Chapter 2: Relations and Functions

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Exercise 2.1 — Exercise 2.1

Q1 If (x/3 + 1, y - 2/3) = (5/3, 1/3), find the values of x and y.

SOLUTION

Since the two ordered pairs are equal, their corresponding elements must be equal.

From the first components:
x/3 + 1 = 5/3
x/3 = 5/3 - 1 = 5/3 - 3/3 = 2/3
x = 2

From the second components:
y - 2/3 = 1/3
y = 1/3 + 2/3 = 3/3 = 1

Answer: x = 2, y = 1

Q2 If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B).

SOLUTION

n(A) = 3 and n(B) = 3 (since B = {3, 4, 5} has 3 elements).

By the property of Cartesian products:
n(A × B) = n(A) × n(B) = 3 × 3 = 9

Answer: n(A × B) = 9

Q3 If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

SOLUTION

G × H = {(g, h) : g ∈ G, h ∈ H}
G × H = {(7,5), (7,4), (7,2), (8,5), (8,4), (8,2)}

H × G = {(h, g) : h ∈ H, g ∈ G}
H × G = {(5,7), (5,8), (4,7), (4,8), (2,7), (2,8)}

Note: G × H ≠ H × G, since (7,5) ≠ (5,7), etc.

Answer: G × H = {(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)}; H × G = {(5,7),(5,8),(4,7),(4,8),(2,7),(2,8)}

Q4 State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly.
(i) If P = {m, n} and Q = {n, m}, then P × Q = {(m, n),(n, m)}.
(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that x ∈ A and y ∈ B.
(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ φ) = φ.

SOLUTION

(i) FALSE.
P = {m, n} and Q = {n, m} = {m, n}.
P × Q = {(m, n), (m, m), (n, n), (n, m)}.
The statement is false. The correct statement is: P × Q = {(m,n),(m,m),(n,n),(n,m)}.

(ii) TRUE.
By definition, A × B = {(x, y) : x ∈ A, y ∈ B}. If A and B are both non-empty, then A × B is non-empty and consists exactly of such ordered pairs.

(iii) TRUE.
B ∩ φ = φ (intersection of any set with the empty set is empty).
A × φ = φ (Cartesian product with the empty set is empty).
Therefore A × (B ∩ φ) = A × φ = φ.

Answer: (i) False; correct statement: P × Q = {(m,n),(m,m),(n,n),(n,m)}. (ii) True. (iii) True.

Q5 If A = {–1, 1}, find A × A × A.

SOLUTION

A = {-1, 1}.

A × A × A = {(a, b, c) : a, b, c ∈ A}

Listing all ordered triplets:

(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1),

(1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)

There are $2^3 = 8$ elements in A × A × A.

Answer: $A \times A \times A = \{(-1,-1,-1),(-1,-1,1),(-1,1,-1),(-1,1,1),(1,-1,-1),(1,-1,1),(1,1,-1),(1,1,1)\}$

Q6 If A × B = {(a, x),(a, y), (b, x), (b, y)}. Find A and B.

SOLUTION

In any Cartesian product A × B, the first elements of the ordered pairs form set A and the second elements form set B.

A = set of all first elements = {a, b}

B = set of all second elements = {x, y}

Answer: A = {a, b}, B = {x, y}

Q7 Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that

(i) A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) A × C is a subset of B × D.

SOLUTION

(i) LHS: B ∩ C = {1,2,3,4} ∩ {5,6} = φ (no common elements).

So A × (B ∩ C) = A × φ = φ.

RHS: A × B = {(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)}

A × C = {(1,5),(1,6),(2,5),(2,6)}

(A × B) ∩ (A × C) = φ (no common ordered pairs).

LHS = RHS = φ. Verified.

(ii) A × C = {(1,5),(1,6),(2,5),(2,6)}

B × D = {(b,d) : b ∈ {1,2,3,4}, d ∈ {5,6,7,8}}

This includes all pairs (1,5),(1,6),(2,5),(2,6) since 1,2 ∈ B and 5,6 ∈ D.

Therefore every element of A × C belongs to B × D.
Hence A × C ⊆ B × D. Verified.

Answer: (i) Both sides equal φ; verified. (ii) A × C = {(1,5),(1,6),(2,5),(2,6)} ⊆ B × D; verified.

Q8 Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.

SOLUTION

A × B = {(1,3),(1,4),(2,3),(2,4)}.
n(A × B) = 4, so the number of subsets = 2^4 = 16.

The 16 subsets of A × B are:
φ,
{(1,3)}, {(1,4)}, {(2,3)}, {(2,4)},
{(1,3),(1,4)}, {(1,3),(2,3)}, {(1,3),(2,4)}, {(1,4),(2,3)}, {(1,4),(2,4)}, {(2,3),(2,4)},
{(1,3),(1,4),(2,3)}, {(1,3),(1,4),(2,4)}, {(1,3),(2,3),(2,4)}, {(1,4),(2,3),(2,4)},
{(1,3),(1,4),(2,3),(2,4)}

Answer: A × B = {(1,3),(1,4),(2,3),(2,4)}; it has 16 subsets.

Q9 Let A and B be two sets such that n(A) = 3 and n(B) = 2. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, y and z are distinct elements.

SOLUTION

The second elements of the ordered pairs in A × B form set B.
The second elements here are 1, 2, 1, so B = {1, 2}.
This confirms n(B) = 2. ✓

The first elements of the ordered pairs form (part of) set A.
The first elements are x, y, z — all distinct.
So A = {x, y, z}, confirming n(A) = 3. ✓

Answer: A = {x, y, z}, B = {1, 2}

Q10 The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of A × A.

SOLUTION

$n(A \times A) = 9 = 3^2$, so $n(A) = 3$.

Since $(-1, 0) \in A \times A$, both -1 and 0 must be in A.
Since $(0, 1) \in A \times A$, both 0 and 1 must be in A.
So A contains -1, 0, and 1. Since n(A) = 3, we have A = {-1, 0, 1}.

$A \times A = \{(-1,-1),(-1,0),(-1,1),(0,-1),(0,0),(0,1),(1,-1),(1,0),(1,1)\}$

The given elements are (-1, 0) and (0, 1). The remaining 7 elements are:
(-1,-1), (-1,1), (0,-1), (0,0), (1,-1), (1,0), (1,1).

Answer: A = {-1, 0, 1}; remaining elements of A × A: (-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0),(1,1)

Exercise 2.2 — Exercise 2.2

Q1 Let A = {1, 2, 3,...,14}. Define a relation R from A to A by R = {(x, y) : 3x – y = 0, where x, y ∈ A}. Write down its domain, codomain and range.

SOLUTION

We need 3x - y = 0, i.e., y = 3x, with both x and y in A = {1,2,...,14}.

Finding valid pairs:
x = 1 => y = 3. (1,3) ∈ R
x = 2 => y = 6. (2,6) ∈ R
x = 3 => y = 9. (3,9) ∈ R
x = 4 => y = 12. (4,12) ∈ R
x = 5 => y = 15. 15 ∉ A, so stop here.

R = {(1,3),(2,6),(3,9),(4,12)}

Domain = set of first elements = {1, 2, 3, 4}
Codomain = A = {1, 2, 3, ..., 14}
Range = set of second elements = {3, 6, 9, 12}

Answer: R = {(1,3),(2,6),(3,9),(4,12)}; Domain = {1,2,3,4}; Codomain = {1,2,...,14}; Range = {3,6,9,12}

Q2 Define a relation R on the set N of natural numbers by R = {(x, y) : y = x + 5, x is a natural number less than 4; x, y ∈ N}. Depict this relationship using roster form. Write down the domain and the range.

SOLUTION

x is a natural number less than 4, so x ∈ {1, 2, 3}.
y = x + 5 for each such x.

x = 1 => y = 6. (1,6) ∈ R
x = 2 => y = 7. (2,7) ∈ R
x = 3 => y = 8. (3,8) ∈ R

R = {(1,6),(2,7),(3,8)}

Domain = {1, 2, 3}
Range = {6, 7, 8}

Answer: R = {(1,6),(2,7),(3,8)}; Domain = {1,2,3}; Range = {6,7,8}

Q3 A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(x, y): the difference between x and y is odd; x ∈ A, y ∈ B}. Write R in roster form.

SOLUTION

We need |x - y| to be odd, which happens when one of x, y is even and the other is odd.

Check each pair (x, y) with x ∈ A = {1,2,3,5}, y ∈ B = {4,6,9}:

x=1 (odd): 4(even) => 1-4=-3, odd ✓; 6(even) => 1-6=-5, odd ✓; 9(odd) => 1-9=-8, even ✗
x=2 (even): 4(even) => 2-4=-2, even ✗; 6(even) => 2-6=-4, even ✗; 9(odd) => 2-9=-7, odd ✓
x=3 (odd): 4(even) => 3-4=-1, odd ✓; 6(even) => 3-6=-3, odd ✓; 9(odd) => 3-9=-6, even ✗
x=5 (odd): 4(even) => 5-4=1, odd ✓; 6(even) => 5-6=-1, odd ✓; 9(odd) => 5-9=-4, even ✗

R = {(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)}

Answer: R = {(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)}

Q4 The Fig 2.7 shows a relationship between the sets P and Q. Write this relation (i) in set-builder form (ii) roster form. What is its domain and range?

SOLUTION

From Fig 2.7: P = {5, 6, 7} and Q = {3, 4, 5}.
The arrows show: 5 → 3, 6 → 4, 7 → 5.
The pattern is y = x - 2.

(i) Set-builder form:
R = {(x, y) : y = x - 2, x ∈ P, y ∈ Q}
or equivalently R = {(x, y) : x - y = 2, x ∈ P, y ∈ Q}

(ii) Roster form:
R = {(5, 3), (6, 4), (7, 5)}

Domain = {5, 6, 7}
Range = {3, 4, 5}

Answer: R = {(x,y): y = x – 2, x ∈ P, y ∈ Q}; Roster: {(5,3),(6,4),(7,5)}; Domain = {5,6,7}; Range = {3,4,5}

Q5 Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by {(a, b): a, b ∈ A, b is exactly divisible by a}.
(i) Write R in roster form
(ii) Find the domain of R
(iii) Find the range of R.

SOLUTION

We need b divisible by a, with a, b ∈ A = {1,2,3,4,6}.

a=1: 1	1✓, 1	2✓, 1	3✓, 1	4✓, 1	6✓ => (1,1),(1,2),(1,3),(1,4),(1,6)
a=2: 2	1? No. 2	2✓, 2	3? No. 2	4✓, 2	6✓ => (2,2),(2,4),(2,6)
a=3: 3	1? No. 3	2? No. 3	3✓, 3	4? No. 3	6✓ => (3,3),(3,6)
a=4: 4	1? No. 4	2? No. 4	3? No. 4	4✓, 4	6? No. => (4,4)
a=6: 6	1? No. 6	2? No. 6	3? No. 6	4? No. 6	6✓ => (6,6)

(i) R = {(1,1),(1,2),(1,3),(1,4),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(6,6)}

(ii) Domain = set of all first elements = {1, 2, 3, 4, 6}

(iii) Range = set of all second elements = {1, 2, 3, 4, 6}

Answer: (i) R = {(1,1),(1,2),(1,3),(1,4),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(6,6)}; (ii) Domain = {1,2,3,4,6}; (iii) Range = {1,2,3,4,6}

Q6 Determine the domain and range of the relation R defined by R = {(x, x + 5) : x ∈ {0, 1, 2, 3, 4, 5}}.

SOLUTION

The relation R = {(x, x+5) : x ∈ {0,1,2,3,4,5}}.

Listing all pairs:
x=0: (0,5); x=1: (1,6); x=2: (2,7); x=3: (3,8); x=4: (4,9); x=5: (5,10)

R = {(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)}

Domain = {0, 1, 2, 3, 4, 5}
Range = {5, 6, 7, 8, 9, 10}

Answer: Domain = {0,1,2,3,4,5}; Range = {5,6,7,8,9,10}

Q7 Write the relation R = {(x, x^3) : x is a prime number less than 10} in roster form.

SOLUTION

Prime numbers less than 10: 2, 3, 5, 7.

$2^3 = 8$, $3^3 = 27$, $5^3 = 125$, $7^3 = 343$.

R = {(2,8),(3,27),(5,125),(7,343)}

Answer: R = {(2,8),(3,27),(5,125),(7,343)}

Q8 Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B.

SOLUTION

n(A) = 3, n(B) = 2.
n(A × B) = 3 × 2 = 6.

A relation from A to B is any subset of A × B.
Number of subsets of A × B = $2^6 = 64$.

Therefore, the number of relations from A to B is 64.

Answer: 64

Q9 Let R be the relation on Z defined by R = {(a, b): a, b ∈ Z, a – b is an integer}. Find the domain and range of R.

SOLUTION

For any a, b ∈ Z, the difference a - b is always an integer (since the integers are closed under subtraction).

Therefore, every pair (a, b) with a, b ∈ Z belongs to R.
This means R = Z × Z.

Domain = Z (all integers, since any integer can be a first element).
Range = Z (all integers, since any integer can be a second element).

Answer: Domain = Z; Range = Z

Exercise 2.3 — Exercise 2.3

Q1 Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.
(i) {(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)}
(ii) {(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)}
(iii) {(1,3),(1,5),(2,5)}

SOLUTION

(i) {(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)}
Each first element (2,5,8,11,14,17) appears exactly once, so each has a unique image.
This is a function.
Domain = {2, 5, 8, 11, 14, 17}
Range = {1}

(ii) {(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)}
Each first element (2,4,6,8,10,12,14) appears exactly once with a unique image.
This is a function.
Domain = {2, 4, 6, 8, 10, 12, 14}
Range = {1, 2, 3, 4, 5, 6, 7}

(iii) {(1,3),(1,5),(2,5)}
The element 1 in the domain maps to both 3 and 5 (two different images).
This violates the definition of a function.
This is NOT a function.

Answer: (i) Function; Domain={2,5,8,11,14,17}, Range={1}. (ii) Function; Domain={2,4,6,8,10,12,14}, Range={1,2,3,4,5,6,7}. (iii) Not a function (element 1 has two images).

Q2 Find the domain and range of the following real functions:
(i) $f(x) = -x$
(ii) $f(x) = \sqrt{(9 - x^2)}$

SOLUTION

(i) f(x) = -x
f(x) is defined for all real numbers x.
Domain = R.
For every real number x, -x is also real. As x ranges over all of R, -x also takes all

real values.
Range = R.

(ii) $f(x) = \sqrt{(9 - x^2)}$
For f(x) to be defined (real-valued), we need $9 - x^2 \geq 0$.
$9 - x^2 \geq 0 \Rightarrow x^2 \leq 9 \Rightarrow -3 \leq x \leq 3$.
Domain = [-3, 3].

When $x \in [-3, 3]$: x^2 ranges from 0 to 9, so $9 - x^2$ ranges from 0 to 9.
Therefore $\sqrt{(9 - x^2)}$ ranges from 0 to 3.
Range = [0, 3].

Answer: (i) Domain = R, Range = R. (ii) Domain = [–3, 3], Range = [0, 3].

Q3 A function f is defined by f(x) = 2x – 5. Write down the values of
(i) f(0), (ii) f(7), (iii) f(–3).

SOLUTION

(i) f(0) = 2(0) - 5 = 0 - 5 = -5

(ii) f(7) = 2(7) - 5 = 14 - 5 = 9

(iii) f(-3) = 2(-3) - 5 = -6 - 5 = -11

Answer: f(0) = –5; f(7) = 9; f(–3) = –11

Q4 The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by t(C) = 9C/5 + 32. Find (i) t(0) (ii) t(28) (iii) t(–10) (iv) The value of C, when t(C) = 212.

SOLUTION

(i) t(0) = 9(0)/5 + 32 = 0 + 32 = 32

(ii) t(28) = 9(28)/5 + 32 = 252/5 + 32 = 50.4 + 32 = 82.4

(iii) t(-10) = 9(-10)/5 + 32 = -90/5 + 32 = -18 + 32 = 14

(iv) t(C) = 212
9C/5 + 32 = 212
9C/5 = 212 - 32 = 180
9C = 900
C = 100

Answer: (i) t(0) = 32 (ii) t(28) = 82.4 (iii) t(–10) = 14 (iv) C = 100

Q5 Find the range of each of the following functions.
(i) $f(x) = 2 – 3x$, $x \in R$, $x > 0$.
(ii) $f(x) = x^2 + 2$, x is a real number.
(iii) $f(x) = x$, x is a real number.

SOLUTION

(i) $f(x) = 2 - 3x$, $x > 0$ (strictly).
When $x > 0$: $3x > 0$, so $-3x < 0$, so $2 - 3x < 2$.
As x increases from 0 (exclusive) to ∞, f(x) decreases from 2 (exclusive) to -∞.
Range = $(-\infty, 2)$.

(ii) $f(x) = x^2 + 2$, $x \in R$.
For all real x: $x^2 \geq 0$, so $x^2 + 2 \geq 2$.
Minimum value is 2 (attained at $x = 0$), and there is no upper bound.
Range = $[2, \infty)$.

(iii) $f(x) = x$, $x \in R$.
The function assigns to each real number itself. So f takes every real value.
Range = R.

Answer: (i) Range = $(–\infty, 2)$. (ii) Range = $[2, \infty)$. (iii) Range = R.

Misc — Miscellaneous Exercise

Q1 The relation f is defined by $f(x) = x^2$, $0 \le x \le 3$; $f(x) = 3x$, $3 \le x \le 10$.
The relation g is defined by $g(x) = x^2$, $0 \le x \le 2$; $g(x) = 3x$, $2 \le x \le 10$.
Show that f is a function and g is not a function.

SOLUTION

For f:
The two pieces overlap only at x = 3.
From piece 1: $f(3) = 3^2 = 9$.
From piece 2: $f(3) = 3(3) = 9$.
Both pieces give the same value at x = 3, so every x in [0,10] has a unique image under f.
Therefore f is a function.

For g:
The two pieces overlap at x = 2.
From piece 1: $g(2) = 2^2 = 4$.
From piece 2: $g(2) = 3(2) = 6$.
The two pieces give different values (4 and 6) at x = 2.
So the element 2 in the domain has two images, which violates the definition of a function.
Therefore g is not a function.

Answer: f is a function (both pieces agree at the overlap x = 3, giving f(3) = 9). g is not a function (at x = 2, g gives two values: 4 and 6).

Q2 If $f(x) = x^2$, find $(f(1.1) – f(1))/(1.1 – 1)$.

SOLUTION

$f(1.1) = (1.1)^2 = 1.21$
$f(1) = (1)^2 = 1$

$(f(1.1) - f(1))/(1.1 - 1) = (1.21 - 1)/(0.1) = 0.21/0.1 = 2.1$

Answer: 2.1

Q3 Find the domain of the function $f(x) = (x^2 + 2x + 1)/(x^2 – 8x + 12)$.

SOLUTION

$f(x)$ is defined for all real x except where the denominator equals zero.

Set the denominator equal to zero:
$x^2 - 8x + 12 = 0$
$(x - 6)(x - 2) = 0$
$x = 2$ or $x = 6$.

The function is undefined at $x = 2$ and $x = 6$.
Domain = $R - \{2, 6\}$.

Answer: Domain = $R – \{2, 6\}$

Q4 Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x – 1)}$.

SOLUTION

For $f(x) = \sqrt{(x - 1)}$ to be real, we need $x - 1 \geq 0$, i.e., $x \geq 1$.
Domain = $[1, \infty)$.

When $x \geq 1$: $x - 1 \geq 0$, so $\sqrt{(x - 1)} \geq 0$.
As x increases from 1, $\sqrt{(x-1)}$ increases from 0 to ∞.
Range = $[0, \infty)$.

Answer: Domain = $[1, \infty)$; Range = $[0, \infty)$

Q5 Find the domain and the range of the real function f defined by $f(x) = |x – 1|$.

SOLUTION

$f(x) = |x - 1|$ is defined for all real numbers x.
Domain = R.

For any $x \in R$: $|x - 1| \geq 0$.
At $x = 1$: $f(1) = 0$ (minimum value).
For all other values of x, $|x - 1| > 0$ and can be made arbitrarily large.
Range = $[0, \infty)$.

Answer: Domain = R; Range = $[0, \infty)$

Q6 Let $f = \{(x, x^2/(1 + x^2)) : x \in R\}$ be a function from R into R. Determine the range of f.

SOLUTION

Let $y = x^2/(1 + x^2)$.

We solve for x in terms of y:
$y(1 + x^2) = x^2$
$y + yx^2 = x^2$
$y = x^2 - yx^2 = x^2(1 - y)$
$x^2 = y/(1 - y)$, provided $y \neq 1$.

Since $x^2 \geq 0$, we need $y/(1 - y) \geq 0$.
This holds when both $y \geq 0$ and $1 - y > 0$, i.e., $0 \leq y < 1$.
(The case $y < 0$ and $1 - y < 0$ would require $y > 1$ and $y < 0$ simultaneously — impossible.)

Also, from $y = x^2/(1 + x^2)$: since $1 + x^2 > x^2$, we have $y < 1$ always, and $y = 0$ when $x = 0$.
As $x \to \infty$, $y \to 1$ (but never reaches 1).

Range = [0, 1).

Answer: Range = [0, 1)

Q7 Let $f, g : R \to R$ be defined, respectively by $f(x) = x + 1$, $g(x) = 2x – 3$. Find $f + g$, $f – g$ and f/g.

SOLUTION

$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$

$(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4$

$(f/g)(x) = f(x)/g(x) = (x + 1)/(2x - 3)$, provided $g(x) \neq 0$, i.e., $x \neq 3/2$.

Answer: $(f+g)(x) = 3x – 2$; $(f–g)(x) = –x + 4$; $(f/g)(x) = (x+1)/(2x–3)$, $x \neq 3/2$

Q8 Let f = {(1,1),(2,3),(0,–1),(–1,–3)} be a function from Z to Z defined by f(x) = ax + b, for some integers a, b. Determine a, b.

SOLUTION

Since f is linear: f(x) = ax + b.

Using (0, -1): f(0) = a(0) + b = b = -1. So b = -1.

Using (1, 1): f(1) = a(1) + b = a + (-1) = a - 1 = 1. So a = 2.

Verification: f(x) = 2x - 1.
f(2) = 4 - 1 = 3 ✓; f(-1) = -2 - 1 = -3 ✓.

Answer: a = 2, b = –1

Q9 Let R be a relation from N to N defined by R = {(a, b) : a, b ∈ N and a = b²}. Are the following true?
(i) (a, a) ∈ R, for all a ∈ N
(ii) (a, b) ∈ R, implies (b, a) ∈ R
(iii) (a, b) ∈ R, (b, c) ∈ R implies (a, c) ∈ R.
Justify your answer in each case.

SOLUTION

(i) (a, a) ∈ R means a = a². This gives a² - a = 0 => a(a-1) = 0 => a = 0 or a = 1.
Since 0 ∉ N, this is true only for a = 1, not for all a ∈ N.
For example, (2, 2) ∉ R since 2 ≠ 2² = 4.
Statement (i) is FALSE.

(ii) (a, b) ∈ R means a = b².
(b, a) ∈ R would require b = a².
If a = b², then a² = b⁴. For b = a², we would need b = b⁴, i.e., b⁴ - b = 0, true only for b = 1.
Counterexample: (4, 2) ∈ R since 4 = 2². But (2, 4) ∉ R since 2 ≠ 4² = 16.
Statement (ii) is FALSE.

(iii) (a, b) ∈ R means a = b², and (b, c) ∈ R means b = c².
(a, c) ∈ R would require a = c².
We have a = b² = (c²)² = c⁴. So a = c⁴, not c² in general.
Counterexample: (16, 4) ∈ R (16 = 4²), (4, 2) ∈ R (4 = 2²). But (16, 2) ∉ R since 16 ≠ 2² = 4.
Statement (iii) is FALSE.

Answer: (i) False — not true for all a ∈ N (e.g. a=2: 2 ≠ 4). (ii) False — e.g. (4,2) ∈ R but (2,4) ∉ R. (iii) False — e.g. (16,4) ∈ R, (4,2) ∈ R but (16,2) ∉ R.

Q10 Let A = {1,2,3,4}, B = {1,5,9,11,15,16} and f = {(1,5),(2,9),(3,1),(4,5),(2,11)}. Are the following true?
(i) f is a relation from A to B
(ii) f is a function from A to B.
Justify your answer in each case.

SOLUTION

(i) A relation from A to B is any subset of A × B.
Check: every first element (1,2,3,4) belongs to A, and every second element (5,9,1,5,11) belongs to B.
So f ⊆ A × B, which makes f a relation from A to B.
Statement (i) is TRUE.

(ii) For f to be a function, each element of A must have exactly one image in B.
Here, 2 appears as a first element in two pairs: (2, 9) and (2, 11).
So the element 2 ∈ A has two different images: 9 and 11.
This violates the definition of a function.
Statement (ii) is FALSE.

Answer: (i) True — f ⊆ A × B, so it is a relation. (ii) False — element 2 maps to both 9 and 11.

Q11 Let f be the subset of Z × Z defined by f = {(ab, a + b) : a, b ∈ Z}. Is f a function from Z to Z? Justify your answer.

SOLUTION

For f to be a function from Z to Z, each first element (ab) must have a unique second element (a+b).

Consider a = 0, b = 0: ordered pair (0·0, 0+0) = (0, 0) ∈ f.
Consider a = 0, b = 1: ordered pair (0·1, 0+1) = (0, 1) ∈ f.

The same first element 0 appears with two different second elements: 0 and 1.

Therefore f is NOT a function from Z to Z.

Answer: f is not a function from Z to Z. The element 0 (= 0×0 = 0×1) maps to both 0+0=0 and 0+1=1, violating the uniqueness condition.

Q12 Let A = {9, 10, 11, 12, 13} and let f : A→N be defined by f(n) = the highest prime factor of n. Find the range of f.

SOLUTION

Find the highest prime factor of each element of A:

f(9) = 9 = 3^2, highest prime factor = 3.
f(10) = 10 = 2 × 5, highest prime factor = 5.
f(11) = 11 is prime, highest prime factor = 11.
f(12) = 12 = 2^2 × 3, highest prime factor = 3.
f(13) = 13 is prime, highest prime factor = 13.

Range of f = {3, 5, 11, 13}.

Answer: Range of f = {3, 5, 11, 13}