Chapter 12: Surface Areas and Volumes

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Exercise 12.1 — Surface Area of a Combination of Solids

Q1 2 cubes each of volume 64 cm³ are joined end to end. Find the surface area of the resulting cuboid.

SOLUTION

Volume of each cube = 64 cm³
Side of cube = $\sqrt[3]{64}$ = 4 cm

When two cubes are joined end to end, the resulting cuboid has:
Length = 4 + 4 = 8 cm, Breadth = 4 cm, Height = 4 cm

Surface area = 2(lb + bh + hl)
= 2(8 × 4 + 4 × 4 + 4 × 8)
= 2(32 + 16 + 32)
= 2 × 80
= 160 cm²

Answer: 160 cm²

Q2 A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

SOLUTION

Diameter of hemisphere = 14 cm => radius r = 7 cm
Total height = 13 cm
Height of cylindrical part h = 13 − 7 = 6 cm

Inner surface area = CSA of hemisphere + CSA of cylinder
= 2πr² + 2πrh
= 2 × (22/7) × 7² + 2 × (22/7) × 7 × 6
= 2 × 22 × 7 + 2 × 22 × 6
= 308 + 264

= 572 cm²

Answer: 572 cm²

Q3 A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

SOLUTION

Radius r = 3.5 cm

Height of cone h = total height − r = 15.5 − 3.5 = 12 cm

Slant height $l = \sqrt{(r^2 + h^2)} = \sqrt{(3.5^2 + 12^2)} = \sqrt{(12.25 + 144)} = \sqrt{156.25} = 12.5$ cm

Total surface area = CSA of cone + CSA of hemisphere

$= \pi rl + 2\pi r^2$

$= \pi r(l + 2r)$

$= (22/7) \times 3.5 \times (12.5 + 7)$

$= (22/7) \times 3.5 \times 19.5$

$= 11 \times 19.5$

$= 214.5$ cm²

Answer: 214.5 cm²

Q4 A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

SOLUTION

Side of cube = 7 cm

Greatest diameter of hemisphere = side of cube = 7 cm => radius r = 3.5 cm

Surface area = Total surface area of cube − base area of hemisphere + CSA of hemisphere

$= 6 \times 7^2 - \pi r^2 + 2\pi r^2$

$= 6 \times 49 + \pi r^2$

$= 294 + (22/7) \times 3.5^2$

$= 294 + (22/7) \times 12.25$

$= 294 + 22 \times 1.75$

$= 294 + 38.5$

$= 332.5$ cm²

Answer: 332.5 cm²

Q5 A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

SOLUTION

Let the edge of the cube = l
Diameter of hemisphere = l => radius r = l/2

Surface area of remaining solid
= Surface area of cube − base area of hemisphere + CSA of hemisphere
= $6l^2 - \pi r^2 + 2\pi r^2$
= $6l^2 + \pi r^2$
= $6l^2 + \pi(l/2)^2$
= $6l^2 + \pi l^2/4$
= $l^2(24 + \pi)/4$ sq. units

Answer: $l^2(24 + \pi)/4$ sq. units

Q6 A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

SOLUTION

Diameter = 5 mm => radius r = 2.5 mm
Length of cylindrical part h = 14 − 2 × 2.5 = 14 − 5 = 9 mm

Surface area = CSA of cylinder + 2 × CSA of hemisphere
= $2\pi rh + 2 \times 2\pi r^2$
= $2\pi r(h + 2r)$
= $2 \times (22/7) \times 2.5 \times (9 + 5)$
= $2 \times (22/7) \times 2.5 \times 14$
= $2 \times 22 \times 2.5 \times 2$
= 220 mm²

Answer: 220 mm²

Q7 A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m². (Note that the base of the tent will not be covered with canvas.)

SOLUTION

Diameter = 4 m => radius r = 2 m
Height of cylindrical part h = 2.1 m
Slant height of cone l = 2.8 m

Area of canvas = CSA of cylinder + CSA of cone
= 2πrh + πrl
= πr(2h + l)
= (22/7) × 2 × (2 × 2.1 + 2.8)
= (22/7) × 2 × (4.2 + 2.8)
= (22/7) × 2 × 7
= 22 × 2
= 44 m²

Cost of canvas = 44 × 500 = Rs 22,000

Answer: Canvas area = 44 m², Cost = Rs 22,000

Q8 From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².

SOLUTION

Diameter = 1.4 cm => radius r = 0.7 cm, height h = 2.4 cm

Slant height of cone l = √(r² + h²) = √(0.7² + 2.4²) = √(0.49 + 5.76) = √6.25 = 2.5 cm

Total surface area of remaining solid
= CSA of cylinder + area of circular base + CSA of cone
= 2πrh + πr² + πrl
= πr(2h + r + l)
= (22/7) × 0.7 × (2 × 2.4 + 0.7 + 2.5)
= (22 × 0.1) × (4.8 + 0.7 + 2.5)
= 2.2 × 8

= 17.6 cm²

≈ 18 cm² (to the nearest cm²)

Answer: 18 cm² (approx.)

Q9 A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.

SOLUTION

Radius r = 3.5 cm, height of cylinder h = 10 cm

The two flat circular ends are replaced by hemispherical surfaces.

Total surface area = CSA of cylinder + 2 × CSA of hemisphere
= 2πrh + 2 × 2πr²
= 2πr(h + 2r)
= 2 × (22/7) × 3.5 × (10 + 2 × 3.5)
= 2 × (22/7) × 3.5 × 17
= 2 × 22 × 0.5 × 17
= 22 × 17
= 374 cm²

Answer: 374 cm²

Exercise 12.2 — Volume of a Combination of Solids

Q1 A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

SOLUTION

Radius r = 1 cm, height of cone h = r = 1 cm

Volume = Volume of hemisphere + Volume of cone
= $(2/3)\pi r^3 + (1/3)\pi r^2 h$
= $(2/3)\pi(1)^3 + (1/3)\pi(1)^2(1)$
= $2\pi/3 + \pi/3$
= $3\pi/3$
= π cm³

Answer: π cm³

Q2 Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

SOLUTION

Diameter = 3 cm => radius r = 1.5 cm
Height of each cone = 2 cm
Height of cylindrical part = 12 − 2 − 2 = 8 cm

Volume = Volume of cylinder + 2 × Volume of cone
= $\pi r^2 h + 2 \times (1/3)\pi r^2 h_{cone}$
= $\pi r^2(h + 2h_{cone}/3)$
= $(22/7) \times (1.5)^2 \times (8 + 2\times2/3)$
= $(22/7) \times (9/4) \times (28/3)$
= $(22 \times 9 \times 28)/(7 \times 4 \times 3)$
= $(22 \times 9 \times 4)/12$
= 22×3
= 66 cm³

Answer: 66 cm³

Q3 A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.

SOLUTION

Diameter = 2.8 cm => radius r = 1.4 cm
Height of cylindrical part h = 5 − 2 × 1.4 = 5 − 2.8 = 2.2 cm

Volume of one gulab jamun = Volume of cylinder + Volume of sphere
= $\pi r^2 h + (4/3)\pi r^3$

$\pi r^2 h = (22/7) \times (1.4)^2 \times 2.2 = (22/7) \times 1.96 \times 2.2 = 13.55$ cm³ (approx.)

$(4/3)\pi r^3 = (4/3) \times (22/7) \times (1.4)^3 = (4/3) \times (22/7) \times 2.744 = 11.50$ cm³ (approx.)

Volume of one gulab jamun = 13.55 + 11.50 = 25.05 cm³ (approx.)

Volume of 45 gulab jamuns = 45 × 25.05 = 1127.28 cm³

Volume of syrup = 30% of 1127.28 = 0.3 × 1127.28 ≈ 338 cm³

Answer: ≈ 338 cm³

Q4 A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.

SOLUTION

Volume of cuboid = 15 × 10 × 3.5 = 525 cm³

Volume of one conical depression = $(1/3)\pi r^2 h$
= $(1/3) \times (22/7) \times (0.5)^2 \times 1.4$
= $(1/3) \times (22/7) \times 0.25 \times 1.4$
= $(1/3) \times 22 \times 0.05$
= 11/30 cm³

Volume of 4 depressions = 4 × 11/30 = 22/15 cm³

Volume of wood = 525 − 22/15 = (7875 − 22)/15 = 7853/15 ≈ 523.53 cm³

Answer: 523.53 cm³ (approx.)

Q5 A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

SOLUTION

Height of cone H = 8 cm, radius R = 5 cm
Radius of each lead shot r = 0.5 cm

Volume of cone = $(1/3)\pi R^2H = (1/3) \times (22/7) \times 25 \times 8 = 4400/21$ cm³

Volume of water that overflows = $(1/4) \times 4400/21 = 1100/21$ cm³

Volume of one lead shot = $(4/3)\pi r^3$
$= (4/3) \times (22/7) \times (0.5)^3$
$= (4/3) \times (22/7) \times (1/8)$
$= 88/168$
$= 11/21$ cm³

Number of lead shots = $(1100/21) \div (11/21)$
$= 1100/21 \times 21/11$
$= 100$

Answer: 100 lead shots

Q6 A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8 g mass. (Use $\pi = 3.14$)

SOLUTION

Lower cylinder: height $h_1 = 220$ cm, diameter = 24 cm => radius $r_1 = 12$ cm
Upper cylinder: height $h_2 = 60$ cm, radius $r_2 = 8$ cm
Use $\pi = 3.14$

Volume of lower cylinder = $\pi r_1^2 h_1$
$= 3.14 \times 144 \times 220$
$= 3.14 \times 31680$
$= 99475.2$ cm³

Volume of upper cylinder = $\pi r_2^2 h_2$
$= 3.14 \times 64 \times 60$

= 3.14 × 3840
= 12057.6 cm³

Total volume = 99475.2 + 12057.6 = 111532.8 cm³

Mass = Volume × density = 111532.8 × 8
= 892262.4 g
= 892262.4/1000 kg
≈ 892.26 kg

Answer: ≈ 892.26 kg

Q7 A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

SOLUTION

Common radius r = 60 cm
Height of cone = 120 cm, height of cylinder = 180 cm

Volume of cylinder = $\pi r^2 h$ = (22/7) × 60² × 180
= (22/7) × 3600 × 180 = 14256000/7 cm³

Volume of cone = (1/3)$\pi r^2 h$ = (1/3) × (22/7) × 3600 × 120
= (22/7) × 3600 × 40 = 3168000/7 cm³

Volume of hemisphere = (2/3)πr^3 = (2/3) × (22/7) × 60³
= (2/3) × (22/7) × 216000 = 3168000/7 cm³

Volume of solid = 3168000/7 + 3168000/7 = 6336000/7 cm³

Volume of water left = Volume of cylinder − Volume of solid
= 14256000/7 − 6336000/7
= 7920000/7
≈ 1131428.57 cm³

Answer: 7920000/7 cm³ ≈ 1131428.57 cm³

Q8 A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm³. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

SOLUTION

Cylindrical neck: height h = 8 cm, diameter = 2 cm => radius r_1 = 1 cm
Spherical part: diameter = 8.5 cm => radius r_2 = 4.25 cm
Use π = 3.14

Volume of sphere = $(4/3)\pi r_2^3$
4.25^2 = 18.0625
4.25^3 = 18.0625 × 4.25 = 76.765625
Volume of sphere = (4/3) × 3.14 × 76.765625
= 4.1867 × 76.765625 ≈ 321.39 cm³

Volume of cylinder = $\pi r_1^2 h$ = 3.14 × 1^2 × 8 = 25.12 cm³

Total volume = 321.39 + 25.12 = 346.51 cm³

The child's answer of 345 cm³ is incorrect. The correct volume is approximately 346.51 cm³.

Answer: ≈ 346.51 cm³; the child's answer of 345 cm³ is not correct