Chapter 14: Probability

Exercise 14.1 — Classical (Theoretical) Probability

Q1 Complete the following statements:
(i) Probability of an event E + Probability of the event 'not E' = ___.
(ii) The probability of an event that cannot happen is ___. Such an event is called ___.
(iii) The probability of an event that is certain to happen is ___. Such an event is called ___.
(iv) The sum of the probabilities of all the elementary events of an experiment is ___.
(v) The probability of an event is greater than or equal to ___ and less than or equal to ___.

SOLUTION

(i) P(E) + P(not E) = 1

(ii) The probability of an event that cannot happen is 0.
Such an event is called an impossible event.

(iii) The probability of an event that is certain to happen is 1.
Such an event is called a sure event or a certain event.

(iv) The sum of the probabilities of all the elementary events of an experiment is 1.

(v) The probability of an event is greater than or equal to 0 and less than or equal to 1.

Answer: (i) 1 (ii) 0, impossible event (iii) 1, sure/certain event (iv) 1 (v) 0, 1

Q2 Which of the following experiments have equally likely outcomes? Explain.
(i) A driver attempts to start a car. The car starts or does not start.
(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.
(iii) A trial is made to answer a true-false question. The answer is right or wrong.
(iv) A baby is born. It is a boy or a girl.

SOLUTION

(i) Not equally likely. Starting a car depends on many factors (fuel, battery, engine condition, etc.).
The two outcomes are not equally likely.

(ii) Not equally likely. Shooting a basketball depends on the player's skill, distance, angle, etc.
The two outcomes are not equally likely.

(iii) Equally likely. If the answer is chosen randomly, each option (True/False) has probability 1/2.
The two outcomes are equally likely.

(iv) Equally likely. A baby is equally likely to be a boy or a girl.
The two outcomes are equally likely.

Answer: (iii) and (iv) have equally likely outcomes

Q3 Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?

SOLUTION

When a coin is tossed, the two outcomes — Head and Tail — are equally likely.
P(Head) = 1/2 and P(Tail) = 1/2.

Since both outcomes have the same probability, neither team has an advantage over the other.
=> Tossing a coin is a fair way of deciding which team gets the ball.

Answer: Both outcomes (Head, Tail) are equally likely with P = 1/2 each, so neither team has an advantage

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Q4 Which of the following cannot be the probability of an event?
(A) 2/3
(B) -1.5
(C) 15%
(D) 0.7

SOLUTION

The probability of an event always satisfies 0 <= P(E) <= 1.

(A) 2/3 = 0.667 — valid (lies between 0 and 1)
(B) -1.5 — not valid (negative, less than 0)
(C) 15% = 0.15 — valid (lies between 0 and 1)
(D) 0.7 — valid (lies between 0 and 1)

=> (B) -1.5 cannot be the probability of an event.

Answer: (B) -1.5

Q5 If P(E) = 0.05, what is the probability of 'not E'?

SOLUTION

P(E) + P(not E) = 1
P(not E) = 1 - P(E)
P(not E) = 1 - 0.05
P(not E) = 0.95

Answer: P(not E) = 0.95

Q6 A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out
(i) an orange flavoured candy?
(ii) a lemon flavoured candy?

SOLUTION

The bag contains only lemon flavoured candies.

(i) There are no orange flavoured candies in the bag.
Getting an orange flavoured candy is an impossible event.
P(orange flavoured candy) = 0

(ii) Since the bag contains only lemon flavoured candies, she will always take out a lemon candy.
This is a sure event.
P(lemon flavoured candy) = 1

Answer: (i) 0 (ii) 1

Q7 It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

SOLUTION

Let E = event that 2 students have the same birthday.
Then 'not E' = event that 2 students do not have the same birthday.

Given: P(not E) = 0.992

P(E) = 1 - P(not E)
P(E) = 1 - 0.992
P(E) = 0.008

Answer: 0.008

Q8 A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is
(i) red?
(ii) not red?

SOLUTION

Total number of balls = 3 red + 5 black = 8

(i) Number of red balls = 3
P(red) = 3/8

(ii) P(not red) = 1 - P(red) = 1 - 3/8 = 5/8

Alternatively: Number of non-red (black) balls = 5
P(not red) = 5/8

Answer: (i) 3/8 (ii) 5/8

Q9 A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be
(i) red?
(ii) white?
(iii) not green?

SOLUTION

Total marbles = 5 red + 8 white + 4 green = 17

(i) P(red) = 5/17

(ii) P(white) = 8/17

(iii) Number of green marbles = 4
P(green) = 4/17
P(not green) = 1 - 4/17 = 13/17

Alternatively: Non-green marbles = 5 + 8 = 13
P(not green) = 13/17

Answer: (i) 5/17 (ii) 8/17 (iii) 13/17

Q10 A piggy bank contains hundred 50p coins, fifty Rs 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin
(i) will be a 50p coin?
(ii) will not be a Rs 5 coin?

SOLUTION

Number of 50p coins = 100
Number of Rs 1 coins = 50
Number of Rs 2 coins = 20
Number of Rs 5 coins = 10
Total coins = 100 + 50 + 20 + 10 = 180

(i) P(50p coin) = 100/180 = 5/9

(ii) Non-Rs 5 coins = 100 + 50 + 20 = 170
P(not a Rs 5 coin) = 170/180 = 17/18

Answer: (i) 5/9 (ii) 17/18

Q11 Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish. What is the probability that the fish taken out is a male fish?

SOLUTION

Number of male fish = 5
Number of female fish = 8
Total fish = 5 + 8 = 13

P(male fish) = 5/13

Answer: 5/13

Q12 A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see figure), and these are equally likely outcomes. What is the probability that it will point at
(i) 8?
(ii) an odd number?
(iii) a number greater than 2?
(iv) a number less than 9?

SOLUTION

Total outcomes = {1, 2, 3, 4, 5, 6, 7, 8}, so total = 8

(i) Favourable outcome = {8} => 1 outcome
P(8) = 1/8

(ii) Odd numbers = {1, 3, 5, 7} => 4 outcomes
P(odd) = 4/8 = 1/2

(iii) Numbers greater than 2 = {3, 4, 5, 6, 7, 8} => 6 outcomes
P(> 2) = 6/8 = 3/4

(iv) Numbers less than 9 = {1, 2, 3, 4, 5, 6, 7, 8} => all 8 outcomes
P(< 9) = 8/8 = 1 (certain event)

Answer: (i) 1/8 (ii) 1/2 (iii) 3/4 (iv) 1

Q13 A die is thrown once. Find the probability of getting
(i) a prime number;
(ii) a number lying between 2 and 6;
(iii) an odd number.

SOLUTION

Possible outcomes when a die is thrown = {1, 2, 3, 4, 5, 6}, total = 6

(i) Prime numbers on a die = {2, 3, 5} => 3 outcomes
P(prime) = 3/6 = 1/2

(ii) Numbers strictly between 2 and 6 = {3, 4, 5} => 3 outcomes
P(between 2 and 6) = 3/6 = 1/2

(iii) Odd numbers = {1, 3, 5} => 3 outcomes
P(odd) = 3/6 = 1/2

Answer: (i) 1/2 (ii) 1/2 (iii) 1/2

Q14 One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting
(i) a king of red colour
(ii) a face card
(iii) a red face card
(iv) the jack of hearts
(v) a spade
(vi) the queen of diamonds

SOLUTION

Total cards = 52
4 suits (Hearts, Diamonds — red; Clubs, Spades — black), 13 cards each.
Face cards per suit: Jack, Queen, King (3 each => 12 total).

(i) Kings of red colour: King of Hearts + King of Diamonds = 2
P(red king) = 2/52 = 1/26

(ii) Face cards = 4 x 3 = 12
P(face card) = 12/52 = 3/13

(iii) Red face cards: 3 from Hearts + 3 from Diamonds = 6
P(red face card) = 6/52 = 3/26

(iv) Jack of Hearts: exactly 1 in the deck

P(Jack of Hearts) = 1/52

(v) Spades = 13
P(spade) = 13/52 = 1/4

(vi) Queen of Diamonds: exactly 1 in the deck
P(Queen of Diamonds) = 1/52

Answer: (i) 1/26 (ii) 3/13 (iii) 3/26 (iv) 1/52 (v) 1/4 (vi) 1/52

Q15 Five cards — the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.
(i) What is the probability that the card is the queen?
(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

SOLUTION

The five cards are: 10, J, Q, K, A of diamonds. Total = 5

(i) There is 1 Queen among the 5 cards.
P(Queen) = 1/5

(ii) Queen is drawn and put aside. Remaining cards = {10, J, K, A} => 4 cards

(a) There is 1 Ace among the remaining 4 cards.
P(Ace) = 1/4

(b) There are no Queens left.
P(Queen) = 0/4 = 0

Answer: (i) 1/5 (ii)(a) 1/4 (ii)(b) 0

Q16 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

SOLUTION

Defective pens = 12
Good pens = 132
Total pens = 12 + 132 = 144

P(good pen) = 132/144

Simplifying by dividing by 12:
P(good pen) = 11/12

Answer: 11/12

Q17 (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?
(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

SOLUTION

(i) Total bulbs = 20, Defective = 4
P(defective) = 4/20 = 1/5

(ii) The bulb drawn in (i) was not defective and is not replaced.
Remaining bulbs = 20 - 1 = 19
Defective bulbs remaining = 4 (unchanged)
Non-defective bulbs remaining = 19 - 4 = 15

P(not defective) = 15/19

Answer: (i) 1/5 (ii) 15/19

Q18 A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears
(i) a two-digit number
(ii) a perfect square number
(iii) a number divisible by 5.

SOLUTION

Total discs = 90, numbered 1 to 90.

(i) Two-digit numbers from 1 to 90: 10, 11, ..., 90
Count = 90 - 10 + 1 = 81
P(two-digit number) = 81/90 = 9/10

(ii) Perfect squares from 1 to 90:
$1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$
($10^2 = 100 > 90$, so 9 perfect squares in range)

P(perfect square) = 9/90 = 1/10

(iii) Numbers divisible by 5 from 1 to 90: 5, 10, 15, ..., 90
Count = 90/5 = 18
P(divisible by 5) = 18/90 = 1/5

Answer: (i) 9/10 (ii) 1/10 (iii) 1/5

Q19 A child has a die whose six faces show the letters as given below:
A B C D E A
The die is thrown once. What is the probability of getting (i) A? (ii) D?

SOLUTION

Six faces: A, B, C, D, E, A. Total outcomes = 6

(i) Letter A appears on 2 faces.
P(A) = 2/6 = 1/3

(ii) Letter D appears on 1 face.
P(D) = 1/6

Answer: (i) 1/3 (ii) 1/6

Q20 Suppose you drop a die at random on the rectangular region shown in the figure (3 m x 2 m rectangle). What is the probability that it will land inside the circle with diameter 1 m?

SOLUTION

Area of the rectangular region = 3 x 2 = 6 m²

Diameter of circle = 1 m => radius r = 1/2 m
Area of circle = pi x r² = pi x (1/2)² = pi/4 m²

P(landing inside circle) = Area of circle / Area of rectangle
= (pi/4) / 6
= pi/24

Answer: pi/24

Q21 A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that
(i) she will buy it?
(ii) she will not buy it?

SOLUTION

Total pens = 144
Defective pens = 20
Good pens = 144 - 20 = 124

(i) She buys only good pens.
P(she will buy) = 124/144 = 31/36
(Dividing numerator and denominator by 4)

(ii) She will not buy a defective pen.
P(she will not buy) = 20/144 = 5/36

Verification: 31/36 + 5/36 = 36/36 = 1 ✓

Answer: (i) 31/36 (ii) 5/36

Q22 Refer to Example 13. (i) Complete the following table:

Event: 'Sum on 2 dice'	2	3	4	5	6	7	8	9	10	11	12
Probability (given some)	1/36	_	_	_	_	_	5/36	_	_	_	1/36

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, ..., 12. Therefore, each of them has a probability 1/11.' Do you agree with this argument? Justify your answer.

SOLUTION

When two dice are thrown: total outcomes = 6 x 6 = 36.

(i) Count favourable outcomes for each sum:

Sum 2: (1,1) => 1 way. P = 1/36
Sum 3: (1,2), (2,1) => 2 ways. P = 2/36 = 1/18
Sum 4: (1,3), (2,2), (3,1) => 3 ways. P = 3/36 = 1/12
Sum 5: (1,4), (2,3), (3,2), (4,1) => 4 ways. P = 4/36 = 1/9
Sum 6: (1,5), (2,4), (3,3), (4,2), (5,1) => 5 ways. P = 5/36

Sum 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) => 6 ways. P = 6/36 = 1/6
Sum 8: (2,6), (3,5), (4,4), (5,3), (6,2) => 5 ways. P = 5/36
Sum 9: (3,6), (4,5), (5,4), (6,3) => 4 ways. P = 4/36 = 1/9
Sum 10: (4,6), (5,5), (6,4) => 3 ways. P = 3/36 = 1/12
Sum 11: (5,6), (6,5) => 2 ways. P = 2/36 = 1/18
Sum 12: (6,6) => 1 way. P = 1/36

Verification: 1+2+3+4+5+6+5+4+3+2+1 = 36 ✓

(ii) No, the argument is incorrect.
The 11 outcomes (sums 2 to 12) are NOT equally likely.
For example, P(sum = 7) = 6/36 = 1/6, but P(sum = 2) = 1/36.
Only when outcomes are equally likely can we divide probability equally.
So each sum does NOT have probability 1/11.

Answer: (i) Probabilities: 1/36, 1/18, 1/12, 1/9, 5/36, 1/6, 5/36, 1/9, 1/12, 1/18, 1/36
(ii) Argument is incorrect — the 11 sums are not equally likely

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Q23 A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

SOLUTION

When a coin is tossed 3 times, total outcomes = $2^3 = 8$.
Sample space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Hanif wins if all tosses are the same: {HHH, TTT} => 2 outcomes
P(Hanif wins) = 2/8 = 1/4

P(Hanif loses) = 1 - P(Hanif wins) = 1 - 1/4 = 3/4

Answer: 3/4

Q24 A die is thrown twice. What is the probability that
(i) 5 will not come up either time?
(ii) 5 will come up at least once?
[Hint: Throwing a die twice and throwing two dice simultaneously are treated as the same experiment]

SOLUTION

When a die is thrown twice: total outcomes = 6 x 6 = 36.

(i) For 5 not to come up either time, each throw must show one of {1, 2, 3, 4, 6} => 5 choices each.
Favourable outcomes = 5 x 5 = 25
P(5 not up either time) = 25/36

(ii) P(5 at least once) = 1 - P(5 not up either time)
= 1 - 25/36
= 11/36

Verification by counting: outcomes with 5 appearing at least once:
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6) => 6; (1,5),(2,5),(3,5),(4,5),(6,5) => 5
Total = 11. P = 11/36 ✓

Answer: (i) 25/36 (ii) 11/36

Q25 Which of the following arguments are correct and which are not correct? Give reasons for your answer.
(i) If two coins are tossed simultaneously there are three possible outcomes — two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is 1/3.
(ii) If a die is thrown, there are two possible outcomes — an odd number or an even number. Therefore, the probability of getting an odd number is 1/2.

SOLUTION

(i) Not correct.
When two coins are tossed, the sample space is {HH, HT, TH, TT} => 4 equally likely outcomes.
P(two heads) = 1/4
P(two tails) = 1/4
P(one head and one tail) = 2/4 = 1/2
The three events are NOT equally likely ('one of each' can happen in 2 ways).
=> Assigning probability 1/3 to each is incorrect.

(ii) Correct.
When a die is thrown, outcomes = {1, 2, 3, 4, 5, 6}.
Odd numbers = {1, 3, 5} => 3 outcomes; Even numbers = {2, 4, 6} => 3 outcomes.
Both groups are equally likely subsets of the 6 equally likely outcomes.
P(odd) = 3/6 = 1/2 — this is correct.

Answer: (i) Not correct — the three outcomes are not equally likely (ii) Correct