Chapter 11: Areas Related to Circles

Exercise 11.1 — Areas of Sector and Segment of a Circle

Q1 Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

SOLUTION

Given: r = 6 cm, θ = 60°. Use π = 22/7.

Area of sector = (θ/360) × πr²
= (60/360) × (22/7) × 6²
= (1/6) × (22/7) × 36
= (22 × 36) / (7 × 6)
= 792/42
= 132/7 cm²

Answer: 132/7 cm² ≈ 18.86 cm²

Q2 Find the area of a quadrant of a circle whose circumference is 22 cm.

SOLUTION

Given: Circumference = 22 cm. A quadrant has θ = 90°.

Step 1: Find the radius.
2πr = 22
2 × (22/7) × r = 22
r = 22 × 7 / (2 × 22) = 7/2 = 3.5 cm

Step 2: Area of quadrant = (90/360) × πr²
= (1/4) × (22/7) × (7/2)²
= (1/4) × (22/7) × (49/4)
= (22 × 49) / (7 × 16)
= (22 × 7) / 16
= 154/16
= 77/8 cm²

Answer: 77/8 cm² = 9.625 cm²

Q3 The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

SOLUTION

Given: r = 14 cm. The minute hand completes 360° in 60 minutes.

Angle swept in 5 minutes = (5/60) × 360° = 30°

Area swept = (θ/360) × πr²
= (30/360) × (22/7) × 14²
= (1/12) × (22/7) × 196
= (22 × 196) / (7 × 12)
= (22 × 28) / 12
= 616/12
= 154/3 cm²

Answer: 154/3 cm² ≈ 51.33 cm²

Q4 A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:
(i) minor segment
(ii) major sector
(Use π = 3.14)

SOLUTION

Given: r = 10 cm, θ = 90°, π = 3.14.

(i) Minor segment:

Area of minor sector = (90/360) × 3.14 × 10²
= (1/4) × 3.14 × 100
= 78.5 cm²

Triangle OAB has a right angle at O (the chord subtends 90° at centre), with OA = OB = 10 cm.
Area of △OAB = (1/2) × OA × OB = (1/2) × 10 × 10 = 50 cm²

Area of minor segment = Area of minor sector − Area of △OAB
= 78.5 − 50 = 28.5 cm²

(ii) Major sector:
Angle of major sector = 360° − 90° = 270°

Area of major sector = (270/360) × 3.14 × 10²
= (3/4) × 3.14 × 100
= 235.5 cm²

∴ (i) Minor segment = 28.5 cm² (ii) Major sector = 235.5 cm²

Answer: (i) 28.5 cm² (ii) 235.5 cm²

Q5 In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
(i) the length of the arc
(ii) area of the sector formed by the arc
(iii) area of the segment formed by the corresponding chord

SOLUTION

Given: r = 21 cm, θ = 60°.

(i) Length of the arc:
Arc length = (θ/360) × 2πr
= (60/360) × 2 × (22/7) × 21
= (1/6) × 2 × 22 × 3
= (1/6) × 132
= 22 cm

(ii) Area of the sector:
= (60/360) × (22/7) × 21²
= (1/6) × (22/7) × 441
= (22 × 63) / 6
= 1386/6
= 231 cm²

(iii) Area of the segment:
Since θ = 60° and OA = OB = r = 21 cm, triangle OAB is equilateral (all sides = 21 cm).

Area of equilateral △OAB = (√3/4) × 21²
= (√3/4) × 441
= (441√3)/4 cm²

Using √3 ≈ 1.732:
= (441 × 1.732)/4 = 763.812/4 ≈ 190.95 cm²

Area of segment = Area of sector − Area of △OAB

= 231 − (441√3)/4
≈ 231 − 190.95 ≈ 40.05 cm²

∴ (i) Arc length = 22 cm (ii) Sector area = 231 cm² (iii) Segment area = (231 − 441√3/4) cm² ≈ 40.05 cm²

Answer: (i) 22 cm (ii) 231 cm² (iii) (231 − 441√3/4) cm² ≈ 40.05 cm²

Q6 A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.
(Use π = 3.14 and √3 = 1.73)

SOLUTION

Given: r = 15 cm, θ = 60°, π = 3.14, √3 = 1.73.

Minor segment = Area of minor sector − Area of △OAB

Area of minor sector = (60/360) × 3.14 × 15²
= (1/6) × 3.14 × 225
= 706.5/6
= 117.75 cm²

Since θ = 60° and OA = OB = 15 cm, △OAB is equilateral (side = 15 cm).
Area of △OAB = (√3/4) × 15²
= (1.73/4) × 225
= 389.25/4
= 97.3125 cm²
≈ 97.31 cm²

Area of minor segment = 117.75 − 97.31 = 20.44 cm²

Area of major segment = πr² − Area of minor segment
= 3.14 × 225 − 20.44
= 706.5 − 20.44
= 686.06 cm²

∴ Minor segment ≈ 20.44 cm² Major segment ≈ 686.06 cm²

Answer: Minor segment ≈ 20.44 cm²; Major segment ≈ 686.06 cm²

Q7 A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.
(Use π = 3.14 and √3 = 1.73)

SOLUTION

Given: r = 12 cm, θ = 120°, π = 3.14, √3 = 1.73.

Area of sector = (120/360) × 3.14 × 12²
= (1/3) × 3.14 × 144
= 3.14 × 48
= 150.72 cm²

For △OAB with angle 120° at O (OA = OB = 12 cm), draw OM ⊥ AB.
∠AOM = 120°/2 = 60°
AM = OA × sin 60° = 12 × (1.73/2) = 12 × 0.865 = 10.38 cm
AB = 2 × AM = 20.76 cm
OM = OA × cos 60° = 12 × 0.5 = 6 cm

Area of △OAB = (1/2) × AB × OM
= (1/2) × 20.76 × 6
= 62.28 cm²

Area of segment = Area of sector − Area of △OAB
= 150.72 − 62.28
= 88.44 cm²

∴ Area of the segment = 88.44 cm²

Answer: 88.44 cm²

Q8 A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 11.8). Find:
(i) the area of that part of the field in which the horse can graze.
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m.
(Use π = 3.14)

SOLUTION

The horse is tied at a corner of the square field. The corner angle = 90°.
The horse can graze in a sector of angle 90° (quarter circle) limited by the field boundary.

(i) With rope length r = 5 m:

Grazing area = (90/360) × 3.14 × 5²
= (1/4) × 3.14 × 25
= 78.5/4
= 19.625 m²

(ii) With rope length r = 10 m:
New grazing area = (90/360) × 3.14 × 10²
= (1/4) × 3.14 × 100
= 78.5 m²

Increase in grazing area = 78.5 − 19.625 = 58.875 m²

∴ (i) Grazing area = 19.625 m² (ii) Increase = 58.875 m²

Answer: (i) 19.625 m² (ii) 58.875 m²

Q9 A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. 11.9. Find:
(i) the total length of the silver wire required.
(ii) the area of each sector of the brooch.

SOLUTION

Given: Diameter = 35 mm, so r = 35/2 = 17.5 mm. 5 diameters divide into 10 equal sectors.

(i) Total wire:
Circumference of circle = πd = (22/7) × 35 = 22 × 5 = 110 mm
Length of 5 diameters = 5 × 35 = 175 mm
Total wire = 110 + 175 = 285 mm

(ii) Each sector has angle = 360°/10 = 36°.
Area of each sector = (36/360) × π × r²
= (1/10) × (22/7) × (35/2)²
= (1/10) × (22/7) × (1225/4)
= (22 × 1225) / (10 × 7 × 4)
= 26950/280
= 96.25 mm²

∴ (i) Total wire = 285 mm (ii) Area of each sector = 96.25 mm²

Answer: (i) 285 mm (ii) 96.25 mm²

Q10 An umbrella has 8 ribs which are equally spaced (see Fig. 11.10). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

SOLUTION

Given: r = 45 cm, 8 equally spaced ribs.
Angle between consecutive ribs = 360°/8 = 45°.

Area between two consecutive ribs = area of one sector
= (45/360) × (22/7) × 45²
= (1/8) × (22/7) × 2025
= (22 × 2025) / (7 × 8)
= 44550/56
= 22275/28 cm²
≈ 795.54 cm²

∴ Area between two consecutive ribs = 22275/28 cm² ≈ 795.54 cm²

Answer: 22275/28 cm² ≈ 795.54 cm²

Q11 A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

SOLUTION

Given: r = 25 cm, θ = 115° for each wiper. Use π = 22/7.

Area cleaned by one wiper = (115/360) × (22/7) × 25²
= (115/360) × (22/7) × 625
= (23/72) × (22/7) × 625
= (23 × 22 × 625) / (72 × 7)
= 316250/504
= 158125/252 cm²
≈ 627.48 cm²

Total area cleaned by 2 wipers = 2 × 158125/252
= 158125/126 cm²
≈ 1254.96 cm²

∴ Total area cleaned = 158125/126 cm² ≈ 1254.96 cm²

Answer: 158125/126 cm² ≈ 1254.96 cm²

Q12 To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned.
(Use π = 3.14)

SOLUTION

Given: θ = 80°, r = 16.5 km, π = 3.14.

Area of sector = (80/360) × 3.14 × 16.5²
= (2/9) × 3.14 × 272.25
= (2 × 3.14 × 272.25) / 9
= 1709.73/9
≈ 189.97 km²

∴ Area of sea over which ships are warned ≈ 189.97 km²

Answer: ≈ 189.97 km²

Q13 A round table cover has six equal designs as shown in Fig. 11.11. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of 0.35 per cm². (Use √3 = 1.7)

SOLUTION

Given: r = 28 cm, 6 equal designs, rate = 0.35 per cm², √3 = 1.7.

The 6 equal designs are 6 equal segments, each formed by a chord subtending 60° at the centre (since 360°/6 = 60°).

Area of each sector (60°) = (60/360) × (22/7) × 28²
= (1/6) × (22/7) × 784
= (22 × 784) / (6 × 7)
= 17248/42
= 8624/21 cm²

Since θ = 60° and OA = OB = 28 cm, △OAB is equilateral (side = 28 cm).
Area of △OAB = (√3/4) × 28²
= (1.7/4) × 784
= 1331.2/4
= 332.8 cm²

Area of each design (segment) = 8624/21 − 332.8
= 410.667 − 332.8

= 77.867 cm²

Total area of 6 designs = 6 × 77.867 = 467.2 cm²

Cost = 467.2 × 0.35 = 163.52

Note: Using √3 = 1.7 exactly:
Each △ area = (1.7 × 784)/4 = 1332.8/4 = 333.2 cm²
Each segment = 8624/21 − 333.2 = 410.667 − 333.2 = 77.467 cm²
Total = 6 × 77.467 = 464.8 cm²
Cost = 464.8 × 0.35 = 162.68

∴ Cost of making the designs = 162.68

Answer: 162.68

Q14 Tick the correct answer in the following:
Area of a sector of angle p (in degrees) of a circle with radius R is
(A) p/(180) × 2R
(B) p/(180) × πR²
(C) p/(360) × 2πR
(D) p/(720) × 2πR²

SOLUTION

The formula for area of a sector of angle p° with radius R is:
Area = (p/360) × πR²

Check option (D): p/(720) × 2πR²
= (p × 2πR²) / 720
= (p × πR²) / 360
= (p/360) × πR² ✓

Check option (A): p/(180) × 2R — this has no π and wrong power of R. ✗
Check option (B): p/(180) × πR² — factor is 1/180 instead of 1/360. ✗
Check option (C): p/(360) × 2πR — this is the arc length formula, not area. ✗

∴ The correct answer is (D).

Answer: (D)