Chapter 11: Thermodynamics

Exercises — Exercises

Q1 A geyser heats water flowing at the rate of 3.0 litres per minute from 27 °C to 77 °C. If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heat of combustion is 4.0×10^4 J/g?

SOLUTION

Given:

Flow rate of water = 3.0 L/min = 3.0 kg/min (density of water ≈ 1 kg/L)

Initial temperature T1 = 27 °C, final temperature T2 = 77 °C

Temperature rise $\Delta T = 77 - 27 = 50$ °C = 50 K

Specific heat of water $c = 4186$ J kg^{-1} K^{-1}

Heat of combustion of fuel $H = 4.0 \times 10^4$ J/g

Heat required per minute to warm the water:

$Q = m \times c \times \Delta T = 3.0 \times 4186 \times 50 = 627\,900$ J/min

All this heat must come from burning the fuel. If the fuel consumption rate is r g/min:

$r \times H = Q$

$r = Q / H = 627\,900 / (4.0 \times 10^4) \approx 15.7$ g/min

Answer: Rate of fuel consumption ≈ 15.7 g/min

Q2 What amount of heat must be supplied to 2.0×10^{-2} kg of nitrogen (at room temperature) to raise its temperature by 45 °C at constant pressure? (Molecular mass of N2 = 28; R = 8.3 J mol^{-1} K^{-1}.)

SOLUTION

Given:

Mass of nitrogen $m = 2.0 \times 10^{-2}$ kg = 20 g

Molecular mass M = 28 g/mol

Temperature rise $\Delta T = 45$ °C = 45 K

Process: constant pressure

R = 8.3 J mol^{-1} K^{-1}

Number of moles:

$\mu = m / M = 20 / 28 = 5/7 \approx 0.714$ mol

N2 is a diatomic molecule. At room temperature only translational and rotational degrees of freedom are active (f = 5), so:

$C_v = (5/2)\,R$ and $C_p = C_v + R = (7/2)\,R$

$C_p = (7/2) \times 8.3 = 29.05\ J\ mol^{-1}\ K^{-1}$

Heat at constant pressure:

$Q = \mu \times C_p \times \Delta T = (5/7) \times 29.05 \times 45$

$Q = 0.7143 \times 29.05 \times 45 \approx 933.8\ J$

Answer: $Q \approx 933.8\ J \approx 9.34 \times 10^2\ J$

Q3 Explain why

(a) Two bodies at different temperatures T1 and T2 if brought in thermal contact do not necessarily settle to the mean temperature (T1 + T2)/2.

(b) The coolant in a chemical or a nuclear plant (i.e., the liquid used to prevent the different parts of a plant from getting too hot) should have high specific heat.

(c) Air pressure in a car tyre increases during driving.

(d) The climate of a harbour town is more temperate than that of a town in a desert at the same latitude.

SOLUTION

(a) When two bodies at temperatures T1 and T2 (with T1 > T2) are brought into thermal contact, heat flows from the hotter to the cooler body until thermal equilibrium is reached. The equilibrium temperature depends on the heat capacities (mass × specific heat) of both bodies. Only if the two bodies have equal heat capacities does the final temperature equal (T1 + T2)/2. In general, the body with the larger heat capacity changes temperature by a smaller amount, so the equilibrium temperature is closer to the initial temperature of that body and is not simply the mean.

(b) A coolant absorbs the excess heat generated in the plant to keep temperatures within safe limits. If the coolant has a high specific heat capacity, it can absorb a large amount of heat per unit mass per degree of temperature rise. This means a smaller mass of coolant is needed for the same heat-removal task, and the coolant's own temperature rise is smaller, making it more effective at keeping the plant cool.

(c) During driving, the tyres flex repeatedly and also experience friction with the road surface. Work done against friction and internal stresses in the rubber is

converted into heat, raising the temperature of the air inside the tyre. By the ideal gas law (PV = nRT), at constant volume (the tyre volume is roughly fixed) an increase in temperature T leads directly to an increase in pressure P. Hence the air pressure in the tyre increases during driving.

(d) Water has a much higher specific heat capacity (≈ 4186 J kg^{-1} K^{-1}) than sand or soil (≈ 800 J kg^{-1} K^{-1}). The large body of sea water near a harbour town absorbs a great deal of heat in summer without becoming very hot, and releases that stored heat slowly in winter without becoming very cold. This moderates the temperature extremes felt in the town. A desert town lacks this large thermal reservoir, so it heats up sharply during the day and cools down sharply at night, giving a more extreme climate.

Answer: (a) Final temperature depends on heat capacities — equal only when both are equal. (b) High specific heat => large heat absorption per unit mass per degree rise => efficient cooling. (c) Friction and flexing heat the air inside; at constant volume, P rises with T (ideal gas law). (d) The sea acts as a large thermal reservoir with high specific heat, moderating temperature swings.

Q4 A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume?

SOLUTION

Given:

Gas: H2 (diatomic), μ = 3 mol

Walls and piston are heat-insulating => adiabatic compression

Final volume V2 = V1/2

For an adiabatic process of an ideal gas:

$P V^{\gamma} = \text{constant} \Rightarrow P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

H2 is diatomic; at room temperature (rotational modes active, vibrational modes not):

$C_v = (5/2) R, \quad C_p = (7/2) R$

$\gamma = C_p / C_v = 7/5 = 1.4$

Pressure ratio:

$P_2 / P_1 = (V_1 / V_2)^{\gamma} = (V_1 / (V_1/2))^{1.4} = 2^{1.4}$

$2^{1}.4 = 2^{(7/5)} = (2^{7})^{(1/5)} = 128^{0}.2 \approx 2.639$

Answer: The pressure increases by a factor of $2^{1}.4 \approx 2.64$

Q5 In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case? (Take 1 cal = 4.19 J)

SOLUTION

Step 1 — Find ΔU using the adiabatic path A → B.
In the adiabatic process, Q = 0.
Work done on the system = 22.3 J, so work done by the system $W_{adiabatic}$ = -22.3 J.
First Law: ΔQ = ΔU + W => 0 = ΔU + (-22.3)
ΔU = +22.3 J

Internal energy is a state function, so ΔU = 22.3 J for any path from A to B.

Step 2 — Apply First Law to the non-adiabatic path.
Heat absorbed: Q = 9.35 cal × 4.19 J/cal = 39.18 J

First Law: Q = ΔU + W (W = work done by system)
W = Q - ΔU = 39.18 - 22.3 = 16.88 J ≈ 16.9 J

Answer: Net work done by the system ≈ 16.9 J

Q6 Two cylinders A and B of equal capacity are connected to each other via a stopcock. A contains a gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened. Answer the following:
(a) What is the final pressure of the gas in A and B?
(b) What is the change in internal energy of the gas?
(c) What is the change in the temperature of the gas?
(d) Do the intermediate states of the system (before settling to the final equilibrium state) lie on its P-V-T surface?

SOLUTION

(a) Final pressure:

When the stopcock is opened, the gas in A expands freely into the evacuated cylinder B. The total volume available doubles (from V to 2V). No work is done against any external pressure (free expansion into vacuum) and no heat is exchanged (system is insulated). By the ideal gas law, at the same temperature (see part c):

$P_{final} \times 2V = P_{initial} \times V \Rightarrow P_{final} = P_{initial} / 2$

Standard pressure $P_{initial} = 1.013 \times 10^5$ Pa

$P_{final} = 0.5 \times 1.013 \times 10^5$ Pa ≈ 0.5 atm

(b) Change in internal energy:

This is a free expansion (Joule expansion) into vacuum. No work is done by the gas (W = 0, since external pressure = 0) and no heat flows (Q = 0, system is thermally insulated).

First Law: $\Delta U = Q - W = 0 - 0 = 0$

Change in internal energy = 0.

(c) Change in temperature:

For an ideal gas, internal energy depends only on temperature. Since $\Delta U = 0$, the temperature of the gas does not change.

$\Delta T = 0$ (temperature remains at 273.15 K = 0 °C).

(d) Intermediate states and the P-V-T surface:

The P-V-T surface (equation of state PV = μRT) describes only equilibrium states. During the free expansion, the gas rushes through the stopcock in a non-quasi-static, turbulent manner. Pressure and temperature are not uniform throughout the gas. These intermediate non-equilibrium states cannot be described by unique values of P, V, T and therefore do NOT lie on the equilibrium P-V-T surface.

Answer: (a) $P_{final} \approx 0.5$ atm ($\approx 0.5 \times 1.013 \times 10^5$ Pa). (b) $\Delta U = 0$. (c) $\Delta T = 0$. (d) No — intermediate states are non-equilibrium and do not lie on the P-V-T surface.

Q7 An electric heater supplies heat to a system at a rate of 100 W. If the system performs work at a rate of 75 joules per second, at what rate is the internal energy increasing?

SOLUTION

Given:

Rate of heat supplied: $dQ/dt = 100\ W = 100\ J/s$

Rate of work done by system: $dW/dt = 75\ J/s$

By the First Law of Thermodynamics:

$dU/dt = dQ/dt - dW/dt$

$dU/dt = 100 - 75 = 25\ J/s = 25\ W$

Answer: The internal energy is increasing at a rate of 25 J/s (25 W)

Q8 A thermodynamic system is taken from an original state to an intermediate state by the linear process shown in Fig. 11.11. Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F.

SOLUTION

From Fig. 11.11 (P-V diagram):

Point D: $P_D = 600\ N/m^2$, $V_D = 2.0\ m^3$

Point E: $P_E = 300\ N/m^2$, $V_E = 5.0\ m^3$

Point F: $P_F = 300\ N/m^2$, $V_F = 2.0\ m^3$

Process D → E (linear on P-V diagram — a straight line):

Work = area under the P-V curve = area of a trapezoid

$W_{DE} = ½ \times (P_D + P_E) \times (V_E - V_D)$

$W_{DE} = ½ \times (600 + 300) \times (5.0 - 2.0)$

$W_{DE} = ½ \times 900 \times 3.0 = 1350\ J$

Process E → F (isobaric at $P = 300\ N/m^2$, volume decreases from 5.0 to 2.0 m^3):

$W_{EF} = P_E \times (V_F - V_E) = 300 \times (2.0 - 5.0) = 300 \times (-3.0) = -900\ J$

(Negative because the gas is compressed — work is done on the gas.)

Total work done by the gas:

$W_{total} = W_{DE} + W_{EF} = 1350 + (-900) = 450\ J$

Answer: Total work done by the gas from D to E to F = 450 J