Chapter 8: Introduction to Trigonometry

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Exercise 8.1 — Trigonometric Ratios

Q1 In △ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine: (i) sin A, cos A (ii) sin C, cos C

SOLUTION

Using the Pythagorean theorem: $AC^2 = AB^2 + BC^2 = 24^2 + 7^2 = 576 + 49 = 625$
=> AC = 25 cm

(i) For angle A: opposite side = BC = 7, adjacent = AB = 24, hypotenuse = AC = 25
sin A = BC/AC = 7/25
cos A = AB/AC = 24/25

(ii) For angle C: opposite side = AB = 24, adjacent = BC = 7, hypotenuse = AC = 25
sin C = AB/AC = 24/25
cos C = BC/AC = 7/25

Answer: sin A = 7/25, cos A = 24/25; sin C = 24/25, cos C = 7/25

Q2 In Fig. 8.13, find tan P – cot R. (Triangle PQR right-angled at Q, with PQ = 12 cm, QR = 5 cm, PR = 13 cm)

SOLUTION

In △PQR, right-angled at Q: PQ = 12, QR = 5
PR = $\sqrt{(PQ^2 + QR^2)} = \sqrt{(144 + 25)} = \sqrt{169} = 13$ cm

For angle P: opposite = QR = 5, adjacent = PQ = 12
tan P = QR/PQ = 5/12

For angle R: opposite = PQ = 12, adjacent = QR = 5
cot R = QR/PQ = 5/12

tan P – cot R = 5/12 – 5/12 = 0

Answer: tan P – cot R = 0

Q3 If sin A = 3/4, calculate cos A and tan A.

SOLUTION

Given: sin A = 3/4, so opposite = 3k, hypotenuse = 4k for some positive k.

By Pythagoras: adjacent² = hypotenuse² – opposite² = 16k² – 9k² = 7k²
=> adjacent = √7 · k

cos A = adjacent/hypotenuse = √7k / 4k = √7/4

tan A = sin A / cos A = (3/4) / (√7/4) = 3/√7 = 3√7/7

Answer: cos A = √7/4, tan A = 3/√7

Q4 Given 15 cot A = 8, find sin A and sec A.

SOLUTION

15 cot A = 8 => cot A = 8/15 => tan A = 15/8
So opposite = 15k, adjacent = 8k for some positive k.

By Pythagoras: hypotenuse² = 15² + 8² = 225 + 64 = 289
=> hypotenuse = 17k

sin A = opposite/hypotenuse = 15/17
sec A = hypotenuse/adjacent = 17/8

Answer: sin A = 15/17, sec A = 17/8

Q5 Given sec θ = 13/12, calculate all other trigonometric ratios.

SOLUTION

sec θ = 13/12 => cos θ = 12/13 => hypotenuse = 13k, adjacent = 12k.

By Pythagoras: opposite² = 13² – 12² = 169 – 144 = 25
=> opposite = 5k

sin θ = 5/13
cos θ = 12/13
tan θ = sin θ / cos θ = 5/12

cot θ = 12/5
cosec θ = 13/5
sec θ = 13/12 (given)

Answer: sin θ = 5/13, cos θ = 12/13, tan θ = 5/12, cot θ = 12/5, cosec θ = 13/5

Q6 If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

SOLUTION

Consider right triangles with the same hypotenuse (or set up triangles ABC and PQR).

If cos A = cos B, then:
AB/AC = PQ/PR (adjacent/hypotenuse in respective triangles)
=> AB/PQ = AC/PR = k (say)

Also by Pythagoras: BC = √(AC² – AB²) and QR = √(PR² – PQ²)
BC/QR = √(AC² – AB²) / √(PR² – PQ²) = √(k²PR² – k²PQ²) / √(PR² – PQ²) = k

So AB/PQ = AC/PR = BC/QR
=> △ABC ~ △PQR (by SSS similarity)
=> ∠A = ∠B

Answer: ∠A = ∠B (proved using similar triangles)

Q7 If cot θ = 7/8, evaluate: (i) (1 + sin θ)(1 – sin θ) / [(1 + cos θ)(1 – cos θ)] (ii) cot² θ

SOLUTION

cot θ = 7/8 => adjacent = 7k, opposite = 8k
hypotenuse = √(8² + 7²) = √(64 + 49) = √113
sin θ = 8/√113, cos θ = 7/√113

(i) (1 + sin θ)(1 – sin θ) / [(1 + cos θ)(1 – cos θ)]
= (1 – sin²θ) / (1 – cos²θ)
= cos²θ / sin²θ
= cot²θ
= (7/8)² = 49/64

(ii) cot²θ = (7/8)² = 49/64

Answer: (i) 49/64 (ii) 49/64

Q8 If 3 cot A = 4, check whether (1 – tan²A) / (1 + tan²A) = cos²A – sin²A or not.

SOLUTION

3 cot A = 4 => cot A = 4/3 => tan A = 3/4

Triangle: opposite = 3, adjacent = 4, hypotenuse = 5

sin A = 3/5, cos A = 4/5

LHS = (1 – tan²A) / (1 + tan²A) = (1 – 9/16) / (1 + 9/16) = (7/16) / (25/16) = 7/25

RHS = cos²A – sin²A = (4/5)² – (3/5)² = 16/25 – 9/25 = 7/25

LHS = RHS = 7/25, so the identity holds.

Answer: Yes, (1 – tan²A)/(1 + tan²A) = cos²A – sin²A = 7/25

Q9 In triangle ABC, right-angled at B, if tan A = 1/√3, find the value of: (i) sin A cos C + cos A sin C (ii) cos A cos C – sin A sin C

SOLUTION

tan A = 1/√3 => A = 30°, and since A + C = 90°, C = 60°.

sin A = 1/2, cos A = √3/2, sin C = √3/2, cos C = 1/2

(i) sin A cos C + cos A sin C

= (1/2)(1/2) + (√3/2)(√3/2)

= 1/4 + 3/4 = 1

(This equals sin(A + C) = sin 90° = 1)

(ii) cos A cos C – sin A sin C

= (√3/2)(1/2) – (1/2)(√3/2)

= √3/4 – √3/4 = 0

(This equals cos(A + C) = cos 90° = 0)

Answer: (i) 1 (ii) 0

Q10 In △PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

SOLUTION

Let QR = x cm, then PR = (25 – x) cm.

By Pythagoras: $PR^2 = PQ^2 + QR^2$

$(25 – x)^2 = 5^2 + x^2$

$625 – 50x + x^2 = 25 + x^2$

$600 = 50x$

x = 12, so QR = 12 cm and PR = 13 cm

Verify: $PR^2 = 13^2 = 169$, $PQ^2 + QR^2 = 25 + 144 = 169$ ✓

For angle P: opposite = QR = 12, adjacent = PQ = 5, hypotenuse = PR = 13

sin P = 12/13

cos P = 5/13

tan P = 12/5

Answer: sin P = 12/13, cos P = 5/13, tan P = 12/5

Q11 State whether the following are true or false. Justify your answer. (i) The value of tan A is always less than 1. (ii) sec A = 12/5 for some value of angle A. (iii) cos A is the abbreviation used for the cosecant of angle A. (iv) cot A is the product of cot and A. (v) sin θ = 4/3 for some angle θ.

SOLUTION

(i) FALSE. tan A = opposite/adjacent. When the opposite side is greater than the adjacent side, tan A > 1. For example, tan 60° = √3 > 1.

(ii) TRUE. sec A = hypotenuse/adjacent ≥ 1 for any acute angle A. Since 12/5 = 2.4 > 1, there exists an angle A where sec A = 12/5.

(iii) FALSE. cos A stands for cosine of angle A, not cosecant. The abbreviation for cosecant is cosec A.

(iv) FALSE. cot A is not a product of 'cot' and A; 'cot' has no independent meaning separate from the angle it modifies. cot A is the cotangent of angle A.

(v) FALSE. sin θ = perpendicular/hypotenuse. Since the hypotenuse is always the longest side, sin θ ≤ 1 for any angle θ. 4/3 > 1, so sin θ = 4/3 is impossible.

Answer: (i) False (ii) True (iii) False (iv) False (v) False

Exercise 8.2 — Trigonometric Ratios of Specific Angles

Q1 Evaluate the following: (i) $\sin 60° \cos 30° + \sin 30° \cos 60°$ (ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$ (iii) $\cos 45° / (\sec 30° + \operatorname{cosec} 30°)$ (iv) $(\sin 30° + \tan 45° - \operatorname{cosec} 60°) / (\sec 30° + \cos 60° + \cot 45°)$ (v) $(5\cos^2 60° + 4\sec^2 30° - \tan^2 45°) / (\sin^2 30° + \cos^2 30°)$

SOLUTION

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$
$= (\sqrt{3}/2)(\sqrt{3}/2) + (1/2)(1/2)$
$= 3/4 + 1/4 = 1$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$
$= 2(1)^2 + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2$
$= 2 + 3/4 - 3/4 = 2$

(iii) $\cos 45° / (\sec 30° + \operatorname{cosec} 30°)$
$= (1/\sqrt{2}) / (2/\sqrt{3} + 2)$
$= (1/\sqrt{2}) / [2(1/\sqrt{3} + 1)]$
$= (1/\sqrt{2}) / [2(\sqrt{3} + 1)/\sqrt{3}]$
$= \sqrt{3} / [2\sqrt{2}(\sqrt{3} + 1)]$
Rationalising by multiplying by $(\sqrt{3} - 1)/(\sqrt{3} - 1)$:
$= \sqrt{3}(\sqrt{3} - 1) / [2\sqrt{2}(3 - 1)]$
$= (3 - \sqrt{3}) / (4\sqrt{2})$

(iv) $(\sin 30° + \tan 45° - \operatorname{cosec} 60°) / (\sec 30° + \cos 60° + \cot 45°)$
$= (1/2 + 1 - 2/\sqrt{3}) / (2/\sqrt{3} + 1/2 + 1)$
Numerator $= 3/2 - 2/\sqrt{3} = (3\sqrt{3} - 4) / (2\sqrt{3})$
Denominator $= 3/2 + 2/\sqrt{3} = (3\sqrt{3} + 4) / (2\sqrt{3})$
$= (3\sqrt{3} - 4) / (3\sqrt{3} + 4)$

(v) $(5\cos^2 60° + 4\sec^2 30° - \tan^2 45°) / (\sin^2 30° + \cos^2 30°)$
$= (5 \times 1/4 + 4 \times 4/3 - 1) / 1$
$= 5/4 + 16/3 - 1$
$= 15/12 + 64/12 - 12/12$
$= 67/12$

Answer: (i) 1 (ii) 2 (iii) $(3 - \sqrt{3})/(4\sqrt{2})$ (iv) $(3\sqrt{3} - 4)/(3\sqrt{3} + 4)$ (v) 67/12

Q2 Choose the correct option and justify your choice: (i) $2 \tan 30°/(1 – \tan^2 30°)$ = (A) sin 60° (B) cos 60° (C) tan 60° (D) sin 30° (ii) $(1 – \tan^2 45°)/(1 + \tan^2 45°)$ = (A) tan 90° (B) 1 (C) sin 45° (D) 0 (iii) sin 2A = 2 sin A is true when A = (A) 0° (B) 30° (C) 45° (D) 60° (iv) $2 \tan 30°/(1 + \tan^2 30°)$ = (A) cos 60° (B) sin 60° (C) tan 60° (D) sin 30°

SOLUTION

(i) $2 \tan 30° / (1 – \tan^2 30°)$
$\tan 30° = 1/\sqrt{3}$, so $\tan^2 30° = 1/3$
$= 2(1/\sqrt{3}) / (1 – 1/3) = (2/\sqrt{3}) / (2/3) = (2/\sqrt{3}) \times (3/2) = 3/\sqrt{3} = \sqrt{3}$
This equals $\tan 60° = \sqrt{3}$
Answer: (C) tan 60°

(ii) $(1 – \tan^2 45°) / (1 + \tan^2 45°)$
$\tan 45° = 1$, so $\tan^2 45° = 1$
$= (1 – 1) / (1 + 1) = 0/2 = 0$
Answer: (D) 0

(iii) $\sin 2A = 2 \sin A$
$2 \sin A \cos A = 2 \sin A$
$2 \sin A \cos A – 2 \sin A = 0$
$2 \sin A (\cos A – 1) = 0$
$\Rightarrow \sin A = 0$ or $\cos A = 1 \Rightarrow A = 0°$
Answer: (A) 0°

(iv) $2 \tan 30° / (1 + \tan^2 30°)$
$= 2(1/\sqrt{3}) / (1 + 1/3) = (2/\sqrt{3}) / (4/3) = (2/\sqrt{3}) \times (3/4) = 6/(4\sqrt{3}) = 3/(2\sqrt{3}) = \sqrt{3}/2$
This equals $\sin 60° = \sqrt{3}/2$
Answer: (B) sin 60°

Answer: (i) C (ii) D (iii) A (iv) B

Q3 If $\tan (A + B) = \sqrt{3}$ and $\tan (A – B) = 1/\sqrt{3}$; $0° < A + B \leq 90°$; $A > B$, find A and B.

SOLUTION

$\tan (A + B) = \sqrt{3} = \tan 60° \Rightarrow A + B = 60°$ ... (1)
$\tan (A – B) = 1/\sqrt{3} = \tan 30° \Rightarrow A – B = 30°$ ... (2)

Adding (1) and (2): $2A = 90° \Rightarrow A = 45°$
Subtracting (2) from (1): $2B = 30° \Rightarrow B = 15°$

Verification: A + B = 60° ≤ 90° ✓, A > B (45° > 15°) ✓

Answer: A = 45°, B = 15°

Q4 State whether the following are true or false. Justify your answer. (i) sin (A + B) = sin A + sin B. (ii) The value of sin θ increases as θ increases. (iii) The value of cos θ increases as θ increases. (iv) sin θ = cos θ for all values of θ. (v) cot A is not defined for A = 0°.

SOLUTION

(i) FALSE. For example, sin (30° + 60°) = sin 90° = 1, but sin 30° + sin 60° = 1/2 + √3/2 ≠ 1.

(ii) TRUE. As θ increases from 0° to 90°, sin θ increases from 0 to 1. sin 0° = 0 < sin 30° = 0.5 < sin 60° ≈ 0.866 < sin 90° = 1.

(iii) FALSE. As θ increases from 0° to 90°, cos θ decreases from 1 to 0. cos 0° = 1 > cos 30° ≈ 0.866 > cos 60° = 0.5 > cos 90° = 0.

(iv) FALSE. sin θ = cos θ only when θ = 45°. For example, sin 30° = 1/2 but cos 30° = √3/2.

(v) TRUE. cot A = cos A / sin A. At A = 0°, sin 0° = 0, so cot 0° = cos 0°/sin 0° = 1/0, which is undefined.

Answer: (i) False (ii) True (iii) False (iv) False (v) True

Exercise 8.3 — Trigonometric Identities

Q1 Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

SOLUTION

Using the identity: $\text{cosec}^2A = 1 + \cot^2A$

$\Rightarrow 1/\sin^2A = 1 + \cot^2A$

$\Rightarrow \sin^2A = 1 / (1 + \cot^2A)$

$\Rightarrow \sin A = 1 / \sqrt{(1 + \cot^2A)}$ (positive since A is acute)

Since $\cos A = \sin A \cdot \cot A$:

$\cos A = \cot A / \sqrt{(1 + \cot^2A)}$

$\sec A = 1/\cos A = \sqrt{(1 + \cot^2A)} / \cot A$

$\tan A = 1/\cot A$

Summary:

$\sin A = 1/\sqrt{(1 + \cot^2A)}$

$\sec A = \sqrt{(1 + \cot^2A)} / \cot A$

$\tan A = 1/\cot A$

Answer: $\sin A = 1/\sqrt{(1 + \cot^2A)}$, $\sec A = \sqrt{(1 + \cot^2A)}/\cot A$, $\tan A = 1/\cot A$

Q2 Write all the other trigonometric ratios of angle A in terms of sec A.

SOLUTION

Given: sec A

$\cos A = 1/\sec A$

Using $\sin^2A + \cos^2A = 1$:

$\sin^2A = 1 - \cos^2A = 1 - 1/\sec^2A = (\sec^2A - 1)/\sec^2A$

$\sin A = \sqrt{(\sec^2A - 1)} / \sec A$

$\tan^2A = \sec^2A - 1 \Rightarrow \tan A = \sqrt{(\sec^2A - 1)}$

$\cot A = 1/\tan A = 1/\sqrt{(\sec^2A - 1)}$

$\text{cosec } A = 1/\sin A = \sec A / \sqrt{(\sec^2A - 1)}$

Summary:

$\cos A = 1/\sec A$

$\sin A = \sqrt{(\sec^2A - 1)} / \sec A$

$\tan A = \sqrt{(\sec^2A - 1)}$

cot A = 1/√(sec²A – 1)

cosec A = sec A / √(sec²A – 1)

Answer: cos A = 1/sec A, sin A = √(sec²A–1)/sec A, tan A = √(sec²A–1), cot A = 1/√(sec²A–1), cosec A = sec A/√(sec²A–1)

Q3 Choose the correct option. Justify your choice. (i) 9 sec²A – 9 tan²A = (A) 1 (B) 9 (C) 8 (D) 0 (ii) (1 + tan θ + sec θ)(1 + cot θ – cosec θ) = (A) 0 (B) 1 (C) 2 (D) –1 (iii) (sec A + tan A)(1 – sin A) = (A) sec A (B) sin A (C) cosec A (D) cos A (iv) (1 + tan²A) / (1 + cot²A) = (A) sec²A (B) –1 (C) cot²A (D) tan²A

SOLUTION

(i) 9 sec²A – 9 tan²A = 9(sec²A – tan²A) = 9 × 1 = 9

Answer: (B) 9

(ii) (1 + tan θ + sec θ)(1 + cot θ – cosec θ)

= (1 + sin θ/cos θ + 1/cos θ)(1 + cos θ/sin θ – 1/sin θ)

= [(cos θ + sin θ + 1)/cos θ] × [(sin θ + cos θ – 1)/sin θ]

= (cos θ + sin θ + 1)(cos θ + sin θ – 1) / (sin θ cos θ)

= [(cos θ + sin θ)² – 1] / (sin θ cos θ)

= [cos²θ + 2 sin θ cos θ + sin²θ – 1] / (sin θ cos θ)

= [1 + 2 sin θ cos θ – 1] / (sin θ cos θ)

= 2 sin θ cos θ / (sin θ cos θ) = 2

Answer: (C) 2

(iii) (sec A + tan A)(1 – sin A)

= (1/cos A + sin A/cos A)(1 – sin A)

= [(1 + sin A)/cos A] × (1 – sin A)

= (1 – sin²A) / cos A

= cos²A / cos A = cos A

Answer: (D) cos A

(iv) (1 + tan²A) / (1 + cot²A)

= sec²A / cosec²A

= (1/cos²A) / (1/sin²A)

= sin²A / cos²A = tan²A

Answer: (D) tan²A

Answer: (i) B (ii) C (iii) D (iv) D

Q4 Prove the following identities, where the angles involved are acute angles for which the expressions are defined: (i) $(\text{cosec}\,\theta - \cot\theta)^2 = (1 - \cos\theta)/(1 + \cos\theta)$ (ii) $\cos A/(1 + \sin A) + (1 + \sin A)/\cos A = 2\sec A$ (iii) $\tan\theta/(1 - \cot\theta) + \cot\theta/(1 - \tan\theta) = 1 + \sec\theta\,\text{cosec}\,\theta$ [Hint: Write the expression in terms of sin θ and cos θ] (iv) $(1 + \sec A)/\sec A = \sin^2 A/(1 - \cos A)$ [Hint: Simplify LHS and RHS separately] (v) $(\cos A - \sin A + 1)/(\cos A + \sin A - 1) = \text{cosec}\,A + \cot A$, using the identity $\text{cosec}^2 A = 1 + \cot^2 A$ (vi) $\sqrt{[(1 + \sin A)/(1 - \sin A)]} = \sec A + \tan A$ (vii) $(\sin\theta - 2\sin^3\theta)/(2\cos^3\theta - \cos\theta) = \tan\theta$ (viii) $(\sin A + \text{cosec}\,A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$ (ix) $(\text{cosec}\,A - \sin A)(\sec A - \cos A) = 1/(\tan A + \cot A)$ [Hint: Simplify LHS and RHS separately] (x) $(1 + \tan^2 A)/(1 + \cot^2 A) = [(1 - \tan A)/(1 - \cot A)]^2 = \tan^2 A$

SOLUTION

(i) LHS = $(\text{cosec}\,\theta - \cot\theta)^2$
$= (1/\sin\theta - \cos\theta/\sin\theta)^2$
$= [(1 - \cos\theta)/\sin\theta]^2$
$= (1 - \cos\theta)^2/\sin^2\theta$
$= (1 - \cos\theta)^2/(1 - \cos^2\theta)$
$= (1 - \cos\theta)^2/[(1 - \cos\theta)(1 + \cos\theta)]$
$= (1 - \cos\theta)/(1 + \cos\theta) = \text{RHS}$

(ii) LHS = $\cos A/(1 + \sin A) + (1 + \sin A)/\cos A$

Taking LCM = $\cos A(1 + \sin A)$:
$= [\cos^2 A + (1 + \sin A)^2]/[\cos A(1 + \sin A)]$
$= [\cos^2 A + 1 + 2\sin A + \sin^2 A]/[\cos A(1 + \sin A)]$
$= [1 + 1 + 2\sin A]/[\cos A(1 + \sin A)]$
(using $\sin^2 A + \cos^2 A = 1$)
$= [2 + 2\sin A]/[\cos A(1 + \sin A)]$
$= 2(1 + \sin A)/[\cos A(1 + \sin A)]$
$= 2/\cos A = 2\sec A = \text{RHS}$

(iii) Let $s = \sin\theta$, $c = \cos\theta$.

LHS = $(s/c)/(1 - c/s) + (c/s)/(1 - s/c)$
$= (s/c)/[(s - c)/s] + (c/s)/[(c - s)/c]$
$= s^2/[c(s - c)] + c^2/[s(c - s)]$
$= s^2/[c(s - c)] - c^2/[s(s - c)]$
$= [s^3 - c^3]/[cs(s - c)]$

Factor numerator using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$:

$= (s - c)(s^2 + sc + c^2) / [cs(s - c)]$
$= (s^2 + sc + c^2) / (cs)$
$= s^2/(cs) + sc/(cs) + c^2/(cs)$
$= s/c + 1 + c/s$
$= \tan\theta + 1 + \cot\theta$
$= 1 + \sin\theta/\cos\theta + \cos\theta/\sin\theta$
$= 1 + (\sin^2\theta + \cos^2\theta)/(\sin\theta\cos\theta)$
$= 1 + 1/(\sin\theta\cos\theta)$
$= 1 + (1/\cos\theta)(1/\sin\theta)$
$= 1 + \sec\theta\,\text{cosec}\,\theta = \text{RHS}$

(iv) LHS $= (1 + \sec A) / \sec A$
$= 1/\sec A + 1$
$= \cos A + 1$
$= 1 + \cos A$

RHS $= \sin^2 A / (1 - \cos A)$
$= (1 - \cos^2 A) / (1 - \cos A)$
$= (1 - \cos A)(1 + \cos A) / (1 - \cos A)$
$= 1 + \cos A$

LHS = RHS = $1 + \cos A$ ✓

(v) Divide numerator and denominator by sin A:
LHS $= (\cot A - 1 + \text{cosec}\,A) / (\cot A + 1 - \text{cosec}\,A)$
$= (\text{cosec}\,A + \cot A - 1) / (\cot A - \text{cosec}\,A + 1)$

Use $\text{cosec}^2 A - \cot^2 A = 1$, i.e., $(\text{cosec}\,A - \cot A)(\text{cosec}\,A + \cot A) = 1$,
so $1 = (\text{cosec}\,A - \cot A)(\text{cosec}\,A + \cot A)$.

Replace 1 in denominator:
Denominator $= \cot A - \text{cosec}\,A + (\text{cosec}\,A - \cot A)(\text{cosec}\,A + \cot A)$
$= (\cot A - \text{cosec}\,A)[1 - (\text{cosec}\,A + \cot A)]$
$= -(\text{cosec}\,A - \cot A)[1 - (\text{cosec}\,A + \cot A)]$

Let $k = \text{cosec}\,A + \cot A$. Numerator $= k - 1$.
Denominator $= -(\text{cosec}\,A - \cot A)(1 - k) = (\text{cosec}\,A - \cot A)(k - 1)$

LHS $= (k - 1) / [(\text{cosec}\,A - \cot A)(k - 1)]$
$= 1 / (\text{cosec}\,A - \cot A)$

Multiply numerator and denominator by $(\text{cosec}\,A + \cot A)$:
$= (\text{cosec}\,A + \cot A) / (\text{cosec}^2 A - \cot^2 A)$

= (cosec A + cot A) / 1 = cosec A + cot A = RHS

(vi) LHS = $\sqrt{[(1 + \sin A) / (1 – \sin A)]}$

Multiply numerator and denominator inside the root by (1 + sin A):
= $\sqrt{[(1 + \sin A)^2 / (1 – \sin^2 A)]}$
= $\sqrt{[(1 + \sin A)^2 / \cos^2 A]}$
= (1 + sin A) / cos A (positive since A is acute)
= 1/cos A + sin A/cos A
= sec A + tan A = RHS

(vii) LHS = $(\sin \theta – 2 \sin^3\theta) / (2 \cos^3\theta – \cos \theta)$

Factor numerator: $\sin \theta(1 – 2 \sin^2\theta)$
Factor denominator: $\cos \theta(2 \cos^2\theta – 1)$

Note: $1 – 2 \sin^2\theta = 1 – 2(1 – \cos^2\theta) = 2 \cos^2\theta – 1$

So LHS = $\sin \theta(2 \cos^2\theta – 1) / [\cos \theta(2 \cos^2\theta – 1)]$
= sin θ / cos θ
= tan θ = RHS

(viii) LHS = $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$
= $\sin^2 A + 2 \sin A \text{ cosec } A + \text{cosec}^2 A + \cos^2 A + 2 \cos A \sec A + \sec^2 A$
= $\sin^2 A + 2(1) + \text{cosec}^2 A + \cos^2 A + 2(1) + \sec^2 A$
(since $\sin A \cdot \text{cosec } A = 1$ and $\cos A \cdot \sec A = 1$)
= $(\sin^2 A + \cos^2 A) + 4 + \text{cosec}^2 A + \sec^2 A$
= $1 + 4 + \text{cosec}^2 A + \sec^2 A$
= $5 + (1 + \cot^2 A) + (1 + \tan^2 A)$
(using $\text{cosec}^2 A = 1 + \cot^2 A$ and $\sec^2 A = 1 + \tan^2 A$)
= $5 + 1 + \cot^2 A + 1 + \tan^2 A$
= $7 + \tan^2 A + \cot^2 A$ = RHS

(ix) LHS = (cosec A – sin A)(sec A – cos A)
= (1/sin A – sin A)(1/cos A – cos A)
= $[(1 – \sin^2 A)/\sin A] \times [(1 – \cos^2 A)/\cos A]$
= $(\cos^2 A/\sin A) \times (\sin^2 A/\cos A)$
= sin A cos A

RHS = 1/(tan A + cot A)
= 1/(sin A/cos A + cos A/sin A)
= $1/[(\sin^2 A + \cos^2 A)/(\sin A \cos A)]$
= sin A cos A / 1

= sin A cos A

LHS = RHS = sin A cos A ✓

(x) Part 1: $(1 + \tan^2 A) / (1 + \cot^2 A) = \tan^2 A$

$(1 + \tan^2 A) / (1 + \cot^2 A) = \sec^2 A / \operatorname{cosec}^2 A$
$= (1/\cos^2 A) / (1/\sin^2 A)$
$= \sin^2 A / \cos^2 A = \tan^2 A$ ✓

Part 2: $[(1 – \tan A) / (1 – \cot A)]^2 = \tan^2 A$

$(1 – \tan A) / (1 – \cot A)$
$= (1 – \tan A) / (1 – 1/\tan A)$
$= (1 – \tan A) / [(\tan A – 1)/\tan A]$
$= (1 – \tan A) \times \tan A / (\tan A – 1)$
$= (1 – \tan A) \times \tan A / [–(1 – \tan A)]$
$= –\tan A$

$\Rightarrow [(1 – \tan A)/(1 – \cot A)]^2 = (–\tan A)^2 = \tan^2 A$ ✓

Therefore $(1 + \tan^2 A)/(1 + \cot^2 A) = [(1 – \tan A)/(1 – \cot A)]^2 = \tan^2 A$

Answer: All ten identities are proved.