Chapter 10: Circles

CONTENTS 2 exercises · 17 questions

10.1	Tangent to a Circle — Introduction	4 Q
10.2	Properties of Tangents — Tangent Perpendicular to Radius, Equ...	13 Q

Exercise 10.1 — Tangent to a Circle — Introduction

Q1 How many tangents can a circle have?

SOLUTION

A tangent to a circle is a line that touches the circle at exactly one point.

At every point on the circle, there is exactly one tangent.

Since a circle has infinitely many points on its circumference, a circle can have infinitely many tangents.

Answer: A circle can have infinitely many tangents.

Q2 Fill in the blanks:

(i) A tangent to a circle intersects it in ___ point(s).
(ii) A line intersecting a circle in two points is called a ___.
(iii) A circle can have ___ parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called ___.

SOLUTION

(i) A tangent to a circle intersects it in one point.
Reason: By definition, a tangent touches the circle at exactly one point.

(ii) A line intersecting a circle in two points is called a secant.
Reason: A secant is a line that cuts through the circle, entering at one point and exiting at another.

(iii) A circle can have two parallel tangents at the most.
Reason: Parallel tangents can only be drawn at the two ends of a diameter. Any other tangent will be at a different angle.

(iv) The common point of a tangent to a circle and the circle is called the point of contact (or point of tangency).

Answer: (i) one (ii) secant (iii) two (iv) point of contact

Q3 A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:
(A) 12 cm
(B) 13 cm
(C) 8.5 cm
(D) √119 cm

SOLUTION

Given: Circle with centre O, radius OP = 5 cm.
PQ is a tangent at point P, and OQ = 12 cm.

Since PQ is a tangent at P, and OP is a radius,
OP ⊥ PQ (tangent is perpendicular to radius at point of contact).

∴ Triangle OPQ is right-angled at P.

By Pythagoras theorem:
$OQ^2 = OP^2 + PQ^2$
$12^2 = 5^2 + PQ^2$
$144 = 25 + PQ^2$
$PQ^2 = 144 - 25 = 119$
$PQ = \sqrt{119}$ cm

∴ The answer is (D) √119 cm.

Answer: (D) √119 cm

Q4 Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

SOLUTION

Construction steps:
1. Draw a circle with centre O and any convenient radius.
2. Draw a line 'l' anywhere (this is the given line).
3. Draw a line 'm' parallel to 'l' such that it touches the circle at exactly one point P. This line m is a tangent to the circle, parallel to l.
4. Draw another line 'n' parallel to 'l' such that it intersects the circle at two points A and B. This line n is a secant to the circle, parallel to l.

Note: The tangent m touches the circle at one point, while the secant n cuts through the circle at two points. Both m and n are parallel to the given line l.

This is always possible because for any given direction, we can find exactly two parallel tangents (at opposite ends of a diameter perpendicular to that direction) and infinitely many parallel secants (lines in between the two tangents).

Answer: Construction as described — one tangent (touching at one point) and one secant (intersecting at two points), both parallel to the given line.

Exercise 10.2 — Properties of Tangents — Tangent Perpendicular to Radius, Equal Tang

Q1 From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:
(A) 7 cm
(B) 12 cm
(C) 15 cm
(D) 24.5 cm

SOLUTION

Let O be the centre of the circle, and QT be the tangent from Q to the circle touching at T.
Given: QT = 24 cm, OQ = 25 cm.

Since QT is a tangent at T, OT ⊥ QT.
∴ Triangle OTQ is right-angled at T.

By Pythagoras theorem:
$OQ^2 = OT^2 + QT^2$
$25^2 = OT^2 + 24^2$
$625 = OT^2 + 576$
$OT^2 = 625 - 576 = 49$
OT = 7 cm

∴ The radius of the circle is 7 cm.
The answer is (A).

Answer: (A) 7 cm

Q2 In the given figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to:
(A) 60°
(B) 70°
(C) 80°
(D) 90°

SOLUTION

Given: TP and TQ are tangents from external point T to a circle with centre O.
∠POQ = 110°.

Since TP is a tangent at P, OP ⊥ TP => ∠OPT = 90°.

Since TQ is a tangent at Q, OQ ⊥ TQ => ∠OQT = 90°.

In quadrilateral OPTQ, the sum of all angles = 360°.
∠OPT + ∠PTQ + ∠OQT + ∠POQ = 360°
90° + ∠PTQ + 90° + 110° = 360°
290° + ∠PTQ = 360°
∠PTQ = 360° − 290° = 70°

∴ The answer is (B) 70°.

Answer: (B) 70°

Q3 If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠POA is equal to:
(A) 50°
(B) 60°
(C) 70°
(D) 80°

SOLUTION

Given: PA and PB are tangents from P to a circle with centre O.
∠APB = 80°.

Since PA is tangent at A: OA ⊥ PA => ∠OAP = 90°.
Since PB is tangent at B: OB ⊥ PB => ∠OBP = 90°.

In quadrilateral OAPB:
∠OAP + ∠APB + ∠OBP + ∠AOB = 360°
90° + 80° + 90° + ∠AOB = 360°
∠AOB = 360° − 260° = 100°

Now, in triangle OPA and triangle OPB:
OA = OB (radii)
PA = PB (tangents from external point are equal)
OP = OP (common)
∴ Triangle OPA ≅ Triangle OPB (SSS congruence)

∴ ∠POA = ∠POB = ∠AOB / 2 = 100° / 2 = 50°

∴ The answer is (A) 50°.

Answer: (A) 50°

Q4 Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

SOLUTION

Given: A circle with centre O and diameter AB. PQ is a tangent at A and RS is a tangent at B.

To prove: PQ ∥ RS.

Proof:
Since PQ is a tangent at A and OA is a radius,
OA ⊥ PQ (tangent is perpendicular to radius at point of contact)
∴ ∠OAP = 90° ... (i)

Since RS is a tangent at B and OB is a radius,
OB ⊥ RS (tangent is perpendicular to radius at point of contact)
∴ ∠OBS = 90° ... (ii)

From (i) and (ii):
∠OAP = ∠OBS = 90°

But ∠OAP and ∠OBS are alternate interior angles formed by the transversal AB cutting lines PQ and RS.

Since alternate interior angles are equal, PQ ∥ RS.

Hence proved.

Answer: PQ ∥ RS (proved using tangent ⊥ radius and alternate interior angles)

Q5 Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

SOLUTION

Given: A circle with centre O. A tangent XY touches the circle at point P.

To prove: The perpendicular to XY at P passes through the centre O.

Proof (by contradiction):
Assume that the perpendicular to XY at P does not pass through centre O.
Let the perpendicular at P meet at some point O' which is not the centre.

Then ∠O'PY = 90° ... (i) (by our assumption)

But we know that the tangent at any point is perpendicular to the radius through the point of contact.
∴ OP ⊥ XY

∴ ∠OPY = 90° ... (ii)

From (i) and (ii):
∠O'PY = ∠OPY = 90°

This is possible only if O and O' coincide, since only one perpendicular can be drawn to a line at a given point on it.

This contradicts our assumption that O' is different from O.

∴ The perpendicular at the point of contact to the tangent must pass through the centre.

Hence proved.

Answer: Proved by contradiction — only one perpendicular exists at a point on a line, so it must pass through the centre.

Q6 The length of a tangent from a point A at distance $5\sqrt{2}$ cm from the centre of the circle is 5 cm. Find the radius of the circle.

SOLUTION

Let O be the centre and AT be the tangent touching at T.
Given: OA = $5\sqrt{2}$ cm, AT = 5 cm.

Since AT is tangent at T, OT ⊥ AT.
∴ Triangle OTA is right-angled at T.

By Pythagoras theorem:
$OA^2 = OT^2 + AT^2$
$(5\sqrt{2})^2 = OT^2 + 5^2$
$25 \times 2 = OT^2 + 25$
$50 = OT^2 + 25$
$OT^2 = 50 - 25 = 25$
OT = 5 cm

∴ The radius of the circle is 5 cm.

Answer: Radius = 5 cm

Q7 Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

SOLUTION

Let O be the common centre of the two concentric circles.
Let the radii be R = 5 cm (larger) and r = 3 cm (smaller).

Let AB be a chord of the larger circle that is tangent to the smaller circle, touching it at point P.

Since AB is tangent to the smaller circle at P,
OP ⊥ AB (radius to point of tangency is perpendicular to tangent)

Since OP ⊥ AB and OP passes through centre O,
the perpendicular from the centre bisects the chord.
∴ AP = PB = AB/2

In right triangle OPA:
$OA^2 = OP^2 + AP^2$
$5^2 = 3^2 + AP^2$
$25 = 9 + AP^2$
$AP^2 = 25 - 9 = 16$
AP = 4 cm

∴ AB = 2 × AP = 2 × 4 = 8 cm

∴ The length of the chord is 8 cm.

Answer: 8 cm

Q8 A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that AB + CD = AD + BC.

SOLUTION

Given: Quadrilateral ABCD circumscribes a circle. Let the circle touch:
Side AB at point P,
Side BC at point Q,
Side CD at point R,
Side DA at point S.

To prove: AB + CD = AD + BC.

Proof:

We know that tangents drawn from an external point to a circle are equal in length.

From vertex A: AP = AS ... (i)
From vertex B: BP = BQ ... (ii)
From vertex C: CQ = CR ... (iii)
From vertex D: DR = DS ... (iv)

Adding (i), (ii), (iii), and (iv):
AP + BP + CR + DR = AS + BQ + CQ + DS

Regrouping:
(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
AB + CD = AD + BC

Hence proved.

Answer: AB + CD = AD + BC (proved using equal tangent lengths from each vertex)

Q9 In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that ∠AOB = 90°.

SOLUTION

Given: XY and X'Y' are parallel tangents to a circle with centre O, touching at P and Q respectively. AB is a tangent at C, intersecting XY at A and X'Y' at B.

To prove: ∠AOB = 90°.

Proof:
Join OC (O to the point of contact C of tangent AB).

In triangle OPA and triangle OCA:
OP = OC (radii of the same circle)
AP = AC (tangents from external point A are equal)
OA = OA (common side)
∴ Triangle OPA ≅ Triangle OCA (SSS congruence)
∴ ∠POA = ∠COA ... (i) (by CPCT)

In triangle OQB and triangle OCB:
OQ = OC (radii of the same circle)
BQ = BC (tangents from external point B are equal)
OB = OB (common side)

∴ Triangle OQB ≅ Triangle OCB (SSS congruence)
∴ ∠QOB = ∠COB ... (ii) (by CPCT)

Since XY ∥ X'Y' and PQ is a diameter (as the tangents are parallel, P and Q are diametrically opposite),
∠POQ = 180° (P, O, Q are collinear — they lie on the diameter)

∴ ∠POA + ∠COA + ∠COB + ∠QOB = 180°

Using (i) and (ii):
2∠COA + 2∠COB = 180°
2(∠COA + ∠COB) = 180°
∠COA + ∠COB = 90°
∠AOB = 90°

Hence proved.

Answer: ∠AOB = 90° (proved using congruent triangles and the fact that P, O, Q are collinear)

Q10 Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

SOLUTION

Given: PA and PB are tangents from an external point P to a circle with centre O, touching the circle at A and B respectively.

To prove: ∠APB + ∠AOB = 180°.

Proof:
Since PA is a tangent at A and OA is a radius,
OA ⊥ PA
∴ ∠OAP = 90° ... (i)

Since PB is a tangent at B and OB is a radius,
OB ⊥ PB
∴ ∠OBP = 90° ... (ii)

In quadrilateral OAPB, the sum of all interior angles = 360°.
∠OAP + ∠APB + ∠OBP + ∠AOB = 360°

Substituting from (i) and (ii):

$90° + \angle APB + 90° + \angle AOB = 360°$

$\angle APB + \angle AOB + 180° = 360°$

$\angle APB + \angle AOB = 180°$

$\therefore \angle APB$ and $\angle AOB$ are supplementary.

Hence proved.

Answer: $\angle APB + \angle AOB = 180°$ (supplementary, proved using angle sum of quadrilateral OAPB)

Q11 Prove that the parallelogram circumscribing a circle is a rhombus.

SOLUTION

Given: ABCD is a parallelogram circumscribing a circle with centre O.

To prove: ABCD is a rhombus (i.e., AB = BC = CD = DA).

Proof:
Since ABCD is a parallelogram:
AB = CD ... (i) (opposite sides of parallelogram)
BC = AD ... (ii) (opposite sides of parallelogram)

Since ABCD circumscribes a circle, by the property proved in Q8:
AB + CD = AD + BC ... (iii)

Substituting (i) and (ii) in (iii):
AB + AB = BC + BC
2AB = 2BC
AB = BC ... (iv)

From (i), (ii), and (iv):
AB = BC = CD = DA

$\therefore$ ABCD is a rhombus (a parallelogram with all sides equal).

Hence proved.

Answer: ABCD is a rhombus (proved using AB + CD = AD + BC and parallelogram properties)

Q12 A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.

SOLUTION

Given: Triangle ABC circumscribes a circle of radius r = 4 cm.
D is the point of contact on BC with BD = 8 cm and DC = 6 cm.

Let the circle touch AB at E and AC at F.

Using the property that tangents from an external point are equal:
From B: BD = BE = 8 cm
From C: CD = CF = 6 cm
From A: AE = AF = x (say)

∴ The sides are:
BC = BD + DC = 8 + 6 = 14 cm
AB = AE + BE = x + 8
AC = AF + FC = x + 6

Now, using the formula: Area of triangle = (semi-perimeter) × (inradius)
Area = s × r, where s is the semi-perimeter.

Semi-perimeter s = (AB + BC + AC) / 2
s = (x + 8 + 14 + x + 6) / 2
s = (2x + 28) / 2
s = x + 14

∴ Area = s × r = (x + 14) × 4 = 4(x + 14) ... (I)

Also, by Heron's formula:
s − a = s − BC = (x + 14) − 14 = x
s − b = s − AC = (x + 14) − (x + 6) = 8
s − c = s − AB = (x + 14) − (x + 8) = 6

Area = √[s(s−a)(s−b)(s−c)]
Area = √[(x + 14)(x)(8)(6)]
Area = √[48x(x + 14)] ... (II)

Equating (I) and (II):
4(x + 14) = √[48x(x + 14)]

Squaring both sides:
16(x + 14)² = 48x(x + 14)

Since x + 14 ≠ 0, divide both sides by (x + 14):
16(x + 14) = 48x
16x + 224 = 48x
224 = 48x − 16x
224 = 32x
x = 7

∴ AB = x + 8 = 7 + 8 = 15 cm
AC = x + 6 = 7 + 6 = 13 cm

Verification: s = 7 + 14 = 21
Area by inradius = 21 × 4 = 84 cm²
Heron's: √(21 × 7 × 8 × 6) = √(7056) = 84 cm² ✓

Answer: AB = 15 cm, AC = 13 cm

Q13 Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

SOLUTION

Given: Quadrilateral ABCD circumscribes a circle with centre O. The circle touches AB at P, BC at Q, CD at R, and DA at S.

To prove: ∠AOB + ∠COD = 180° and ∠BOC + ∠AOD = 180°.

Proof:
Join OP, OQ, OR, OS, OA, OB, OC, OD.

In triangles OAP and OAS:
OP = OS (radii)
AP = AS (tangents from A)
OA = OA (common)
∴ Triangle OAP ≅ Triangle OAS (SSS)
∴ ∠AOP = ∠AOS ... (1)
Let ∠AOP = ∠AOS = α

In triangles OBP and OBQ:
OP = OQ (radii)
BP = BQ (tangents from B)
OB = OB (common)
∴ Triangle OBP ≅ Triangle OBQ (SSS)

∴ ∠BOP = ∠BOQ ... (2)
Let ∠BOP = ∠BOQ = β

In triangles OCQ and OCR:
OQ = OR (radii)
CQ = CR (tangents from C)
OC = OC (common)
∴ Triangle OCQ ≅ Triangle OCR (SSS)
∴ ∠COQ = ∠COR ... (3)
Let ∠COQ = ∠COR = γ

In triangles ODR and ODS:
OR = OS (radii)
DR = DS (tangents from D)
OD = OD (common)
∴ Triangle ODR ≅ Triangle ODS (SSS)
∴ ∠DOR = ∠DOS ... (4)
Let ∠DOR = ∠DOS = δ

The angles around O sum to 360°:
2α + 2β + 2γ + 2δ = 360°
α + β + γ + δ = 180° ... (5)

Now:
∠AOB = ∠AOP + ∠BOP = α + β
∠COD = ∠COR + ∠DOR = γ + δ

∴ ∠AOB + ∠COD = α + β + γ + δ = 180° [from (5)]

Similarly:
∠BOC = ∠BOQ + ∠COQ = β + γ
∠AOD = ∠AOS + ∠DOS = α + δ

∴ ∠BOC + ∠AOD = β + γ + α + δ = 180° [from (5)]

∴ Opposite sides of the quadrilateral subtend supplementary angles at the centre.

Hence proved.

Answer: ∠AOB + ∠COD = 180° and ∠BOC + ∠AOD = 180° (proved using congruent triangles and angle sum at centre)