Chapter 7: Coordinate Geometry

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Exercise 7.1 — Distance Formula

Q1 Find the distance between the following pairs of points:
(i) (2, 3) and (4, 1)
(ii) (−5, 7) and (−1, 3)
(iii) (a, b) and (−a, −b)

SOLUTION

Distance formula: $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

(i) Points: (2, 3) and (4, 1)
$d = \sqrt{[(4 - 2)^2 + (1 - 3)^2]}$
$= \sqrt{[4 + 4]} = \sqrt{8} = 2\sqrt{2}$

(ii) Points: (−5, 7) and (−1, 3)
$d = \sqrt{[(-1 - (-5))^2 + (3 - 7)^2]}$
$= \sqrt{[(4)^2 + (-4)^2]}$
$= \sqrt{[16 + 16]} = \sqrt{32} = 4\sqrt{2}$

(iii) Points: (a, b) and (−a, −b)
$d = \sqrt{[(-a - a)^2 + (-b - b)^2]}$
$= \sqrt{[(-2a)^2 + (-2b)^2]}$
$= \sqrt{[4a^2 + 4b^2]} = 2\sqrt{(a^2 + b^2)}$

Answer: (i) $2\sqrt{2}$ (ii) $4\sqrt{2}$ (iii) $2\sqrt{(a^2 + b^2)}$

Q2 Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2?

SOLUTION

$d = \sqrt{[(36 - 0)^2 + (15 - 0)^2]}$
$= \sqrt{[1296 + 225]}$
$= \sqrt{1521}$

Check: $39^2 = 1521$, so $\sqrt{1521} = 39$.

Yes, the distance between towns A and B = 39 km (with coordinates in km).

Answer: 39 units; the towns A and B are 39 km apart

Q3 Determine if the points (1, 5), (2, 3) and (−2, −11) are collinear.

SOLUTION

Let A = (1, 5), B = (2, 3), C = (−2, −11).

$AB = \sqrt{[(2 - 1)^2 + (3 - 5)^2]} = \sqrt{[1 + 4]} = \sqrt{5}$

$BC = \sqrt{[(-2 - 2)^2 + (-11 - 3)^2]} = \sqrt{[16 + 196]} = \sqrt{212} = 2\sqrt{53}$

$AC = \sqrt{[(-2 - 1)^2 + (-11 - 5)^2]} = \sqrt{[9 + 256]} = \sqrt{265}$

For collinearity we need AB + BC = AC (or some similar equality).
$AB + BC = \sqrt{5} + 2\sqrt{53} \approx 2.24 + 14.56 = 16.80$
$AC = \sqrt{265} \approx 16.28$

Since AB + BC ≠ AC, the points are NOT collinear.

Answer: The points are not collinear.

Q4 Check whether (5, −2), (6, 4) and (7, −2) are the vertices of an isosceles triangle.

SOLUTION

Let A = (5, −2), B = (6, 4), C = (7, −2).

$AB = \sqrt{[(6 - 5)^2 + (4 - (-2))^2]} = \sqrt{[1 + 36]} = \sqrt{37}$

$BC = \sqrt{[(7 - 6)^2 + (-2 - 4)^2]} = \sqrt{[1 + 36]} = \sqrt{37}$

$AC = \sqrt{[(7 - 5)^2 + (-2 - (-2))^2]} = \sqrt{[4 + 0]} = 2$

AB = BC = $\sqrt{37}$, so two sides are equal.
∴ Triangle ABC is an isosceles triangle.

Answer: Yes, it is an isosceles triangle with AB = BC = $\sqrt{37}$.

Q5 In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 7.8. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.
[From Fig. 7.8: A(3, 4), B(6, 7), C(9, 4), D(6, 1)]

SOLUTION

A = (3, 4), B = (6, 7), C = (9, 4), D = (6, 1)

Sides:
AB = $\sqrt{[(6-3)^2 + (7-4)^2]} = \sqrt{[9 + 9]} = \sqrt{18} = 3\sqrt{2}$
BC = $\sqrt{[(9-6)^2 + (4-7)^2]} = \sqrt{[9 + 9]} = \sqrt{18} = 3\sqrt{2}$
CD = $\sqrt{[(6-9)^2 + (1-4)^2]} = \sqrt{[9 + 9]} = \sqrt{18} = 3\sqrt{2}$
DA = $\sqrt{[(3-6)^2 + (4-1)^2]} = \sqrt{[9 + 9]} = \sqrt{18} = 3\sqrt{2}$

All four sides are equal: AB = BC = CD = DA = $3\sqrt{2}$.

Diagonals:
AC = $\sqrt{[(9-3)^2 + (4-4)^2]} = \sqrt{36} = 6$
BD = $\sqrt{[(6-6)^2 + (1-7)^2]} = \sqrt{36} = 6$

Both diagonals are equal: AC = BD = 6.

All sides equal AND both diagonals equal => ABCD is a square.
∴ Champa is correct.

Answer: Champa is correct. ABCD is a square with side $3\sqrt{2}$ and diagonal 6.

Q6 Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:
(i) (−1, −2), (1, 0), (−1, 2), (−3, 0)
(ii) (−3, 5), (3, 1), (0, 3), (−1, −4)
(iii) (4, 5), (7, 6), (4, 3), (1, 2)

SOLUTION

(i) A = (−1, −2), B = (1, 0), C = (−1, 2), D = (−3, 0)

AB = $\sqrt{[(1+1)^2 + (0+2)^2]} = \sqrt{[4+4]} = 2\sqrt{2}$
BC = $\sqrt{[(-1-1)^2 + (2-0)^2]} = \sqrt{[4+4]} = 2\sqrt{2}$
CD = $\sqrt{[(-3+1)^2 + (0-2)^2]} = \sqrt{[4+4]} = 2\sqrt{2}$
DA = $\sqrt{[(-1+3)^2 + (-2-0)^2]} = \sqrt{[4+4]} = 2\sqrt{2}$

All sides equal. Diagonals:

AC = $\sqrt{[(-1+1)^2 + (2+2)^2]} = \sqrt{16} = 4$
BD = $\sqrt{[(-3-1)^2 + (0-0)^2]} = \sqrt{16} = 4$

All sides equal and diagonals equal => ABCD is a square.

(ii) A = (−3, 5), B = (3, 1), C = (0, 3), D = (−1, −4)

Check if A, B, C are collinear:
AC = $\sqrt{[(0+3)^2 + (3-5)^2]} = \sqrt{[9+4]} = \sqrt{13}$
BC = $\sqrt{[(0-3)^2 + (3-1)^2]} = \sqrt{[9+4]} = \sqrt{13}$
AB = $\sqrt{[(3+3)^2 + (1-5)^2]} = \sqrt{[36+16]} = \sqrt{52} = 2\sqrt{13}$

AC + BC = $\sqrt{13} + \sqrt{13} = 2\sqrt{13}$ = AB
=> A, B, C are collinear. These four points do not form a quadrilateral.

(iii) A = (4, 5), B = (7, 6), C = (4, 3), D = (1, 2)

AB = $\sqrt{[(7-4)^2 + (6-5)^2]} = \sqrt{[9+1]} = \sqrt{10}$
BC = $\sqrt{[(4-7)^2 + (3-6)^2]} = \sqrt{[9+9]} = 3\sqrt{2}$
CD = $\sqrt{[(1-4)^2 + (2-3)^2]} = \sqrt{[9+1]} = \sqrt{10}$
DA = $\sqrt{[(4-1)^2 + (5-2)^2]} = \sqrt{[9+9]} = 3\sqrt{2}$

Opposite sides equal: AB = CD = $\sqrt{10}$ and BC = DA = $3\sqrt{2}$.

Diagonals:
AC = $\sqrt{[(4-4)^2 + (3-5)^2]} = \sqrt{4} = 2$
BD = $\sqrt{[(1-7)^2 + (2-6)^2]} = \sqrt{[36+16]} = \sqrt{52} = 2\sqrt{13}$

Opposite sides equal but diagonals unequal => ABCD is a parallelogram.

Answer: (i) Square (ii) Not a quadrilateral (3 points are collinear) (iii) Parallelogram

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Q7 Find the point on the x-axis which is equidistant from (2, −5) and (−2, 9).

SOLUTION

Let P = (x, 0) be a point on the x-axis.
Given: PA = PB where A = (2, −5) and B = (−2, 9).

$PA^2 = (x-2)^2 + 25$
$PB^2 = (x+2)^2 + 81$

Setting $PA^2 = PB^2$:
$(x-2)^2 + 25 = (x+2)^2 + 81$
$x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$
$-4x + 29 = 4x + 85$

$-8x = 56$

$x = -7$

Verification: $PA = \sqrt{[(-7-2)^2 + 25]} = \sqrt{[81+25]} = \sqrt{106}$

$PB = \sqrt{[(-7+2)^2 + 81]} = \sqrt{[25+81]} = \sqrt{106}$ ✓

∴ The required point is $(-7, 0)$.

Answer: $(-7, 0)$

Q8 Find the values of y for which the distance between the points P(2, −3) and Q(10, y) is 10 units.

SOLUTION

$PQ = 10$, $P = (2, -3)$, $Q = (10, y)$

$\sqrt{[(10-2)^2 + (y+3)^2]} = 10$

$\sqrt{[64 + (y+3)^2]} = 10$

Squaring: $64 + (y+3)^2 = 100$

$(y+3)^2 = 36$

$y + 3 = \pm 6$

Case 1: $y + 3 = 6 \Rightarrow y = 3$

Case 2: $y + 3 = -6 \Rightarrow y = -9$

Verification: $y = 3$: $PQ = \sqrt{[64+36]} = \sqrt{100} = 10$ ✓

$y = -9$: $PQ = \sqrt{[64+36]} = \sqrt{100} = 10$ ✓

Answer: $y = 3$ or $y = -9$

Q9 If Q(0, 1) is equidistant from P(5, −3) and R(x, 6), find the values of x. Also find the distances QR and PR.

SOLUTION

Given: $QP = QR$, $Q = (0, 1)$, $P = (5, -3)$, $R = (x, 6)$.

$QP = \sqrt{[(5-0)^2 + (-3-1)^2]} = \sqrt{[25+16]} = \sqrt{41}$

$QR = \sqrt{[x^2 + (6-1)^2]} = \sqrt{[x^2+25]}$

Setting $QP = QR$:

$\sqrt{41} = \sqrt{(x^2+25)}$

$41 = x^2 + 25$

$x^2 = 16$

$x = \pm 4$

QR = $\sqrt{41}$ (same as QP by construction).

For x = 4: R = (4, 6)

PR = $\sqrt{[(4-5)^2 + (6+3)^2]} = \sqrt{[1+81]} = \sqrt{82}$

For x = −4: R = (−4, 6)

PR = $\sqrt{[(-4-5)^2 + (6+3)^2]} = \sqrt{[81+81]} = \sqrt{162} = 9\sqrt{2}$

Answer: x = 4 or x = −4; QR = $\sqrt{41}$; PR = $\sqrt{82}$ (when x = 4) or $9\sqrt{2}$ (when x = −4)

Q10 Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (−3, 4).

SOLUTION

Let P = (x, y), A = (3, 6), B = (−3, 4).

Given: PA = PB, so $PA^2 = PB^2$.

$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$

Expanding:

$x^2-6x+9 + y^2-12y+36 = x^2+6x+9 + y^2-8y+16$

Cancelling x^2 and y^2:

$-6x - 12y + 45 = 6x - 8y + 25$

$-12x - 4y + 20 = 0$

Dividing by −4:

$3x + y - 5 = 0$

∴ The required relation is 3x + y = 5.

Answer: 3x + y = 5

Exercise 7.2 — Section Formula

Q1 Find the coordinates of the point which divides the join of (−1, 7) and (4, −3) in the ratio 2 : 3.

SOLUTION

Section formula: P dividing (x_1, y_1) and (x_2, y_2) in ratio m : n gives
$x = (m \cdot x_2 + n \cdot x_1)/(m+n)$, $y = (m \cdot y_2 + n \cdot y_1)/(m+n)$

$(x_1, y_1) = (-1, 7)$, $(x_2, y_2) = (4, -3)$, m : n = 2 : 3

$x = (2\times4 + 3\times(-1))/(2+3) = (8-3)/5 = 5/5 = 1$
$y = (2\times(-3) + 3\times7)/(2+3) = (-6+21)/5 = 15/5 = 3$

∴ The required point is (1, 3).

Answer: (1, 3)

Q2 Find the coordinates of the points of trisection of the line segment joining (4, −1) and (−2, −3).

SOLUTION

Let A = (4, −1) and B = (−2, −3).
Trisection points P and Q divide AB into three equal parts.

P divides AB in ratio 1 : 2 (from A):
$x_P = (1\times(-2) + 2\times4)/(1+2) = (-2+8)/3 = 6/3 = 2$
$y_P = (1\times(-3) + 2\times(-1))/(1+2) = (-3-2)/3 = -5/3$
=> P = (2, −5/3)

Q divides AB in ratio 2 : 1 (from A):
$x_Q = (2\times(-2) + 1\times4)/(2+1) = (-4+4)/3 = 0$
$y_Q = (2\times(-3) + 1\times(-1))/(2+1) = (-6-1)/3 = -7/3$
=> Q = (0, −7/3)

∴ The points of trisection are (2, −5/3) and (0, −7/3).

Answer: (2, −5/3) and (0, −7/3)

Q3 To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD. Niharika runs 1/4 th the distance AD on the 2nd line and posts a green flag. Preet runs 1/5 th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

SOLUTION

AD = 100 m (100 flower pots at 1 m intervals).

Niharika (green flag): 2nd line, 1/4 of AD = 25 m => G = (2, 25)
Preet (red flag): 8th line, 1/5 of AD = 20 m => R = (8, 20)

Distance between flags:
GR = $\sqrt{[(8-2)^2 + (20-25)^2]} = \sqrt{[36+25]} = \sqrt{61}$ m

Rashmi's flag (midpoint of GR):
x = (2+8)/2 = 5
y = (25+20)/2 = 45/2 = 22.5

∴ Rashmi posts her blue flag at (5, 22.5) — on the 5th line, 22.5 m from AD.

Answer: Distance = $\sqrt{61}$ m; Rashmi's flag at (5, 22.5) i.e. 5th line, 22.5 m from AD

Q4 Find the ratio in which the line segment joining the points (−3, 10) and (6, −8) is divided by (−1, 6).

SOLUTION

Let (−1, 6) divide A(−3, 10) and B(6, −8) in ratio k : 1.

Using section formula for x-coordinate:
(6k + (−3))/(k+1) = −1
6k − 3 = −k − 1
7k = 2
k = 2/7

Verification with y-coordinate:
y = (−8×(2/7) + 10)/((2/7)+1) = (−16/7 + 70/7)/(9/7) = (54/7)×(7/9) = 6 ✓

∴ The ratio is 2 : 7.

Answer: 2 : 7

Q5 Find the ratio in which the line segment joining A(1, −5) and B(−4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

SOLUTION

Let the x-axis divide AB in ratio k : 1.
A point on the x-axis has y = 0.

y-coordinate by section formula:
(k×5 + 1×(−5))/(k+1) = 0
5k − 5 = 0
k = 1

∴ The ratio is 1 : 1.

x-coordinate:
x = (1×(−4) + 1×1)/(1+1) = (−4+1)/2 = −3/2

∴ The point of division is (−3/2, 0).

Answer: Ratio = 1 : 1; Point of division = (−3/2, 0)

Q6 If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

SOLUTION

Let A = (1, 2), B = (4, y), C = (x, 6), D = (3, 5).

Diagonals of a parallelogram bisect each other.
=> Midpoint of AC = Midpoint of BD

Midpoint of AC = ((1+x)/2, (2+6)/2) = ((1+x)/2, 4)
Midpoint of BD = ((4+3)/2, (y+5)/2) = (7/2, (y+5)/2)

Equating x-coordinates: (1+x)/2 = 7/2 => x = 6
Equating y-coordinates: 4 = (y+5)/2 => y = 3

∴ x = 6 and y = 3.

Answer: x = 6, y = 3

Q7 Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, −3) and B is (1, 4).

SOLUTION

The centre is the midpoint of any diameter.
Let A = (x, y), B = (1, 4), Centre = (2, −3).

Midpoint of AB = Centre:
(x+1)/2 = 2 => x = 3
(y+4)/2 = −3 => y = −10

Verification: midpoint of (3,−10) and (1,4) = (2, −3) ✓

∴ A = (3, −10).

Answer: (3, −10)

Q8 If A and B are (−2, −2) and (2, −4), respectively, find the coordinates of P such that AP = (3/7)AB and P lies on the line segment AB.

SOLUTION

A = (−2, −2), B = (2, −4)
AP = (3/7)AB => AP/AB = 3/7

PB = AB − AP = (4/7)AB
=> AP : PB = 3 : 4

P divides AB in ratio 3 : 4:
x = (3×2 + 4×(−2))/(3+4) = (6−8)/7 = −2/7
y = (3×(−4) + 4×(−2))/(3+4) = (−12−8)/7 = −20/7

∴ P = (−2/7, −20/7).

Answer: (−2/7, −20/7)

Q9 Find the coordinates of the points which divide the line segment joining A(−2, 2) and B(2, 8) into four equal parts.

SOLUTION

A = (−2, 2), B = (2, 8). Three points P_1, P_2, P_3 divide AB into 4 equal parts.

P_1 divides AB in ratio 1 : 3:
x = (1×2 + 3×(−2))/4 = (2−6)/4 = −1

$y = (1\times8 + 3\times2)/4 = (8+6)/4 = 14/4 = 7/2$

$P_1 = (-1, 7/2)$

P_2 is the midpoint (ratio 2 : 2):

$x = (-2+2)/2 = 0$, $y = (2+8)/2 = 5$

$P_2 = (0, 5)$

P_3 divides AB in ratio 3 : 1:

$x = (3\times2 + 1\times(-2))/4 = (6-2)/4 = 1$

$y = (3\times8 + 1\times2)/4 = (24+2)/4 = 26/4 = 13/2$

$P_3 = (1, 13/2)$

$\therefore$ The three points are $(-1, 7/2)$, $(0, 5)$ and $(1, 13/2)$.

Answer: $(-1, 7/2)$, $(0, 5)$ and $(1, 13/2)$

Q10 Find the area of a rhombus if its vertices are (3, 0), (4, 5), (−1, 4) and (−2, −1) taken in order.
[Hint: Area of a rhombus = (1/2) × (product of its diagonals)]

SOLUTION

A = (3, 0), B = (4, 5), C = (−1, 4), D = (−2, −1).

Diagonals of the rhombus are AC and BD.

$AC = \sqrt{[(-1-3)^2 + (4-0)^2]} = \sqrt{[16+16]} = \sqrt{32} = 4\sqrt{2}$

$BD = \sqrt{[(-2-4)^2 + (-1-5)^2]} = \sqrt{[36+36]} = \sqrt{72} = 6\sqrt{2}$

Area = (1/2) × AC × BD

$= (1/2) \times 4\sqrt{2} \times 6\sqrt{2}$

$= (1/2) \times 24 \times 2$

$= (1/2) \times 48 = 24$ sq. units

Answer: 24 sq. units