBE (common angle B)
By AA: triangle ABD ~ triangle CBE.

(iii) triangle AEP ~ triangle ADB:

In triangles AEP and ADB:
angle AEP = 90° = angle ADB (altitudes: CE ⊥ AB, AD ⊥ BC)
angle EAP = angle DAB (same angle at vertex A)
By AA: triangle AEP ~ triangle ADB.

(iv) triangle PDC ~ triangle BEC:
In triangles PDC and BEC:
angle PDC = 90° = angle BEC (altitudes: AD ⊥ BC gives angle PDC = 90°; CE ⊥ AB gives angle BEC = 90°)
angle PCD = angle BCE (same angle C)
By AA: triangle PDC ~ triangle BEC.

Hence all four parts are proved.

Answer: (i) triangle AEP ~ triangle CDP (AA); (ii) triangle ABD ~ triangle CBE (AA); (iii) triangle AEP ~ triangle ADB (AA); (iv) triangle PDC ~ triangle BEC (AA).

Q8 E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that triangle ABE ~ triangle CFB.

SOLUTION

Given: ABCD is a parallelogram. E is on AD produced. BE intersects CD at F.
To prove: triangle ABE ~ triangle CFB.

In parallelogram ABCD, AE lies along AD produced, so AE || BC, and AB || DC.

In triangles ABE and CFB:
angle A = angle C (opposite angles of parallelogram ABCD are equal: angle DAB = angle BCD)
angle AEB = angle CBF (alternate interior angles: AE || BC with transversal BE)
By AA similarity: triangle ABE ~ triangle CFB.

Hence proved.

Answer: Proved: triangle ABE ~ triangle CFB by AA (angle A = angle C as opposite angles of parallelogram; angle AEB = angle CBF as alternate angles since AD || BC).

Q9 In Fig. 6.39, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that: (i) triangle ABC ~ triangle AMP (ii) CA/PA = BC/MP.

SOLUTION

Given: Triangle ABC right-angled at B (angle B = 90°). Triangle AMP right-angled at M (angle M = 90°).

(i) In triangles ABC and AMP:
angle ABC = angle AMP = 90° (given)
angle BAC = angle MAP (common angle A)
By AA similarity criterion: triangle ABC ~ triangle AMP.

(ii) Since triangle ABC ~ triangle AMP, corresponding sides are proportional:
CA/PA = AB/AM = BC/MP
Therefore: CA/PA = BC/MP.

Hence both parts are proved.

Answer: (i) triangle ABC ~ triangle AMP (AA: angle B = angle M = 90°, common angle A). (ii) CA/PA = BC/MP (corresponding sides of similar triangles).

Q10 CD and GH are respectively the bisectors of angle ACB and angle EGF such that D and H lie on sides AB and FE of triangle ABC and triangle EFG respectively. If triangle ABC ~ triangle FEG, show that: (i) CD/GH = AC/FG (ii) triangle DCB ~ triangle HGE (iii) triangle DCA ~ triangle HGF.

SOLUTION

Given: triangle ABC ~ triangle FEG. CD bisects angle ACB with D on AB. GH bisects angle EGF with H on FE.

Since triangle ABC ~ triangle FEG:
angle A = angle F, angle B = angle E, angle C = angle G ... (1)
and AB/FE = BC/EG = CA/GF ... (2)

(i) Prove CD/GH = AC/FG:
CD bisects angle ACB, so angle ACD = angle DCB = angle ACB/2.
GH bisects angle EGF, so angle EGH = angle HGF = angle EGF/2.
Since angle ACB = angle EGF (from similarity), angle ACD = angle FGH.

In triangles ACD and FGH:
angle A = angle F (from similarity)
angle ACD = angle FGH (proved above, each = half of equal angles)

By AA: triangle ACD ~ triangle FGH.
Therefore: CD/GH = AC/FG.

(ii) Prove triangle DCB ~ triangle HGE:
angle DCB = angle ACB/2 and angle HGE = angle EGF/2.
Since angle ACB = angle EGF: angle DCB = angle HGE.
Also angle B = angle E (from similarity).
By AA: triangle DCB ~ triangle HGE.

(iii) Prove triangle DCA ~ triangle HGF:
From part (i), triangle ACD ~ triangle FGH.
This can be written as triangle DCA ~ triangle HGF (same triangles, reordered vertices).
Hence triangle DCA ~ triangle HGF.

All three parts proved.

Answer: (i) CD/GH = AC/FG (triangle ACD ~ triangle FGH by AA). (ii) triangle DCB ~ triangle HGE (AA). (iii) triangle DCA ~ triangle HGF (same as part i, rewritten).

Q11 In Fig. 6.40, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that triangle ABD ~ triangle ECF.

SOLUTION

Given: Triangle ABC is isosceles with AB = AC. AD ⊥ BC. E is on CB produced. EF ⊥ AC.
To prove: triangle ABD ~ triangle ECF.

In triangles ABD and ECF:
angle ADB = 90° = angle EFC (AD ⊥ BC and EF ⊥ AC)
angle ABD = angle ECF (since AB = AC in isosceles triangle ABC, angle ABC = angle ACB; angle ECF = angle ACB = angle ABC = angle ABD)

By AA similarity criterion: triangle ABD ~ triangle ECF.

Hence proved.

Answer: Proved: triangle ABD ~ triangle ECF by AA (angle ADB = angle EFC = 90°; angle ABD = angle ECF since AB = AC => angle ABC = angle ACB).

Q12 Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of triangle PQR (see Fig. 6.41). Show that triangle ABC ~ triangle PQR.

SOLUTION

Given: AB/PQ = BC/QR = AD/PM, where AD is median of triangle ABC and PM is median of triangle PQR.
To prove: triangle ABC ~ triangle PQR.

Since AD is a median of triangle ABC: D is the mid-point of BC, so BD = BC/2.
Since PM is a median of triangle PQR: M is the mid-point of QR, so QM = QR/2.

Given: AB/PQ = BC/QR = AD/PM.
From BC/QR = BD/(QM) (since BD = BC/2 and QM = QR/2, so BC/QR = BD/QM).

So: AB/PQ = BD/QM = AD/PM.

In triangles ABD and PQM:
AB/PQ = BD/QM = AD/PM (all three sides proportional).
By SSS similarity: triangle ABD ~ triangle PQM.

Therefore: angle ABD = angle PQM (corresponding angles of similar triangles)
i.e., angle ABC = angle PQR.

Now in triangles ABC and PQR:
AB/PQ = BC/QR (given)
angle ABC = angle PQR (proved above)

By SAS similarity: triangle ABC ~ triangle PQR.

Hence proved.

Answer: Proved: triangle ABC ~ triangle PQR (first show triangle ABD ~ triangle PQM by SSS to get angle B = angle Q, then use SAS in triangles ABC and PQR).

Q13 D is a point on the side BC of a triangle ABC such that angle ADC = angle BAC. Show that $CA^2 = CB \cdot CD$.

SOLUTION

Given: In triangle ABC, D is on BC and angle ADC = angle BAC.
To prove: $CA^2 = CB \cdot CD$, i.e., CA/CD = CB/CA.

In triangles BAC and ACD:

angle BAC = angle ADC (given)
angle ACB = angle DCA (common angle C)
By AA: triangle BAC ~ triangle ACD.

Therefore corresponding sides are proportional:
CA/CD = CB/CA

=> $CA^2 = CB \cdot CD$.

Hence proved.

Answer: Proved: $CA^2 = CB \cdot CD$ (triangles BAC ~ triangle ACD by AA, giving CA/CD = CB/CA).

Q14 Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that triangle ABC ~ triangle PQR.

SOLUTION

Given: AB/PQ = AC/PR = AD/PM, where AD is median of triangle ABC (D on BC) and PM is median of triangle PQR (M on QR).
To prove: triangle ABC ~ triangle PQR.

Produce AD to E such that DE = AD. Join CE.
Produce PM to N such that MN = PM. Join RN.

Since AD is a median and DE = AD, in quadrilateral ABEC:
D is the mid-point of BC (AD is median) and D is the mid-point of AE (by construction).
So ABEC is a parallelogram. Therefore CE = AB and CE || AB.

Similarly, PQNR is a parallelogram (M mid-point of QR, MN = PM).
Therefore RN = PQ and RN || PQ.

Now: AE = 2·AD and PN = 2·PM.
Given AB/PQ = AC/PR = AD/PM => AB/PQ = AC/PR = AE/PN.

Also CE = AB and RN = PQ, so CE/RN = AB/PQ.

In triangles ACE and PRN:
AC/PR = AE/PN = CE/RN (all equal to AB/PQ).
By SSS similarity: triangle ACE ~ triangle PRN.
Therefore angle CAE = angle RPN ... (3)

Since D lies on AE, angle CAE = angle CAD; similarly angle RPN = angle RPM.
So (3) gives angle CAD = angle RPM ... (4)

By the same construction applied to the other pair, triangle ABD ~ triangle PQM, giving angle BAD = angle QPM ... (5)

Adding (4) and (5):
angle BAC = angle BAD + angle DAC = angle QPM + angle RPM = angle QPR ... (6)

Now in triangles ABC and PQR:
AB/PQ = AC/PR (given)
angle BAC = angle QPR (from (6))
By SAS similarity: triangle ABC ~ triangle PQR.

Hence proved.

Answer: Proved: triangle ABC ~ triangle PQR (extend medians to form parallelograms, prove triangle ACE ~ triangle PRN to get angle BAC = angle QPR, then SAS applies).

Q15 A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

SOLUTION

Let AB = 6 m be the pole and BC = 4 m be its shadow.
Let DE be the tower and EF = 28 m be its shadow.

At the same time of day, the sun's rays are parallel, so the angles of elevation are equal.
In triangles ABC and DEF:
angle B = angle E = 90° (both pole and tower are vertical)
angle ACB = angle DFE (sun's rays are parallel => equal angles of elevation/depression)
By AA: triangle ABC ~ triangle DEF.

Therefore: AB/DE = BC/EF
6/DE = 4/28
DE = 6 × 28/4
DE = 6 × 7
DE = 42 m

The height of the tower is 42 m.

Answer: Height of the tower = 42 m

Q16 If AD and PM are medians of triangles ABC and PQR, respectively where triangle ABC ~ triangle PQR, prove that AB/PQ = AD/PM.

SOLUTION

Given: triangle ABC ~ triangle PQR. AD is a median of triangle ABC (D is the mid-point of BC). PM is a median of triangle PQR (M is the mid-point of QR).
To prove: AB/PQ = AD/PM.

Since triangle ABC ~ triangle PQR:
AB/PQ = BC/QR = CA/RP ... (1)
and angle A = angle P, angle B = angle Q, angle C = angle R ... (2)

From (1): AB/PQ = BC/QR.
Since D is mid-point of BC: BD = BC/2.
Since M is mid-point of QR: QM = QR/2.
=> BC/QR = BD/QM ... (3)

From (1) and (3): AB/PQ = BD/QM ... (4)

In triangles ABD and PQM:
AB/PQ = BD/QM (from (4))
angle ABD = angle PQM (i.e., angle B = angle Q, from (2))

By SAS similarity: triangle ABD ~ triangle PQM.

Therefore corresponding sides are proportional:
AB/PQ = AD/PM (AD and PM are the third sides).

Hence proved: AB/PQ = AD/PM.

Answer: Proved: AB/PQ = AD/PM (triangle ABD ~ triangle PQM by SAS, using AB/PQ = BD/QM and angle B = angle Q).