Chapter 1: Sets

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Exercise 1.1 — Exercise 1.1

Q1 Which of the following are sets? Justify your answer.
(i) The collection of all the months of a year beginning with the letter J.
(ii) The collection of ten most talented writers of India.
(iii) A team of eleven best-cricket batsmen of the world.
(iv) The collection of all boys in your class.
(v) The collection of all natural numbers less than 100.
(vi) A collection of novels written by the writer Munshi Prem Chand.
(vii) The collection of all even integers.
(viii) The collection of questions in this Chapter.
(ix) A collection of most dangerous animals of the world.

SOLUTION

(i) The months of a year beginning with J are January, June, and July — clearly determined. This IS a set.

(ii) 'Most talented' is subjective; different people may choose different writers. This is NOT a well-defined collection, so it is NOT a set.

(iii) 'Best-cricket batsmen' is a matter of opinion and cannot be objectively decided. This is NOT a set.

(iv) The boys in your class can be clearly identified. This IS a set.

(v) Natural numbers less than 100 are precisely determined: 1, 2, 3, ..., 99. This IS a set.

(vi) The novels written by Munshi Prem Chand are historically recorded and can be listed. This IS a set.

(vii) Even integers are precisely defined (integers divisible by 2). This IS a set.

(viii) The questions in this chapter are fixed and listed. This IS a set.

(ix) 'Most dangerous' is subjective and depends on the criterion used. This is NOT a set.

Answer: (i), (iv), (v), (vi), (vii), (viii) are sets. (ii), (iii), (ix) are not sets because the criterion for selection is subjective and not well-defined.

Q2 Let A = {1, 2, 3, 4, 5, 6}. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces:
(i) 5 ... A
(ii) 8 ... A
(iii) 0 ... A
(iv) 4 ... A
(v) 2 ... A
(vi) 10 ... A

SOLUTION

A = {1, 2, 3, 4, 5, 6}.
(i) $5 \in A$, since 5 is an element of A.
(ii) $8 \notin A$, since 8 is not an element of A.
(iii) $0 \notin A$, since 0 is not an element of A.
(iv) $4 \in A$, since 4 is an element of A.
(v) $2 \in A$, since 2 is an element of A.
(vi) $10 \notin A$, since 10 is not an element of A.

Answer: (i) $5 \in A$ (ii) $8 \notin A$ (iii) $0 \notin A$ (iv) $4 \in A$ (v) $2 \in A$ (vi) $10 \notin A$

Q3 Write the following sets in roster form:
(i) A = {x : x is an integer and –3 ≤ x < 7}
(ii) B = {x : x is a natural number less than 6}
(iii) C = {x : x is a two-digit natural number such that the sum of its digits is 8}
(iv) D = {x : x is a prime number which is divisor of 60}
(v) E = The set of all letters in the word TRIGONOMETRY
(vi) F = The set of all letters in the word BETTER

SOLUTION

(i) Integers x with –3 ≤ x < 7: A = {–3, –2, –1, 0, 1, 2, 3, 4, 5, 6}.

(ii) Natural numbers less than 6: B = {1, 2, 3, 4, 5}.

(iii) Two-digit natural numbers with digit sum 8:
17 (1+7=8), 26 (2+6=8), 35 (3+5=8), 44 (4+4=8), 53 (5+3=8), 62 (6+2=8), 71 (7+1=8), 80 (8+0=8).
C = {17, 26, 35, 44, 53, 62, 71, 80}.

(iv) Prime factors of 60: 60 = 2 × 2 × 3 × 5, so the prime divisors are 2, 3, 5.
D = {2, 3, 5}.

(v) Letters in TRIGONOMETRY (each distinct letter listed once):
T, R, I, G, O, N, M, E, Y — E = {T, R, I, G, O, N, O, M, E, T, R, Y} → E = {T, R, I, G, O, N, M, E, Y}.

(vi) Letters in BETTER (each distinct letter listed once): B, E, T, R.
F = {B, E, T, R}.

Answer: (i) A = {–3, –2, –1, 0, 1, 2, 3, 4, 5, 6} (ii) B = {1, 2, 3, 4, 5} (iii) C = {17, 26, 35, 44, 53, 62, 71, 80} (iv) D = {2, 3, 5} (v) E = {T, R, I, G, O, N, M, E, Y} (vi) F = {B, E, T, R}

Q4 Write the following sets in the set-builder form:
(i) {3, 6, 9, 12}
(ii) {2, 4, 8, 16, 32}
(iii) {5, 25, 125, 625}
(iv) {2, 4, 6, ...}
(v) {1, 4, 9, ..., 100}

SOLUTION

(i) {3, 6, 9, 12} — multiples of 3 from 3×1 to 3×4.

{x : x = 3n, n ∈ N and 1 ≤ n ≤ 4}.

(ii) {2, 4, 8, 16, 32} — powers of 2: 2^1, 2^2, 2^3, 2^4, 2^5.
{x : x = 2^n, n ∈ N and 1 ≤ n ≤ 5}.

(iii) {5, 25, 125, 625} — powers of 5: 5^1, 5^2, 5^3, 5^4.
{x : x = 5^n, n ∈ N and 1 ≤ n ≤ 4}.

(iv) {2, 4, 6, ...} — all positive even integers.
{x : x is an even natural number}.

(v) {1, 4, 9, ..., 100} — perfect squares from 1^2 to 10^2.
{x : x = n^2, n ∈ N and 1 ≤ n ≤ 10}.

Answer: (i) {x : x = 3n, n ∈ N, 1 ≤ n ≤ 4} (ii) {x : x = 2^n, n ∈ N, 1 ≤ n ≤ 5} (iii) {x : x = 5^n, n ∈ N, 1 ≤ n ≤ 4} (iv) {x : x is an even natural number} (v) {x : x = n^2, n ∈ N, 1 ≤ n ≤ 10}

Q5 List all the elements of the following sets:
(i) A = {x : x is an odd natural number}
(ii) B = {x : x is an integer, –1/2 < x < 9/2}
(iii) C = {x : x is an integer, x^2 ≤ 4}
(iv) D = {x : x is a letter in the word "LOYAL"}
(v) E = {x : x is a month of a year not having 31 days}
(vi) F = {x : x is a consonant in the English alphabet which precedes k}

SOLUTION

(i) Odd natural numbers: A = {1, 3, 5, 7, 9, 11, ...} — this is an infinite set.

(ii) Integers x with –1/2 < x < 9/2, i.e., –0.5 < x < 4.5:
Integers in this range: 0, 1, 2, 3, 4.
B = {0, 1, 2, 3, 4}.

(iii) Integers x with x^2 ≤ 4: x^2 ≤ 4 –2 ≤ x ≤ 2.
Integers: –2, –1, 0, 1, 2.
C = {–2, –1, 0, 1, 2}.

(iv) Letters in 'LOYAL' (distinct): L, O, Y, A.
D = {L, O, Y, A}.

(v) Months not having 31 days: February (28/29 days), April (30), June (30), September (30), November (30).
E = {February, April, June, September, November}.

(vi) Consonants in the English alphabet that precede k (a, b, c, ..., j), taking only consonants (not vowels a, e, i):
b, c, d, f, g, h, j.
F = {b, c, d, f, g, h, j}.

Answer: (i) A = {1, 3, 5, 7, ...} (ii) B = {0, 1, 2, 3, 4} (iii) C = {–2, –1, 0, 1, 2} (iv) D = {L, O, Y, A} (v) E = {February, April, June, September, November} (vi) F = {b, c, d, f, g, h, j}

Q6 Match each of the set on the left in the roster form with the same set on the right described in set-builder form:
(i) {1, 2, 3, 6} (a) {x : x is a prime number and a divisor of 6}
(ii) {2, 3} (b) {x : x is an odd natural number less than 10}
(iii) {M, A, T, H, E, I, C, S} (c) {x : x is natural number and divisor of 6}
(iv) {1, 3, 5, 7, 9} (d) {x : x is a letter of the word MATHEMATICS}

SOLUTION

(i) {1, 2, 3, 6}: These are the natural number divisors of 6 (1, 2, 3, 6). This matches (c).

(ii) {2, 3}: These are the prime numbers that divide 6 (since 6 = 2 × 3). This matches (a).

(iii) {M, A, T, H, E, I, C, S}: The word MATHEMATICS has letters M, A, T, H, E, M, A, T, I, C, S — distinct letters are M, A, T, H, E, I, C, S. This matches (d).

(iv) {1, 3, 5, 7, 9}: These are the odd natural numbers less than 10. This matches (b).

Answer: (i) → (c), (ii) → (a), (iii) → (d), (iv) → (b)

Exercise 1.2 — Exercise 1.2

Q1 Which of the following are examples of the null set?
(i) Set of odd natural numbers divisible by 2
(ii) Set of even prime numbers
(iii) {x : x is a natural number, x < 5 and x > 7}
(iv) {y : y is a point common to any two parallel lines}

SOLUTION

(i) No odd natural number is divisible by 2 (odd and even are mutually exclusive). This set has NO elements — it IS a null set.

(ii) The even prime numbers: 2 is the only even prime. So this set = {2}, which is NOT empty.

(iii) There is no natural number that is both less than 5 and greater than 7 simultaneously. This set has NO elements — it IS a null set.

(iv) Two parallel lines, by definition, never meet. So they have no common point. This set has NO elements — it IS a null set.

Answer: (i), (iii), and (iv) are null sets. (ii) is not a null set since it contains the element 2.

Q2 Which of the following sets are finite or infinite?
(i) The set of months of a year
(ii) {1, 2, 3, ...}
(iii) {1, 2, 3, ..., 99, 100}
(iv) The set of positive integers greater than 100
(v) The set of prime numbers less than 99

SOLUTION

(i) There are 12 months in a year — a definite finite number. FINITE.

(ii) {1, 2, 3, ...} is the set of all natural numbers, which goes on indefinitely. INFINITE.

(iii) {1, 2, 3, ..., 99, 100} has exactly 100 elements. FINITE.

(iv) Positive integers greater than 100 are 101, 102, 103, ... — there is no last such integer. INFINITE.

(v) There are finitely many primes less than 99 (they are 2, 3, 5, 7, ..., 97 — a

specific finite list). FINITE.

Answer: (i) Finite (ii) Infinite (iii) Finite (iv) Infinite (v) Finite

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Q3 State whether each of the following set is finite or infinite:
(i) The set of lines which are parallel to the x-axis
(ii) The set of letters in the English alphabet
(iii) The set of numbers which are multiples of 5
(iv) The set of animals living on the earth
(v) The set of circles passing through the origin (0, 0)

SOLUTION

(i) For every real number c (other than 0), the line y = c is parallel to the x-axis. There are infinitely many such values of c. INFINITE.

(ii) The English alphabet has 26 letters — a definite finite count. FINITE.

(iii) Multiples of 5 are 5, 10, 15, 20, ... — the sequence never ends. INFINITE.

(iv) While the number of animals currently living on earth is very large, it is a definite finite number at any given time. FINITE.

(v) A circle through the origin can have any centre (a, b) with radius $\sqrt{(a^2+b^2)}$. The centre can be any point in the plane, giving infinitely many circles. INFINITE.

Answer: (i) Infinite (ii) Finite (iii) Infinite (iv) Finite (v) Infinite

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Q4 In the following, state whether A = B or not:
(i) A = {a, b, c, d}, B = {d, c, b, a}
(ii) A = {4, 8, 12, 16}, B = {8, 4, 16, 18}
(iii) A = {2, 4, 6, 8, 10}, B = {x : x is positive even integer and x ≤ 10}
(iv) A = {x : x is a multiple of 10}, B = {10, 15, 20, 25, 30, ...}

SOLUTION

(i) A = {a, b, c, d} and B = {d, c, b, a}. Both sets contain exactly the same four elements (order does not matter in sets). A = B.

(ii) A = {4, 8, 12, 16} and B = {8, 4, 16, 18}. A contains 12 but B does not; B contains 18 but A does not. A ≠ B.

(iii) A = {2, 4, 6, 8, 10} and B = {positive even integers ≤ 10} = {2, 4, 6, 8, 10}. Both sets are identical. A = B.

(iv) A = {multiples of 10} = {10, 20, 30, 40, ...} and B = {10, 15, 20, 25, 30, ...}. B contains 15, which is not a multiple of 10, but 15 $\notin$ A. So A $\neq$ B.

Answer: (i) A = B (ii) A $\neq$ B (iii) A = B (iv) A $\neq$ B

Q5 Are the following pair of sets equal? Give reasons.
(i) A = {2, 3}, B = {x : x is solution of $x^2 + 5x + 6 = 0$}
(ii) A = {x : x is a letter in the word FOLLOW}, B = {y : y is a letter in the word WOLF}

SOLUTION

(i) Solving $x^2 + 5x + 6 = 0$: $x^2 + 5x + 6 = (x + 2)(x + 3) = 0$, so x = –2 or x = –3.
B = {–2, –3}. But A = {2, 3}.
2 $\in$ A but 2 $\notin$ B; –2 $\in$ B but –2 $\notin$ A. Therefore A $\neq$ B.

(ii) Letters in FOLLOW (distinct): F, O, L, W → A = {F, O, L, W}.
Letters in WOLF (distinct): W, O, L, F → B = {W, O, L, F}.
Both sets contain exactly the same elements. Therefore A = B.

Answer: (i) A $\neq$ B (A = {2, 3} but B = {–2, –3}) (ii) A = B (both equal {F, O, L, W})

Q6 From the sets given below, select equal sets:
A = {2, 4, 8, 12}, B = {1, 2, 3, 4}, C = {4, 8, 12, 14}, D = {3, 1, 4, 2},
E = {–1, 1}, F = {0, a}, G = {1, –1}, H = {0, 1}

SOLUTION

Compare all sets element by element:
A = {2, 4, 8, 12} — contains 8 and 12, which appear only in A and C.
B = {1, 2, 3, 4} and D = {3, 1, 4, 2}: Both have elements 1, 2, 3, 4 (same four elements). B = D.
E = {–1, 1} and G = {1, –1}: Both have elements 1 and –1 (same two elements). E = G.
C = {4, 8, 12, 14}: 14 appears nowhere else; C is not equal to any other set.
F = {0, a}: contains a (a letter), not equal to any numerical set here.
H = {0, 1}: contains 0 and 1, not equal to any other set.
A has 8 and 12, not shared with B, D, E, G, F, H.

Equal pairs: B = D and E = G.

Answer: B = D and E = G are the equal pairs.

Exercise 1.3 — Exercise 1.3

Q1 Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces:
(i) {2, 3, 4} ... {1, 2, 3, 4, 5}
(ii) {a, b, c} ... {b, c, d}
(iii) {x : x is a student of Class XI of your school} ... {x : x is student of your school}
(iv) {x : x is a circle in the plane} ... {x : x is a circle in the same plane with radius 1 unit}
(v) {x : x is a triangle in a plane} ... {x : x is a rectangle in the plane}
(vi) {x : x is an equilateral triangle in a plane} ... {x : x is a triangle in the same plane}
(vii) {x : x is an even natural number} ... {x : x is an integer}

SOLUTION

(i) Every element of {2, 3, 4} (namely 2, 3, 4) belongs to {1, 2, 3, 4, 5}. So {2, 3, 4} $\subset$ {1, 2, 3, 4, 5}.

(ii) $a \in \{a, b, c\}$ but $a \notin \{b, c, d\}$. So {a, b, c} $\not\subset$ {b, c, d}.

(iii) Every Class XI student is also a student of the school. So {Class XI students} $\subset$ {all school students}.

(iv) A circle with radius 2 units is a circle in the plane but not a circle of radius 1 unit. So the set of all circles $\not\subset$ the set of circles with radius 1 unit.

(v) A triangle is not a rectangle, so no element of {triangles} belongs to {rectangles}. {triangles} $\not\subset$ {rectangles}.

(vi) Every equilateral triangle is a triangle. So {equilateral triangles} $\subset$ {triangles}.

(vii) Every even natural number is an integer. So {even natural numbers} $\subset$ {integers}.

Answer: (i) $\subset$ (ii) $\not\subset$ (iii) $\subset$ (iv) $\not\subset$ (v) $\not\subset$ (vi) $\subset$ (vii) $\subset$

Q2 Examine whether the following statements are true or false:
(i) {a, b} $\not\subset$ {b, c, a}
(ii) {a, e} $\subset$ {x : x is a vowel in the English alphabet}
(iii) {1, 2, 3} $\subset$ {1, 3, 5}
(iv) {a} $\subset$ {a, b, c}
(v) {a} $\in$ {a, b, c}
(vi) {x : x is an even natural number less than 6} $\subset$ {x : x is a natural number which divides 36}

SOLUTION

(i) $a \in \{a, b\}$ and $a \in \{b, c, a\}$; $b \in \{a, b\}$ and $b \in \{b, c, a\}$. So $\{a, b\} \subset \{b, c, a\}$. The statement '$\{a, b\} \not\subset \{b, c, a\}$' is FALSE.

(ii) Vowels in English: a, e, i, o, u. Both a and e are vowels, so $\{a, e\} \subset \{\text{vowels}\}$. TRUE.

(iii) $2 \in \{1, 2, 3\}$ but $2 \notin \{1, 3, 5\}$. So $\{1, 2, 3\} \not\subset \{1, 3, 5\}$. FALSE.

(iv) $a \in \{a\}$ and $a \in \{a, b, c\}$, so $\{a\} \subset \{a, b, c\}$. TRUE.

(v) The elements of $\{a, b, c\}$ are a, b, c (individual letters). The set $\{a\}$ is not one of these elements; it is a set, not an element. So $\{a\} \notin \{a, b, c\}$. FALSE.

(vi) Even natural numbers less than 6: {2, 4}. Divisors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36. Both 2 and 4 divide 36. So $\{2, 4\} \subset \{\text{divisors of 36}\}$. TRUE.

Answer: (i) False (ii) True (iii) False (iv) True (v) False (vi) True

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Q3 Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are incorrect and why?

(i) $\{3, 4\} \subset A$

(ii) $\{3, 4\} \in A$

(iii) $\{\{3, 4\}\} \subset A$

(iv) $1 \in A$

(v) $1 \subset A$

(vi) $\{1, 2, 5\} \subset A$

(vii) $\{1, 2, 5\} \in A$

(viii) $\{1, 2, 3\} \subset A$

(ix) $\varphi \in A$

(x) $\varphi \subset A$

(xi) $\{\varphi\} \subset A$

SOLUTION

$A = \{1, 2, \{3, 4\}, 5\}$. The elements of A are: 1, 2, {3, 4}, and 5. Note that {3, 4} is a single element (a set) of A, not two separate elements.

(i) $\{3, 4\} \subset A$ means $3 \in A$ and $4 \in A$. But 3 and 4 are not elements of A (only {3, 4} as a whole is). INCORRECT.

(ii) $\{3, 4\} \in A$ means {3, 4} is an element of A, which is true since {3, 4} is listed in A. CORRECT.

(iii) $\{\{3, 4\}\} \subset A$ means $\{3, 4\} \in A$, which is true. So $\{\{3, 4\}\} \subset A$. CORRECT.

(iv) $1 \in A$ — 1 is an element of A. CORRECT.

(v) $1 \subset A$ — 1 is a number (not a set), so $1 \subset A$ is meaningless/incorrect (subset relation is only defined between sets). INCORRECT.

(vi) $\{1, 2, 5\} \subset A$ — 1, 2, 5 are all elements of A. CORRECT.

(vii) $\{1, 2, 5\} \in A$ — the elements of A are 1, 2, {3, 4}, 5; the set {1, 2, 5} is not listed as an element. INCORRECT.

(viii) $\{1, 2, 3\} \subset A$ — $3 \in \{1, 2, 3\}$ but $3 \notin A$ (only $\{3, 4\} \in A$, not 3 itself). INCORRECT.

(ix) $\varphi \in A$ — φ is not listed among the elements 1, 2, {3, 4}, 5 of A. INCORRECT.

(x) $\varphi \subset A$ — the empty set is a subset of every set. CORRECT.

(xi) $\{\varphi\} \subset A$ means $\varphi \in A$. But φ is not an element of A (see (ix)). INCORRECT.

Answer: Incorrect statements: (i), (v), (vii), (viii), (ix), (xi). Correct statements: (ii), (iii), (iv), (vi), (x).

Q4 Write down all the subsets of the following sets:
(i) {a}
(ii) {a, b}
(iii) {1, 2, 3}
(iv) φ

SOLUTION

(i) Subsets of {a}: the empty set and {a} itself.
φ, {a}.

(ii) Subsets of {a, b}: φ, {a}, {b}, {a, b}.

(iii) Subsets of {1, 2, 3}: φ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}.
(Total: $2^3 = 8$ subsets.)

(iv) Subsets of φ: only φ itself.
φ.

Answer: (i) φ, {a} (ii) φ, {a}, {b}, {a, b} (iii) φ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3} (iv) φ

Q5 Write the following as intervals:
(i) $\{x : x \in R, -4 < x \leq 6\}$
(ii) $\{x : x \in R, -12 < x < -10\}$
(iii) $\{x : x \in R, 0 \leq x < 7\}$
(iv) $\{x : x \in R, 3 \leq x \leq 4\}$

SOLUTION

(i) $-4 < x \leq 6$: open at –4, closed at 6. Interval: $(-4, 6]$.

(ii) $-12 < x < -10$: open at both ends. Interval: $(-12, -10)$.

(iii) $0 \leq x < 7$: closed at 0, open at 7. Interval: $[0, 7)$.

(iv) $3 \leq x \leq 4$: closed at both ends. Interval: $[3, 4]$.

Answer: (i) $(-4, 6]$ (ii) $(-12, -10)$ (iii) $[0, 7)$ (iv) $[3, 4]$

Q6 Write the following intervals in set-builder form:
(i) $(-3, 0)$
(ii) $[6, 12]$
(iii) $(6, 12]$
(iv) $[-23, 5)$

SOLUTION

(i) $(-3, 0)$ is the open interval: $\{x : x \in R, -3 < x < 0\}$.

(ii) $[6, 12]$ is the closed interval: $\{x : x \in R, 6 \leq x \leq 12\}$.

(iii) $(6, 12]$ is open at 6, closed at 12: $\{x : x \in R, 6 < x \leq 12\}$.

(iv) $[-23, 5)$ is closed at –23, open at 5: $\{x : x \in R, -23 \leq x < 5\}$.

Answer: (i) $\{x : x \in R, -3 < x < 0\}$ (ii) $\{x : x \in R, 6 \leq x \leq 12\}$ (iii) $\{x : x \in R, 6 < x \leq 12\}$ (iv) $\{x : x \in R, -23 \leq x < 5\}$

Q7 What universal set(s) would you propose for each of the following:
(i) The set of right triangles.
(ii) The set of isosceles triangles.

SOLUTION

(i) The set of right triangles is a subset of the set of all triangles, which is itself a subset of the set of all polygons or all plane figures. Suitable universal sets include:

— The set of all triangles in a plane, or
— The set of all polygons in a plane.

(ii) The set of isosceles triangles is also a subset of the set of all triangles. Suitable universal sets include:
— The set of all triangles in a plane, or
— The set of all polygons in a plane.

Answer: For both (i) and (ii): suitable universal sets are the set of all triangles in a plane, or more broadly the set of all polygons (or all plane figures).

Q8 Given the sets A = {1, 3, 5}, B = {2, 4, 6} and C = {0, 2, 4, 6, 8}, which of the following may be considered as universal set(s) for all the three sets A, B and C?
(i) {0, 1, 2, 3, 4, 5, 6}
(ii) φ
(iii) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
(iv) {1, 2, 3, 4, 5, 6, 7, 8}

SOLUTION

A universal set must contain every element of A, B, and C.

(i) U_1 = {0, 1, 2, 3, 4, 5, 6}: C = {0, 2, 4, 6, 8}. The element 8 $\in$ C but 8 $\notin U_1$. So U_1 cannot be a universal set. NOT suitable.

(ii) φ is the empty set and contains no elements, so A, B, C cannot be subsets of φ. NOT suitable.

(iii) U_3 = {0,1,2,3,4,5,6,7,8,9,10}: Check A $\subset U_3$ (yes), B $\subset U_3$ (yes), C $\subset U_3$ (yes). SUITABLE.

(iv) U_4 = {1,2,3,4,5,6,7,8}: C = {0,2,4,6,8}. The element 0 $\in$ C but 0 $\notin U_4$. NOT suitable.

Answer: Only (iii) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} can serve as a universal set for A, B, and C.

Exercise 1.4 — Exercise 1.4

Q1 Find the union of each of the following pairs of sets:
(i) X = {1, 3, 5}, Y = {1, 2, 3}
(ii) A = {a, e, i, o, u}, B = {a, b, c}
(iii) A = {x : x is a natural number and multiple of 3}, B = {x : x is a natural number less than 6}
(iv) A = {x : x is a natural number and $1 < x \leq 6$}, B = {x : x is a natural number and $6 < x < 10$}
(v) A = {1, 2, 3}, B = φ

SOLUTION

(i) $X \cup Y = \{1, 3, 5\} \cup \{1, 2, 3\} = \{1, 2, 3, 5\}$.

(ii) $A \cup B = \{a, e, i, o, u\} \cup \{a, b, c\} = \{a, b, c, e, i, o, u\}$.

(iii) A = {3, 6, 9, 12, ...} (multiples of 3), B = {1, 2, 3, 4, 5}.
$A \cup B = \{1, 2, 3, 4, 5, 6, 9, 12, 15, ...\}$.
(All natural numbers that are either in B or multiples of 3.)

(iv) A = {2, 3, 4, 5, 6}, B = {7, 8, 9}.
$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$.

(v) $A \cup \varphi = A = \{1, 2, 3\}$. (Union with the empty set leaves the set unchanged.)

Answer: (i) {1, 2, 3, 5} (ii) {a, b, c, e, i, o, u} (iii) {1, 2, 3, 4, 5, 6, 9, 12, 15, ...} (iv) {2, 3, 4, 5, 6, 7, 8, 9} (v) {1, 2, 3}

Q2 Let A = {a, b}, B = {a, b, c}. Is $A \subset B$? What is $A \cup B$?

SOLUTION

A = {a, b} and B = {a, b, c}.
Every element of A (a and b) is also in B. Therefore $A \subset B$. Yes, A is a subset of B.

$A \cup B = \{a, b\} \cup \{a, b, c\} = \{a, b, c\} = B$.

Answer: Yes, $A \subset B$. $A \cup B = \{a, b, c\} = B$.

Q3 If A and B are two sets such that A ⊂ B, then what is A ∪ B?

SOLUTION

Since A ⊂ B, every element of A is already in B.
Therefore, the union A ∪ B contains no new elements beyond those in B.
Hence A ∪ B = B.

Answer: A ∪ B = B.

Q4 If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {5, 6, 7, 8} and D = {7, 8, 9, 10}; find
(i) A ∪ B
(ii) A ∪ C
(iii) B ∪ C
(iv) B ∪ D
(v) A ∪ B ∪ C
(vi) A ∪ B ∪ D
(vii) B ∪ C ∪ D

SOLUTION

(i) A ∪ B = {1, 2, 3, 4} ∪ {3, 4, 5, 6} = {1, 2, 3, 4, 5, 6}.

(ii) A ∪ C = {1, 2, 3, 4} ∪ {5, 6, 7, 8} = {1, 2, 3, 4, 5, 6, 7, 8}.

(iii) B ∪ C = {3, 4, 5, 6} ∪ {5, 6, 7, 8} = {3, 4, 5, 6, 7, 8}.

(iv) B ∪ D = {3, 4, 5, 6} ∪ {7, 8, 9, 10} = {3, 4, 5, 6, 7, 8, 9, 10}.

(v) A ∪ B ∪ C = {1, 2, 3, 4, 5, 6} ∪ {5, 6, 7, 8} = {1, 2, 3, 4, 5, 6, 7, 8}.

(vi) A ∪ B ∪ D = {1, 2, 3, 4, 5, 6} ∪ {7, 8, 9, 10} = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

(vii) B ∪ C ∪ D = {3, 4, 5, 6, 7, 8} ∪ {7, 8, 9, 10} = {3, 4, 5, 6, 7, 8, 9, 10}.

Answer: (i) {1,2,3,4,5,6} (ii) {1,2,3,4,5,6,7,8} (iii) {3,4,5,6,7,8} (iv) {3,4,5,6,7,8,9,10} (v) {1,2,3,4,5,6,7,8} (vi) {1,2,3,4,5,6,7,8,9,10} (vii) {3,4,5,6,7,8,9,10}

Q5 Find the intersection of each pair of sets of question 1 above.

SOLUTION

(i) X ∩ Y = {1, 3, 5} ∩ {1, 2, 3} = {1, 3}.

(ii) A ∩ B = {a, e, i, o, u} ∩ {a, b, c} = {a}.

(iii) A = {multiples of 3} = {3, 6, 9, ...}, B = {1, 2, 3, 4, 5}.

A ∩ B = {3} (the only multiple of 3 that is also a natural number less than 6, apart from... 3 itself, since 6 is not < 6).
A ∩ B = {3}.

(iv) A = {2, 3, 4, 5, 6}, B = {7, 8, 9}. No common elements.
A ∩ B = φ.

(v) A ∩ φ = φ. (Intersection with the empty set is always empty.)

Answer: (i) {1, 3} (ii) {a} (iii) {3} (iv) φ (v) φ

Q6 If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, C = {11, 13, 15} and D = {15, 17}; find
(i) A ∩ B
(ii) B ∩ C
(iii) A ∩ C ∩ D
(iv) A ∩ C
(v) B ∩ D
(vi) A ∩ (B ∪ C)
(vii) A ∩ D
(viii) A ∩ (B ∪ D)
(ix) (A ∩ B) ∩ (B ∪ C)
(x) (A ∪ D) ∩ (B ∪ C)

SOLUTION

(i) A ∩ B = {3,5,7,9,11} ∩ {7,9,11,13} = {7, 9, 11}.

(ii) B ∩ C = {7,9,11,13} ∩ {11,13,15} = {11, 13}.

(iii) A ∩ C = {11}; {11} ∩ D = {11} ∩ {15,17} = φ. So A ∩ C ∩ D = φ.

(iv) A ∩ C = {3,5,7,9,11} ∩ {11,13,15} = {11}.

(v) B ∩ D = {7,9,11,13} ∩ {15,17} = φ.

(vi) B ∪ C = {7,9,11,13,15}. A ∩ (B ∪ C) = {3,5,7,9,11} ∩ {7,9,11,13,15} = {7, 9, 11}.

(vii) A ∩ D = {3,5,7,9,11} ∩ {15,17} = φ.

(viii) B ∪ D = {7,9,11,13,15,17}. A ∩ (B ∪ D) = {3,5,7,9,11} ∩ {7,9,11,13,15,17} = {7, 9, 11}.

(ix) A ∩ B = {7,9,11}. B ∪ C = {7,9,11,13,15}. (A ∩ B) ∩ (B ∪ C) = {7,9,11} ∩ {7,9,11,13,15} = {7, 9, 11}.

(x) A ∪ D = {3,5,7,9,11,15,17}. B ∪ C = {7,9,11,13,15}. (A ∪ D) ∩ (B ∪ C) =

{7,9,11,15}.

Answer: (i) {7,9,11} (ii) {11,13} (iii) φ (iv) {11} (v) φ (vi) {7,9,11} (vii) φ (viii) {7,9,11} (ix) {7,9,11} (x) {7,9,11,15}

Q7 If A = {x : x is a natural number}, B = {x : x is an even natural number}, C = {x : x is an odd natural number} and D = {x : x is a prime number}, find
(i) A ∩ B
(ii) A ∩ C
(iii) A ∩ D
(iv) B ∩ C
(v) B ∩ D
(vi) C ∩ D

SOLUTION

(i) A = {1,2,3,4,...}, B = {2,4,6,8,...}. Every even natural number is a natural number, so B ⊂ A. A ∩ B = B = {x : x is an even natural number}.

(ii) A ∩ C = C = {x : x is an odd natural number}. (Every odd natural number is a natural number.)

(iii) A ∩ D = D = {x : x is a prime number}. (Every prime number is a natural number.)

(iv) B ∩ C: A number cannot be both even and odd simultaneously. B ∩ C = φ.

(v) B ∩ D: Even numbers that are also prime. The only even prime is 2. B ∩ D = {2}.

(vi) C ∩ D: Odd numbers that are also prime — these are all primes except 2. C ∩ D = {3, 5, 7, 11, 13, ...} = {x : x is an odd prime number}.

Answer: (i) B (even naturals) (ii) C (odd naturals) (iii) D (primes) (iv) φ (v) {2} (vi) {x : x is an odd prime}

Q8 Which of the following pairs of sets are disjoint?
(i) {1, 2, 3, 4} and {x : x is a natural number and 4 ≤ x ≤ 6}
(ii) {a, e, i, o, u} and {c, d, e, f}
(iii) {x : x is an even integer} and {x : x is an odd integer}

SOLUTION

(i) {x : x is natural and 4 ≤ x ≤ 6} = {4, 5, 6}. Intersection with {1,2,3,4} = {4} ≠ φ. NOT disjoint.

(ii) {a,e,i,o,u} ∩ {c,d,e,f} = {e} ≠ φ. NOT disjoint.

(iii) Even integers and odd integers share no common element (a number is either even or odd, not both). Their intersection = φ. They ARE disjoint.

Answer: (i) Not disjoint (share element 4) (ii) Not disjoint (share element e) (iii) Disjoint

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Q9 If A = {3, 6, 9, 12, 15, 18, 21}, B = {4, 8, 12, 16, 20}, C = {2, 4, 6, 8, 10, 12, 14, 16}, D = {5, 10, 15, 20}; find
(i) A – B
(ii) A – C
(iii) A – D
(iv) B – A
(v) C – A
(vi) D – A
(vii) B – C
(viii) B – D
(ix) C – B
(x) D – B
(xi) C – D
(xii) D – C

SOLUTION

(i) A – B: elements of A not in B. B ∩ A = {12}. A – B = {3, 6, 9, 15, 18, 21}.

(ii) A – C: elements of A not in C. A ∩ C = {6, 12}. A – C = {3, 9, 15, 18, 21}.

(iii) A – D: elements of A not in D. A ∩ D = {15}. A – D = {3, 6, 9, 12, 18, 21}.

(iv) B – A: elements of B not in A. B ∩ A = {12}. B – A = {4, 8, 16, 20}.

(v) C – A: elements of C not in A. C ∩ A = {6, 12}. C – A = {2, 4, 8, 10, 14, 16}.

(vi) D – A: elements of D not in A. D ∩ A = {15}. D – A = {5, 10, 20}.

(vii) B – C: elements of B not in C. B ∩ C = {4, 8, 12, 16}. B – C = {20}.

(viii) B – D: elements of B not in D. B ∩ D = {20}. B – D = {4, 8, 12, 16}.

(ix) C – B: elements of C not in B. C ∩ B = {4, 8, 12, 16}. C – B = {2, 6, 10, 14}.

(x) D – B: elements of D not in B. D ∩ B = {20}. D – B = {5, 10, 15}.

(xi) C – D: elements of C not in D. C ∩ D = {10}. C – D = {2, 4, 6, 8, 12, 14, 16}.

(xii) D – C: elements of D not in C. D ∩ C = {10}. D – C = {5, 15, 20}.

Answer: (i) {3,6,9,15,18,21} (ii) {3,9,15,18,21} (iii) {3,6,9,12,18,21} (iv) {4,8,16,20} (v) {2,4,8,10,14,16} (vi) {5,10,20} (vii) {20} (viii) {4,8,12,16} (ix) {2,6,10,14} (x) {5,10,15} (xi) {2,4,6,8,12,14,16} (xii) {5,15,20}

Q10 If X = {a, b, c, d} and Y = {f, b, d, g}, find
(i) X – Y
(ii) Y – X
(iii) X ∩ Y

SOLUTION

(i) X – Y: elements of X not in Y. Common elements: b and d. X – Y = {a, c}.

(ii) Y – X: elements of Y not in X. Common elements: b and d. Y – X = {f, g}.

(iii) X ∩ Y = {b, d}.

Answer: (i) X – Y = {a, c} (ii) Y – X = {f, g} (iii) X ∩ Y = {b, d}

Q11 If R is the set of real numbers and Q is the set of rational numbers, then what is R – Q?

SOLUTION

R – Q consists of all real numbers that are NOT rational — these are precisely the irrational numbers.
R – Q = {x : x ∈ R and x ∉ Q} = the set of all irrational numbers.
Examples: √2, √3, π, e, etc.

Answer: R – Q is the set of all irrational numbers.

Q12 State whether each of the following statement is true or false. Justify your answer.
(i) {2, 3, 4, 5} and {3, 6} are disjoint sets.
(ii) {a, e, i, o, u} and {a, b, c, d} are disjoint sets.
(iii) {2, 6, 10, 14} and {3, 7, 11, 15} are disjoint sets.
(iv) {2, 6, 10} and {3, 7, 11} are disjoint sets.

SOLUTION

(i) $\{2,3,4,5\} \cap \{3,6\} = \{3\} \neq \varphi$. They share element 3. FALSE — they are NOT disjoint.

(ii) $\{a,e,i,o,u\} \cap \{a,b,c,d\} = \{a\} \neq \varphi$. They share element a. FALSE — they are NOT disjoint.

(iii) $\{2,6,10,14\} \cap \{3,7,11,15\}$: Check each pair — no common element. The intersection is φ. TRUE — they ARE disjoint.

(iv) $\{2,6,10\} \cap \{3,7,11\}$: Check — no common element. The intersection is φ. TRUE — they ARE disjoint.

Answer: (i) False (ii) False (iii) True (iv) True

Exercise 1.5 — Exercise 1.5

Q1 Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8} and C = {3, 4, 5, 6}. Find
(i) A′
(ii) B′
(iii) (A ∪ C)′
(iv) (A ∪ B)′
(v) (A′)′
(vi) (B – C)′

SOLUTION

U = {1,2,3,4,5,6,7,8,9}, A = {1,2,3,4}, B = {2,4,6,8}, C = {3,4,5,6}.

(i) A′ = U – A = {5, 6, 7, 8, 9}.

(ii) B′ = U – B = {1, 3, 5, 7, 9}.

(iii) A ∪ C = {1,2,3,4,5,6}. (A ∪ C)′ = U – {1,2,3,4,5,6} = {7, 8, 9}.

(iv) A ∪ B = {1,2,3,4,6,8}. (A ∪ B)′ = U – {1,2,3,4,6,8} = {5, 7, 9}.

(v) (A′)′ = A = {1, 2, 3, 4}. (Double complementation law.)

(vi) B – C = {2,4,6,8} – {3,4,5,6} = {2, 8}. (B – C)′ = U – {2,8} = {1, 3, 4, 5, 6, 7, 9}.

Answer: (i) {5,6,7,8,9} (ii) {1,3,5,7,9} (iii) {7,8,9} (iv) {5,7,9} (v) {1,2,3,4} (vi) {1,3,4,5,6,7,9}

Q2 If U = {a, b, c, d, e, f, g, h}, find the complements of the following sets:
(i) A = {a, b, c}
(ii) B = {d, e, f, g}
(iii) C = {a, c, e, g}
(iv) D = {f, g, h, a}

SOLUTION

U = {a, b, c, d, e, f, g, h}.

(i) A′ = U – A = {d, e, f, g, h}.

(ii) B′ = U – B = {a, b, c, h}.

(iii) C′ = U – C = {b, d, f, h}.

(iv) D′ = U – D = {b, c, d, e}.

Answer: (i) {d,e,f,g,h} (ii) {a,b,c,h} (iii) {b,d,f,h} (iv) {b,c,d,e}

Q3 Taking the set of natural numbers as the universal set, write down the complements of the following sets:
(i) {x : x is an even natural number}
(ii) {x : x is an odd natural number}
(iii) {x : x is a positive multiple of 3}
(iv) {x : x is a prime number}
(v) {x : x is a natural number divisible by 3 and 5}
(vi) {x : x is a perfect square}
(vii) {x : x is a perfect cube}
(viii) {x : x + 5 = 8}
(ix) {x : 2x + 5 = 9}
(x) {x : x ≥ 7}
(xi) {x : x ∈ N and 2x + 1 > 10}

SOLUTION

Universal set = N = {1, 2, 3, 4, ...}.

(i) Complement of even naturals = {x : x is an odd natural number} = {1, 3, 5, 7, ...}.

(ii) Complement of odd naturals = {x : x is an even natural number} = {2, 4, 6, 8, ...}.

(iii) Complement of {multiples of 3} = {x ∈ N : x is NOT a multiple of 3} = {1, 2, 4, 5, 7, 8, 10, ...}.

(iv) Complement of {prime numbers} = {x ∈ N : x is NOT a prime} = {1, 4, 6, 8, 9, 10, ...} (1 and all composites).

(v) Divisible by both 3 and 5 means divisible by LCM(3,5) = 15. Set = {15, 30, 45, ...}.
Complement = {x ∈ N : x is not a multiple of 15} = {1, 2, ..., 14, 16, ..., 29, 31, ...}.

(vi) Perfect squares in N: 1, 4, 9, 16, 25, ...
Complement = {x ∈ N : x is not a perfect square} = {2, 3, 5, 6, 7, 8, 10, 11, ...}.

(vii) Perfect cubes in N: 1, 8, 27, 64, 125, ...
Complement = {x ∈ N : x is not a perfect cube} = {2, 3, 4, 5, 6, 7, 9, 10, ...}.

(viii) x + 5 = 8 => x = 3. Set = {3}. Complement = {x ∈ N : x ≠ 3} = {1, 2, 4, 5, 6, ...}.

(ix) $2x + 5 = 9 \Rightarrow 2x = 4 \Rightarrow x = 2$. Set = {2}. Complement = $\{x \in N : x \neq 2\}$ = {1, 3, 4, 5, 6, ...}.

(x) Set = $\{x \in N : x \geq 7\}$ = {7, 8, 9, ...}. Complement = {1, 2, 3, 4, 5, 6}.

(xi) $2x + 1 > 10 \Rightarrow 2x > 9 \Rightarrow x > 4.5$, so $x \geq 5$ for natural numbers. Set = {5, 6, 7, 8, ...}.
Complement = {1, 2, 3, 4}.

Answer: (i) {odd naturals} (ii) {even naturals} (iii) {x∈N: x not multiple of 3} (iv) {1 and composite numbers} (v) {x∈N: x not multiple of 15} (vi) {x∈N: x not a perfect square} (vii) {x∈N: x not a perfect cube} (viii) {x∈N: x≠3} (ix) {x∈N: x≠2} (x) {1,2,3,4,5,6} (xi) {1,2,3,4}

Q4 If U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}. Verify that
(i) $(A \cup B)' = A' \cap B'$
(ii) $(A \cap B)' = A' \cup B'$

SOLUTION

U = {1,2,3,4,5,6,7,8,9}, A = {2,4,6,8}, B = {2,3,5,7}.
A' = U – A = {1,3,5,7,9}.
B' = U – B = {1,4,6,8,9}.

(i) LHS: $A \cup B$ = {2,3,4,5,6,7,8}. $(A \cup B)'$ = U – {2,3,4,5,6,7,8} = {1, 9}.
RHS: $A' \cap B'$ = {1,3,5,7,9} ∩ {1,4,6,8,9} = {1, 9}.
LHS = RHS = {1, 9}. Verified.

(ii) LHS: $A \cap B$ = {2}. $(A \cap B)'$ = U – {2} = {1,3,4,5,6,7,8,9}.
RHS: $A' \cup B'$ = {1,3,5,7,9} ∪ {1,4,6,8,9} = {1,3,4,5,6,7,8,9}.
LHS = RHS = {1,3,4,5,6,7,8,9}. Verified.

Answer: Both De Morgan's laws verified: (i) $(A \cup B)' = A' \cap B' = \{1, 9\}$; (ii) $(A \cap B)' = A' \cup B' = \{1,3,4,5,6,7,8,9\}$.

Q5 Draw appropriate Venn diagram for each of the following:
(i) $(A \cup B)'$
(ii) $A' \cap B'$
(iii) $(A \cap B)'$
(iv) $A' \cup B'$

SOLUTION

Since Venn diagrams are graphical, the shaded regions are described below:

(i) $(A \cup B)'$: Shade the region of U that lies OUTSIDE both A and B — the area in the rectangle U but not in either circle A or B.

(ii) $A' \cap B'$: A' is everything outside A; B' is everything outside B. Their intersection is the region outside BOTH A and B — same as $(A \cup B)'$ (De Morgan's law). Shade the region in U outside both circles.

(iii) $(A \cap B)'$: Shade the region of U that is OUTSIDE the intersection of A and B — i.e., shade everything EXCEPT the overlapping lens-shaped region of A and B.

(iv) $A' \cup B'$: $A' \cup B' = (A \cap B)'$ (De Morgan's law). Shade everything in U EXCEPT the region common to both A and B (same shading as (iii)).

Note: (i) and (ii) produce identical shaded regions; (iii) and (iv) produce identical shaded regions.

Answer: (i) Region in U outside both A and B (ii) Same as (i) (iii) Everything in U except $A \cap B$ (iv) Same as (iii)

Q6 Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A'?

SOLUTION

$A' = U - A$ = all triangles in the plane that do NOT have any angle different from 60°.
A triangle with no angle different from 60° must have all three angles equal to 60°.
Such a triangle is an equilateral triangle.
Therefore, A' = the set of all equilateral triangles in the plane.

Answer: A' is the set of all equilateral triangles in the plane.

Q7 Fill in the blanks to make each of the following a true statement:
(i) $A \cup A' = \ldots$
(ii) $\varphi' \cap A = \ldots$
(iii) $A \cap A' = \ldots$
(iv) $U' \cap A = \ldots$

SOLUTION

(i) $A \cup A' = U$. (The complement law: every element is either in A or in A', so their

union is the universal set.)

(ii) $\varphi' = U$ (the complement of the empty set is the universal set). So $\varphi' \cap A = U \cap A = A$.

(iii) $A \cap A' = \varphi$. (A and its complement have no elements in common.)

(iv) $U' = \varphi$ (the complement of the universal set is the empty set). So $U' \cap A = \varphi \cap A = \varphi$.

Answer: (i) U (ii) A (iii) φ (iv) φ

Misc — Miscellaneous Exercise

Q1 Decide, among the following sets, which sets are subsets of one and another:
A = {x : x ∈ R and x satisfy $x^2 - 8x + 12 = 0$},
B = {2, 4, 6},
C = {2, 4, 6, 8, ...},
D = {6}.

SOLUTION

First find A: solve $x^2 - 8x + 12 = 0$.
$x^2 - 8x + 12 = (x - 2)(x - 6) = 0 \Rightarrow x = 2$ or $x = 6$.
So A = {2, 6}.

C = {2, 4, 6, 8, ...} = all even positive integers.

Now determine subset relations:
D = {6}: 6 ∈ A and 6 ∈ B and 6 ∈ C. So D ⊂ A, D ⊂ B, D ⊂ C.
A = {2, 6}: 2 ∈ B and 6 ∈ B, so A ⊂ B. Also 2 and 6 are even, so A ⊂ C.
B = {2, 4, 6}: 2, 4, 6 are all even, so B ⊂ C. But 4 ∉ A, so B ⊄ A.
C = {2,4,6,8,...}: 4 ∈ C but 4 ∉ A, 4 ∉ B so C ⊄ A and C ⊄ B. Also 2 ∈ C but 2 ∉ D, so C ⊄ D.

Summary: D ⊂ A ⊂ B ⊂ C, and also D ⊂ B and D ⊂ C.

Answer: A = {2, 6}, C = even positive integers. Subset relations: D ⊂ A ⊂ B ⊂ C (and D ⊂ B, D ⊂ C).

Q2 In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.
(i) If x ∈ A and A ∈ B, then x ∈ B
(ii) If A ⊂ B and B ∈ C, then A ∈ C
(iii) If A ⊂ B and B ⊂ C, then A ⊂ C
(iv) If A ⊄ B and B ⊄ C, then A ⊄ C
(v) If x ∈ A and A ⊄ B, then x ∉ B
(vi) If A ⊂ B and x ∉ B, then x ∉ A

SOLUTION

(i) FALSE. Counterexample: Let x = 1, A = {1}, B = {{1}, 2}. Then x = 1 ∈ A and A = {1} ∈ B, but x = 1 ∉ B (B contains {1} as a whole, not 1 itself).

(ii) FALSE. Counterexample: Let A = {1}, B = {1, 2}, C = {{1, 2}, 3}. Then A ⊂ B

and $B \in C$, but $A \notin C$ (the elements of C are {1, 2} and 3, not A = {1}).

(iii) TRUE. Proof: Let $x \in A$. Since $A \subset B$, we have $x \in B$. Since $B \subset C$, we have $x \in C$. So every element of A belongs to C, i.e., $A \subset C$.

(iv) FALSE. Counterexample: Let A = {1, 3}, B = {2, 4}, C = {1, 3, 5}. Then $A \not\subset B$ ($1 \in A$ but $1 \notin B$) and $B \not\subset C$ ($2 \in B$ but $2 \notin C$), yet $A \subset C$ ($1 \in C$ and $3 \in C$). So $A \not\subset C$ need not follow.

(v) FALSE. Counterexample: Let A = {1, 2}, B = {1, 3}. Then $1 \in A$ and $A \not\subset B$ ($2 \in A$ but $2 \notin B$), yet $1 \in B$. So $x \in B$ is possible.

(vi) TRUE. Proof: Suppose $x \notin B$ and $A \subset B$. If $x \in A$, then since $A \subset B$, $x \in B$ — contradiction. Therefore $x \notin A$.

Answer: (i) False (ii) False (iii) True (iv) False (v) False (vi) True

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Q3 Let A, B, and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that B = C.

SOLUTION

We show $B \subset C$ and $C \subset B$.

Let $x \in B$.
Then $x \in A \cup B$ (since $B \subset A \cup B$).
Since $A \cup B = A \cup C$, we have $x \in A \cup C$.
So either $x \in A$ or $x \in C$.

Case 1: $x \in C$. Done — $x \in C$.

Case 2: $x \in A$. Then $x \in A$ and $x \in B$, so $x \in A \cap B$.
Since $A \cap B = A \cap C$, we have $x \in A \cap C$, so $x \in C$.

In both cases $x \in C$. Therefore $B \subset C$.

By a symmetric argument (swapping B and C throughout), $C \subset B$.

Since $B \subset C$ and $C \subset B$, we conclude B = C.

Answer: B = C. (Proved by showing $B \subset C$ and $C \subset B$ using the given conditions.)

Q4 Show that the following four conditions are equivalent:
(i) $A \subset B$
(ii) $A - B = \varphi$
(iii) $A \cup B = B$
(iv) $A \cap B = A$

SOLUTION

We show (i) => (ii) => (iii) => (iv) => (i).

(i) => (ii): Assume $A \subset B$. If $x \in A - B$, then $x \in A$ and $x \notin B$. But $A \subset B$ implies $x \in B$ — contradiction. So $A - B$ has no elements, i.e., $A - B = \varphi$.

(ii) => (iii): Assume $A - B = \varphi$, i.e., no element of A is outside B, so every element of A is in B, meaning $A \subset B$. Then $A \cup B \supset B$ (always). Also, for any $x \in A \cup B$, either $x \in A$ (and thus $x \in B$ since $A \subset B$) or $x \in B$; in both cases $x \in B$. So $A \cup B \subset B$. Hence $A \cup B = B$.

(iii) => (iv): Assume $A \cup B = B$. Since $A \cap B \subset A$ (always), and also $A \cap B \subset B$ (always). For any $x \in A$, $x \in A \cup B = B$, so $x \in A$ and $x \in B$, giving $x \in A \cap B$. Thus $A \subset A \cap B$. Since $A \cap B \subset A$ always, we get $A \cap B = A$.

(iv) => (i): Assume $A \cap B = A$. For any $x \in A$, $x \in A \cap B$ (since $A = A \cap B$), so $x \in B$. Therefore $A \subset B$.

All four conditions are equivalent.

Answer: The four conditions are equivalent. Proof by cyclic implication: (i) => (ii) => (iii) => (iv) => (i).

Q5 Show that if $A \subset B$, then $C - B \subset C - A$.

SOLUTION

Assume $A \subset B$. Let $x \in C - B$.
Then $x \in C$ and $x \notin B$.
Since $A \subset B$, if x were in A, then $x \in B$ — but $x \notin B$, so $x \notin A$.
Therefore $x \in C$ and $x \notin A$, which means $x \in C - A$.
Since every $x \in C - B$ also belongs to $C - A$, we have $C - B \subset C - A$.

Answer: Proved: if $A \subset B$, then $C - B \subset C - A$.

Q6 Show that for any sets A and B,
$A = (A \cap B) \cup (A - B)$ and $A \cup (B - A) = A \cup B$.

SOLUTION

Part 1: Show $A = (A \cap B) \cup (A - B)$.

Let $x \in A$. Either $x \in B$ or $x \notin B$.
If $x \in B$: then $x \in A \cap B \subset (A \cap B) \cup (A - B)$.
If $x \notin B$: then $x \in A$ and $x \notin B$, so $x \in A - B \subset (A \cap B) \cup (A - B)$.
Therefore $A \subset (A \cap B) \cup (A - B)$.

Conversely, $A \cap B \subset A$ and $A - B \subset A$, so $(A \cap B) \cup (A - B) \subset A$.
Hence $A = (A \cap B) \cup (A - B)$.

Part 2: Show $A \cup (B - A) = A \cup B$.

$B - A \subset B \subset A \cup B$ and $A \subset A \cup B$, so $A \cup (B - A) \subset A \cup B$.

Conversely, let $x \in A \cup B$. Either $x \in A$ (then $x \in A \cup (B - A)$) or $x \in B$.
If $x \in B$ and $x \notin A$: then $x \in B - A \subset A \cup (B - A)$.
If $x \in B$ and $x \in A$: then $x \in A \subset A \cup (B - A)$.
So $A \cup B \subset A \cup (B - A)$.

Hence $A \cup (B - A) = A \cup B$.

Answer: Both identities proved: $A = (A \cap B) \cup (A - B)$, and $A \cup (B - A) = A \cup B$.

Q7 Using properties of sets, show that
(i) $A \cup (A \cap B) = A$
(ii) $A \cap (A \cup B) = A$

SOLUTION

(i) Show $A \cup (A \cap B) = A$.
$A \cap B \subset A$, so $A \cup (A \cap B) \subset A \cup A = A$.
Also $A \subset A \cup (A \cap B)$ (since A is contained in any union involving A).
Therefore $A \cup (A \cap B) = A$. (This is the Absorption Law.)

(ii) Show $A \cap (A \cup B) = A$.
$A \subset A \cup B$, so $A \cap (A \cup B) \supset A \cap A = A$. Thus $A \subset A \cap (A \cup B)$.
Also $A \cap (A \cup B) \subset A$ (since the intersection is a subset of the first set).
Therefore $A \cap (A \cup B) = A$. (This is also the Absorption Law.)

Answer: Both absorption laws proved: (i) $A \cup (A \cap B) = A$, (ii) $A \cap (A \cup B) = A$.

Q8 Show that A ∩ B = A ∩ C need not imply B = C.

SOLUTION

We provide a counterexample to disprove B = C.

Let A = {1, 2}, B = {2, 3}, C = {2, 4}.
A ∩ B = {1,2} ∩ {2,3} = {2}.
A ∩ C = {1,2} ∩ {2,4} = {2}.
So A ∩ B = A ∩ C = {2}, yet B = {2,3} ≠ {2,4} = C.

This shows that A ∩ B = A ∩ C does NOT necessarily imply B = C.

Answer: A ∩ B = A ∩ C does not imply B = C. Counterexample: A = {1,2}, B = {2,3}, C = {2,4} gives A ∩ B = A ∩ C = {2} but B ≠ C.

Q9 Let A and B be sets. If A ∩ X = B ∩ X = φ and A ∪ X = B ∪ X for some set X, show that A = B.
(Hint: A = A ∩ (A ∪ X), B = B ∩ (B ∪ X) and use Distributive law)

SOLUTION

Given: A ∩ X = φ, B ∩ X = φ, and A ∪ X = B ∪ X.

Using the hint:
A = A ∩ (A ∪ X) [Absorption Law: A ⊂ A ∪ X, so A ∩ (A ∪ X) = A]
= A ∩ (B ∪ X) [since A ∪ X = B ∪ X]
= (A ∩ B) ∪ (A ∩ X) [Distributive Law]
= (A ∩ B) ∪ φ [since A ∩ X = φ]
= A ∩ B.

Similarly:
B = B ∩ (B ∪ X) [Absorption Law]
= B ∩ (A ∪ X) [since A ∪ X = B ∪ X]
= (B ∩ A) ∪ (B ∩ X) [Distributive Law]
= (A ∩ B) ∪ φ [since B ∩ X = φ]
= A ∩ B.

Therefore A = A ∩ B = B, which gives A = B.

Answer: A = B. (Proved using absorption and distributive laws with the given disjointness and union-equality conditions.)

Q10 Find sets A, B and C such that A ∩ B, B ∩ C and A ∩ C are non-empty sets and A ∩ B ∩ C = φ.

SOLUTION

We need three sets where every pair has a common element, but no single element belongs to all three.

Let A = {1, 2}, B = {2, 3}, C = {1, 3}.
A ∩ B = {2} — non-empty. ✓
B ∩ C = {3} — non-empty. ✓
A ∩ C = {1} — non-empty. ✓
A ∩ B ∩ C = {1,2} ∩ {2,3} ∩ {1,3} = {2} ∩ {1,3} = φ. ✓

Answer: Example: A = {1, 2}, B = {2, 3}, C = {1, 3}. Then A ∩ B = {2}, B ∩ C = {3}, A ∩ C = {1} are all non-empty, but A ∩ B ∩ C = φ.