Chapter 5: Measures of Central Tendency

Exercises — Measures of Central Tendency

Q1 Which average would be suitable in the following cases?
(i) Average size of readymade garments.
(ii) Average intelligence of students in a class.
(iii) Average production in a factory per shift.
(iv) Average wage in an industrial concern.
(v) When the sum of absolute deviations from average is least.
(vi) When quantities of the variable are in ratios.
(vii) In case of open-ended frequency distribution.

SOLUTION

(i) Mode — garments are produced in standard sizes (S, M, L, XL); the most frequently demanded size is what matters for production planning.

(ii) Median — intelligence scores may not be normally distributed and can have outliers; median is not affected by extreme values.

(iii) Arithmetic Mean — production per shift is a quantitative, numerical variable where all values contribute equally; mean gives the overall average output.

(iv) Arithmetic Mean — wages are numerical; mean gives the average wage, used to compare remuneration across firms or industries.

(v) Median — the sum of absolute deviations ($|X - A|$) is minimum when A is the median. This is a mathematical property of the median.

(vi) Geometric Mean — when values are in ratios or percentages (e.g., growth rates, index numbers), geometric mean is the appropriate average.

(vii) Median or Mode — in open-ended distributions (e.g., "below 10" or "above 100"), the exact values in the first and last classes are unknown, so arithmetic mean cannot be computed. Median and mode can still be determined.

Answer: (i) Mode (ii) Median (iii) Arithmetic Mean (iv) Arithmetic Mean (v) Median (vi) Geometric Mean (vii) Median or Mode

Q2 Indicate the most appropriate alternative from the multiple choices provided against each question.
(i) The most suitable average for qualitative measurement is
(a) arithmetic mean (b) median (c) mode (d) geometric mean (e) none of the above
(ii) Which average is affected most by the presence of extreme items?
(a) median (b) mode (c) arithmetic mean (d) none of the above
(iii) The algebraic sum of deviation of a set of n values from A.M. is
(a) n (b) 0 (c) 1 (d) none of the above

SOLUTION

(i) Answer: (b) median
Qualitative data (e.g., ranking of beauty, intelligence, honesty) cannot be added or divided. Median gives the middle-ranked value and is the most suitable measure for such ordinal data. Mode is also used occasionally, but median is preferred for ranked qualitative data.

(ii) Answer: (c) arithmetic mean
The arithmetic mean uses all values in its calculation (sum divided by N). A single extreme value (outlier) directly changes the sum and hence the mean significantly. Median and mode are positional measures and are not affected by extreme values.

(iii) Answer: (b) 0
By definition, the arithmetic mean X-bar is such that the sum of deviations of all observations from it equals zero.
Proof: Sum of (X - X-bar) = Sum(X) - N * X-bar = N * X-bar - N * X-bar = 0.

Answer: (i) b — median; (ii) c — arithmetic mean; (iii) b — 0

Q3 Comment whether the following statements are true or false.
(i) The sum of deviation of items from median is zero.
(ii) An average alone is not enough to compare series.
(iii) Arithmetic mean is a positional value.
(iv) Upper quartile is the lowest value of top 25% of items.
(v) Median is unduly affected by extreme observations.

SOLUTION

(i) False — the sum of deviations of items from the ARITHMETIC MEAN is zero (Σ(X - X-bar) = 0). The sum of deviations from the median is not necessarily zero; however, the sum of ABSOLUTE deviations from the median is minimum.

(ii) True — averages give only a central value. Two distributions may have the same mean but differ greatly in their spread (dispersion). Measures of dispersion (range, standard deviation, etc.) are also needed to compare series meaningfully.

(iii) False — arithmetic mean is NOT a positional value; it is calculated from all observations using the formula X-bar = ΣX/N. Median and mode are positional values (they depend on the position of items in an ordered series).

(iv) True — Q3 (upper quartile / third quartile) is the value below which 75% of the data falls and above which 25% (the top quarter) lies. So Q3 is indeed the lowest value of the top 25% of items.

(v) False — median is NOT unduly affected by extreme observations. Since it is determined by the middle value in an ordered series (a positional measure), very large or very small values at the extremes do not change its value. This is one advantage of median over arithmetic mean.

Answer: (i) False (ii) True (iii) False (iv) True (v) False

Q4 If the arithmetic mean of the data given below is 28, find (a) the missing frequency, and (b) the median of the series:
Profit per retail shop (in Rs): 0-10, 10-20, 20-30, 30-40, 40-50, 50-60
Number of retail shops: 12, 18, 27, ?, 17, 6

SOLUTION

Let the missing frequency (class 30-40) = f. Midpoints (m): 5, 15, 25, 35, 45, 55.
N = 12 + 18 + 27 + f + 17 + 6 = 80 + f
Sf*m = 12x5 + 18x15 + 27x25 + f x35 + 17x45 + 6x55
= 60 + 270 + 675 + 35f + 765 + 330 = 2100 + 35f

(a) Mean = Sf*m / N = 28
(2100 + 35f) / (80 + f) = 28 => 2100 + 35f = 2240 + 28f => 7f = 140 => f = 20
So the missing frequency = 20 and N = 100.

(b) Cumulative frequencies: 12, 30, 57, 77, 94, 100.
N/2 = 50 lies in the class 20-30 (cf before = 30, class frequency = 27), so 20-30 is the median class.
Median = L + ((N/2 - cf)/f) x h = 20 + ((50 - 30)/27) x 10 = 20 + 7.41 = 27.41

Answer: (a) Missing frequency = 20; (b) Median = Rs 27.41

Q5 The following table gives the daily income of ten workers in a factory. Find the arithmetic mean.
Workers: A, B, C, D, E, F, G, H, I, J
Daily Income (in Rs): 120, 150, 180, 200, 250, 300, 220, 350, 370, 260

SOLUTION

N = 10 workers
ΣX = 120 + 150 + 180 + 200 + 250 + 300 + 220 + 350 + 370 + 260
= 2400

Arithmetic Mean = ΣX / N = 2400 / 10 = Rs 240

Answer: Arithmetic Mean = Rs 240

Q6 Following information pertains to the daily income of 150 families. Calculate the arithmetic mean.
Income (in Rs): More than 75, More than 85, More than 95, More than 105, More than 115, More than 125, More than 135, More than 145
Number of families: 150, 140, 115, 95, 70, 60, 40, 25

SOLUTION

This is a more-than cumulative frequency distribution. Convert to an ordinary frequency table:

CI	f	Midpoint (m)	fm
75-85	150-140=10	80	800
85-95	140-115=25	90	2250
95-105	115-95=20	100	2000
105-115	95-70=25	110	2750
115-125	70-60=10	120	1200
125-135	60-40=20	130	2600
135-145	40-25=15	140	2100
145-155	25	150	3750

Σf = 10+25+20+25+10+20+15+25 = 150 (checks out)
Σfm = 800+2250+2000+2750+1200+2600+2100+3750 = 17450

Arithmetic Mean = Σfm / Σf = 17450 / 150 = Rs 116.3

Answer: Arithmetic Mean = Rs 116.3

Q7 The size of land holdings of 380 families in a village is given below. Find the median size of land holdings.
Size of Land Holdings (in acres): Less than 100, 100-200, 200-300, 300-400, 400 and above
Number of families: 40, 89, 148, 64, 39

SOLUTION

N = 40 + 89 + 148 + 64 + 39 = 380
N/2 = 190

Cumulative frequency table:

CI	f	cf
0-100	40	40
100-200	89	129

CI	f	cf
200-300	148	277
300-400	64	341
400-500	39	380

The 190th item falls in class 200-300 (cf crosses 190 between 129 and 277).
Median class: 200-300; L = 200, cf = 129, f = 148, h = 100

Median = L + ((N/2 - cf) / f) × h
= 200 + ((190 - 129) / 148) × 100
= 200 + (61 / 148) × 100
= 200 + 41.22
= 241.22 acres

Answer: Median size of land holding = 241.22 acres

Q8 The following series relates to the daily income of workers employed in a firm. Compute (a) highest income of lowest 50% workers (b) minimum income earned by the top 25% workers and (c) maximum income earned by lowest 25% workers.
Daily Income (in Rs): 10-14, 15-19, 20-24, 25-29, 30-34, 35-39
Number of workers: 5, 10, 15, 20, 10, 5

SOLUTION

Convert inclusive class intervals to exclusive by subtracting 0.5 from lower limit and adding 0.5 to upper limit:

CI (exclusive)	f	cf
9.5-14.5	5	5
14.5-19.5	10	15
19.5-24.5	15	30
24.5-29.5	20	50
29.5-34.5	10	60
34.5-39.5	5	65

N = 65

(a) Highest income of lowest 50% = Median (N/2 = 32.5th item)
32.5 falls in class 24.5-29.5 (cf crosses 32.5 between 30 and 50).
L = 24.5, cf = 30, f = 20, h = 5
Median = 24.5 + ((32.5 - 30) / 20) × 5 = 24.5 + 0.625 = Rs 25.12

(b) Minimum income of top 25% = Q3 (3N/4 = 48.75th item)
48.75 falls in class 24.5-29.5 (30 < 48.75 < 50).
L = 24.5, cf = 30, f = 20, h = 5
Q3 = 24.5 + ((48.75 - 30) / 20) × 5 = 24.5 + 4.6875 = Rs 29.19

(c) Maximum income of lowest 25% = Q1 (N/4 = 16.25th item)
16.25 falls in class 19.5-24.5 (15 < 16.25 < 30).
L = 19.5, cf = 15, f = 15, h = 5
Q1 = 19.5 + ((16.25 - 15) / 15) × 5 = 19.5 + 0.4167 = Rs 19.92

Answer: (a) Median = Rs 25.12 (b) Q3 = Rs 29.19 (c) Q1 = Rs 19.92

Q9 The following table gives production yield in kg. per hectare of wheat of 150 farms in a village. Calculate the mean, median and mode values.
Production yield (kg. per hectare): 50-53, 53-56, 56-59, 59-62, 62-65, 65-68, 68-71, 71-74, 74-77
Number of farms: 3, 8, 14, 30, 36, 28, 16, 10, 5

SOLUTION

N = 3+8+14+30+36+28+16+10+5 = 150
Class width h = 3

MEAN (Direct Method):

CI	f	m	fm
50-53	3	51.5	154.5
53-56	8	54.5	436.0
56-59	14	57.5	805.0
59-62	30	60.5	1815.0
62-65	36	63.5	2286.0
65-68	28	66.5	1862.0
68-71	16	69.5	1112.0
71-74	10	72.5	725.0
74-77	5	75.5	377.5

Σfm = 154.5+436+805+1815+2286+1862+1112+725+377.5 = 9573
Mean = Σfm / N = 9573 / 150 = 63.82 kg/hectare

MEDIAN:
Cumulative frequencies:
50-53: cf=3; 53-56: cf=11; 56-59: cf=25; 59-62: cf=55; 62-65: cf=91; 65-68: cf=119; 68-71:

cf=135; 71-74: cf=145; 74-77: cf=150
N/2 = 75 → falls in class 62-65 (cf crosses 75 between 55 and 91).
L = 62, cf = 55, f = 36, h = 3
Median = 62 + ((75 - 55) / 36) × 3 = 62 + (20/36) × 3 = 62 + 1.67 = 63.67 kg/hectare

MODE:
Modal class = 62-65 (highest frequency = 36).
D1 = 36 - 30 = 6 (freq of modal class minus freq of preceding class)
D2 = 36 - 28 = 8 (freq of modal class minus freq of succeeding class)
L = 62, h = 3
Mode = L + (D1 / (D1 + D2)) × h
= 62 + (6 / (6 + 8)) × 3
= 62 + (6/14) × 3
= 62 + 1.286
= 63.29 kg/hectare

Answer: Mean = 63.82 kg/hectare; Median = 63.67 kg/hectare; Mode = 63.29 kg/hectare