Chapter 9: Some Applications of Trigonometry

Exercise 9.1 — Heights and Distances

Q1 A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.

SOLUTION

Let AB be the vertical pole and AC be the rope.
Given: AC = 20 m, angle ACB = 30°.

In right triangle ABC:
sin 30° = AB/AC
1/2 = AB/20
AB = 20 × (1/2) = 10 m

∴ The height of the pole is 10 m.

Answer: 10 m

Q2 A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

SOLUTION

Let AB be the original tree. It breaks at point C.
The broken part CB bends to touch the ground at D.
So CD = CB (broken part), angle CDA = 30°, AD = 8 m.

In right triangle ACD:
tan 30° = AC/AD
1/√3 = AC/8
AC = 8/√3 = 8√3/3 m

Also: cos 30° = AD/CD
√3/2 = 8/CD
CD = 16/√3 = 16√3/3 m

Total height of tree = AC + CD
= 8√3/3 + 16√3/3

$= 24\sqrt{3}/3$

$= 8\sqrt{3}$ m

$\therefore$ The height of the tree is $8\sqrt{3}$ m.

Answer: $8\sqrt{3}$ m

Q3 A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

SOLUTION

Case 1: For children below 5 years

Height = 1.5 m, angle of inclination = 30°.

Let the length of slide = l_1

$\sin 30° = 1.5/l_1$

$1/2 = 1.5/l_1$

$l_1 = 1.5 \times 2 = 3$ m

Case 2: For elder children

Height = 3 m, angle of inclination = 60°.

Let the length of slide = l_2

$\sin 60° = 3/l_2$

$\sqrt{3}/2 = 3/l_2$

$l_2 = 6/\sqrt{3} = 6\sqrt{3}/3 = 2\sqrt{3}$ m

$\therefore$ Length of slide for younger children = 3 m and for elder children = $2\sqrt{3}$ m.

Answer: 3 m and $2\sqrt{3}$ m

Q4 The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

SOLUTION

Let AB be the tower and C be the point on the ground.

Given: BC = 30 m, angle ACB = 30°.

In right triangle ABC:

tan 30° = AB/BC
1/√3 = AB/30
AB = 30/√3 = 30√3/3 = 10√3 m

∴ The height of the tower is 10√3 m.

Answer: 10√3 m

Q5 A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

SOLUTION

Let A be the position of the kite and B be the point on the ground where string is tied.
Height of kite above ground = 60 m.
Angle of inclination = 60°.

Let the length of string AB = l.

sin 60° = 60/l
√3/2 = 60/l
l = 120/√3 = 120√3/3 = 40√3 m

∴ The length of the string is 40√3 m.

Answer: 40√3 m

Q6 A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

SOLUTION

Let AB be the building (30 m tall) and the boy's eye level is at 1.5 m.
The effective height to consider = 30 - 1.5 = 28.5 m.

Let C be the initial position and D be the final position of the boy.
Let DB = y (distance from building at final position).
Let CD = x (distance walked).
So CB = x + y.

From position D (angle of elevation = 60°):
tan 60° = 28.5/y
√3 = 28.5/y
y = 28.5/√3 = 9.5√3 m

From position C (angle of elevation = 30°):
tan 30° = 28.5/(x + y)
1/√3 = 28.5/(x + y)
x + y = 28.5√3 m

Distance walked = x = (x + y) - y
= 28.5√3 - 9.5√3
= 19√3 m

∴ The boy walked 19√3 m towards the building.

Answer: 19√3 m

Q7 From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower on the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

SOLUTION

Let AB be the building (20 m) and BC be the transmission tower on top of it.
Let D be the point on the ground.

Angle of elevation of bottom of tower (B) = 45°.
Angle of elevation of top of tower (C) = 60°.
Let AD = d (horizontal distance).

From triangle ABD:
tan 45° = AB/AD
1 = 20/d
d = 20 m

From triangle ACD:
tan 60° = AC/AD
√3 = AC/20
AC = 20√3 m

Height of tower = BC = AC - AB
= 20√3 - 20

= 20(√3 - 1) m

∴ The height of the transmission tower is 20(√3 - 1) m.

Answer: 20(√3 - 1) m

Q8 A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

SOLUTION

Let AB be the pedestal (height = h) and BC be the statue (1.6 m) on top of it.
Let D be the point on the ground.
Angle of elevation of top of pedestal (B) = 45°.
Angle of elevation of top of statue (C) = 60°.
Let AD = d (horizontal distance).

From triangle ABD:
tan 45° = AB/AD
1 = h/d
d = h ... (i)

From triangle ACD:
tan 60° = AC/AD
√3 = (h + 1.6)/d
√3 d = h + 1.6 ... (ii)

Substituting d = h from (i) into (ii):
√3 h = h + 1.6
√3 h - h = 1.6
h(√3 - 1) = 1.6
h = 1.6/(√3 - 1)

Rationalizing: h = 1.6(√3 + 1)/((√3 - 1)(√3 + 1))
= 1.6(√3 + 1)/(3 - 1)
= 1.6(√3 + 1)/2
= 0.8(√3 + 1) m

∴ The height of the pedestal is 0.8(√3 + 1) m.

Answer: 0.8(√3 + 1) m

Q9 The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

SOLUTION

Let AB be the tower (50 m) and CD be the building (h m).
Let the distance between their feet BD = d.

Angle of elevation of top of building from foot of tower = 30°.
=> tan 30° = CD/BD
=> 1/√3 = h/d
=> d = h√3 ... (i)

Angle of elevation of top of tower from foot of building = 60°.
=> tan 60° = AB/BD
=> √3 = 50/d
=> d = 50/√3 ... (ii)

From (i) and (ii):
h√3 = 50/√3
h = 50/(√3 × √3)
h = 50/3 m

∴ The height of the building is 50/3 m (approximately 16.67 m).

Answer: 50/3 m

Q10 Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

SOLUTION

Let AB and CD be two poles of equal height h, standing on either side of the road.
Let P be the point on the road between them.
Width of road: BD = 80 m.
Let BP = x, then PD = 80 - x.

Angle of elevation of A from P = 60°.
tan 60° = AB/BP
√3 = h/x

$h = x\sqrt{3}$ ... (i)

Angle of elevation of C from P = 30°.
$\tan 30° = CD/PD$
$1/\sqrt{3} = h/(80 - x)$
$h = (80 - x)/\sqrt{3}$ ... (ii)

From (i) and (ii):
$x\sqrt{3} = (80 - x)/\sqrt{3}$
$x\sqrt{3} \times \sqrt{3} = 80 - x$
$3x = 80 - x$
$4x = 80$
$x = 20$ m

From (i): $h = 20\sqrt{3}$ m

BP = 20 m, PD = 80 - 20 = 60 m.

∴ Height of each pole = $20\sqrt{3}$ m. The point is 20 m from one pole and 60 m from the other.

Answer: Height = $20\sqrt{3}$ m; distances = 20 m and 60 m

Q11 A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.

SOLUTION

Let AB be the TV tower (height = h) on one bank.
Let C be the point on the other bank directly opposite (BC = width of canal = d).
Let D be the point 20 m further from C (so CD = 20 m, BD = d + 20).

From point C: angle of elevation = 60°.
$\tan 60° = h/d$
$\sqrt{3} = h/d$
$h = d\sqrt{3}$ ... (i)

From point D: angle of elevation = 30°.
$\tan 30° = h/(d + 20)$
$1/\sqrt{3} = h/(d + 20)$

$h = (d + 20)/\sqrt{3}$ ... (ii)

From (i) and (ii):
$d\sqrt{3} = (d + 20)/\sqrt{3}$
$d\sqrt{3} \times \sqrt{3} = d + 20$
$3d = d + 20$
$2d = 20$
$d = 10$ m

From (i): $h = 10\sqrt{3}$ m

∴ Height of tower = $10\sqrt{3}$ m and width of canal = 10 m.

Answer: Height = $10\sqrt{3}$ m, width of canal = 10 m

Q12 From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

SOLUTION

Let AB be the building (7 m) and CD be the cable tower.
From the top of the building (A), angle of elevation of top of tower (D) = 60°.
From A, angle of depression of foot of tower (C) = 45°.

Draw AE perpendicular to CD (horizontal line from A).
Then AE = BC (horizontal distance) and CE = AB = 7 m.
Let DE = CD - CE = CD - 7.

From angle of depression of C:
$\tan 45° = AB/BC$ (in triangle ABC)
$1 = 7/BC$
$BC = 7$ m
So AE = 7 m.

From angle of elevation of D:
$\tan 60° = DE/AE$
$\sqrt{3} = DE/7$
$DE = 7\sqrt{3}$ m

Height of tower CD = CE + DE = $7 + 7\sqrt{3} = 7(1 + \sqrt{3})$ m

∴ The height of the cable tower is $7(1 + \sqrt{3})$ m.

Answer: $7(1 + \sqrt{3})$ m

Q13 As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

SOLUTION

Let AB be the lighthouse (75 m).
Let C and D be the two ships, with C closer to the lighthouse.
Angle of depression of C = 45°, angle of depression of D = 30°.

In right triangle ABC:
$\tan 45° = AB/BC$
$1 = 75/BC$
$BC = 75$ m

In right triangle ABD:
$\tan 30° = AB/BD$
$1/\sqrt{3} = 75/BD$
$BD = 75\sqrt{3}$ m

Distance between ships = CD = BD - BC
$= 75\sqrt{3} - 75$
$= 75(\sqrt{3} - 1)$ m

∴ The distance between the two ships is $75(\sqrt{3} - 1)$ m.

Answer: $75(\sqrt{3} - 1)$ m

Q14 A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

SOLUTION

Height of girl's eyes = 1.2 m.
Height of balloon from ground = 88.2 m.
Effective height above eye level = 88.2 - 1.2 = 87 m.

Let A be the initial position and B be the final position of the balloon.
Let C be the position of the girl.

Let CD be the horizontal distance to A, and CE be the horizontal distance to B.

When angle of elevation = 60°:
tan 60° = 87/CD
√3 = 87/CD
CD = 87/√3 = 29√3 m

When angle of elevation = 30°:
tan 30° = 87/CE
1/√3 = 87/CE
CE = 87√3 m

Distance travelled by balloon = CE - CD
= 87√3 - 29√3
= 58√3 m

∴ The balloon travelled 58√3 m.

Answer: 58√3 m

Q15 A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

SOLUTION

Let AB be the tower (height = h) and the car moves along the highway towards B. Let C be the initial position of the car and D be its position after 6 seconds.

Angle of depression of C = 30° => angle ACB = 30°.
Angle of depression of D = 60° => angle ADB = 60°.

In triangle ABD:
tan 60° = h/BD
BD = h/√3 ... (i)

In triangle ABC:
tan 30° = h/BC
BC = h√3 ... (ii)

Distance CD = BC - BD = h√3 - h/√3
= h(√3 - 1/√3)

= h(3 - 1)/√3
= 2h/√3

The car covers CD = 2h/√3 in 6 seconds.
Speed = (2h/√3)/6 = h/(3√3)

Time to cover BD = h/√3 from point D to foot of tower:
Time = BD/Speed = (h/√3) / (h/(3√3))
= (h/√3) × (3√3/h)
= 3 seconds

∴ The car takes 3 seconds to reach the foot of the tower from point D.

Answer: 3 seconds

Q16 The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

SOLUTION

Let AB be the tower (height = h).
Let C and D be two points at distances 4 m and 9 m from the base B.
The angles of elevation from C and D are complementary.

Let angle ACB = theta, then angle ADB = 90° - theta.

From point C (at distance 4 m):
tan(theta) = h/4 ... (i)

From point D (at distance 9 m):
tan(90° - theta) = h/9
cot(theta) = h/9
1/tan(theta) = h/9 ... (ii)

Multiplying (i) and (ii):
tan(theta) × 1/tan(theta) = (h/4) × (h/9)
1 = h²/36
h² = 36
h = 6 m (taking positive value)

∴ The height of the tower is 6 m. Hence proved.

Answer: Height of tower = 6 m (proved)