# Chapter 13: Statistics

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## Exercise 13.1 — Exercise 13.1

**Q1** Find the mean deviation about the mean for the data: 4, 7, 8, 9, 10, 12, 13, 17.

SOLUTION

Step 1 – Find the mean.

n = 8, Sum = 4 + 7 + 8 + 9 + 10 + 12 + 13 + 17 = 80

Mean $\bar{x}$ = 80/8 = 10

| Step 2 – Find the absolute deviations | $x_i - \bar{x}$ | . | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 4 − 10 | = 6, | 7 − 10 | = 3, | 8 − 10 | = 2, | 9 − 10 | = 1 |
| | 10 − 10 | = 0, | 12 − 10 | = 2, | 13 − 10 | = 3, | 17 − 10 | = 7 |

Step 3 – Sum of absolute deviations = 6 + 3 + 2 + 1 + 0 + 2 + 3 + 7 = 24.

Step 4 – Mean deviation about mean = 24/8 = 3.

Answer: M.D.($\bar{x}$) = 3

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**Q2** Find the mean deviation about the mean for the data: 38, 70, 48, 40, 42, 55, 63, 46, 54, 44.

SOLUTION

Step 1 – Find the mean.

n = 10, Sum = 38 + 70 + 48 + 40 + 42 + 55 + 63 + 46 + 54 + 44 = 500

Mean $\bar{x}$ = 500/10 = 50

Step 2 – Absolute deviations $|x_i - 50|$:

12, 20, 2, 10, 8, 5, 13, 4, 4, 6

Step 3 – Sum = 12 + 20 + 2 + 10 + 8 + 5 + 13 + 4 + 4 + 6 = 84.

Step 4 – M.D.($\bar{x}$) = 84/10 = 8.4.

Answer: M.D.($\bar{x}$) = 8.4

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Q3 Find the mean deviation about the median for the data: 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17.

SOLUTION

Step 1 – Arrange in ascending order:
10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18 (n = 12, even)

Step 2 – Median = mean of 6th and 7th observations = (13 + 14)/2 = 13.5.

Step 3 – Absolute deviations |xi − 13.5|:
3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5

Step 4 – Sum = 28.

M.D.(M) = 28/12 = 7/3 ≈ 2.33.

Answer: M.D.(M) = 7/3 ≈ 2.33

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Q4 Find the mean deviation about the median for the data: 36, 72, 46, 42, 60, 45, 53, 46, 51, 49.

SOLUTION

Step 1 – Arrange in ascending order:
36, 42, 45, 46, 46, 49, 51, 53, 60, 72 (n = 10, even)

Step 2 – Median = mean of 5th and 6th observations = (46 + 49)/2 = 47.5.

Step 3 – Absolute deviations |xi − 47.5|:
11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

Step 4 – Sum = 70.

M.D.(M) = 70/10 = 7.

Answer: M.D.(M) = 7

## Q5 Find the mean deviation about the mean for the data:

xi : 5 10 15 20 25

fi : 7 4 6 3 5

### SOLUTION

Step 1 – Find N and the mean.

N = 7 + 4 + 6 + 3 + 5 = 25

Σfi·xi = 5×7 + 10×4 + 15×6 + 20×3 + 25×5 = 35 + 40 + 90 + 60 + 125 = 350

Mean $\bar{x}$ = 350/25 = 14

Step 2 – Absolute deviations and weighted deviations:

| | | | | |
|---|---|---|---|---|
| xi = 5: | 5 − 14 | = 9, fi· | xi − $\bar{x}$ | = 7 × 9 = 63 |
| xi = 10: | 10 − 14 | = 4, fi· | xi − $\bar{x}$ | = 4 × 4 = 16 |
| xi = 15: | 15 − 14 | = 1, fi· | xi − $\bar{x}$ | = 6 × 1 = 6 |
| xi = 20: | 20 − 14 | = 6, fi· | xi − $\bar{x}$ | = 3 × 6 = 18 |
| xi = 25: | 25 − 14 | = 11, fi· | xi − $\bar{x}$ | = 5 × 11 = 55 |

Step 3 – Σfi·|xi − $\bar{x}$| = 63 + 16 + 6 + 18 + 55 = 158.

M.D.($\bar{x}$) = 158/25 = 6.32.

**Answer: M.D.($\bar{x}$) = 6.32**

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## Q6 Find the mean deviation about the mean for the data:

xi : 10 30 50 70 90

fi : 4 24 28 16 8

### SOLUTION

Step 1 – Find N and the mean.

N = 4 + 24 + 28 + 16 + 8 = 80

Σfi·xi = 10×4 + 30×24 + 50×28 + 70×16 + 90×8

= 40 + 720 + 1400 + 1120 + 720 = 4000

Mean $\bar{x}$ = 4000/80 = 50

Step 2 – Weighted absolute deviations:

| | | | | |
|---|---|---|---|---|
| xi = 10: | 10 − 50 | = 40, fi· | xi − $\bar{x}$ | = 4 × 40 = 160 |
| xi = 30: | 30 − 50 | = 20, fi· | xi − $\bar{x}$ | = 24 × 20 = 480 |

| | | | | |
|---|---|---|---|---|
| xi = 10: | 10 − 50 | = 40, fi· | xi − x̄ | = 4 × 40 = 160 |
| xi = 50: | 50 − 50 | = 0, fi· | xi − x̄ | = 28 × 0 = 0 |
| xi = 70: | 70 − 50 | = 20, fi· | xi − x̄ | = 16 × 20 = 320 |
| xi = 90: | 90 − 50 | = 40, fi· | xi − x̄ | = 8 × 40 = 320 |

Step 3 – Σfi·|xi − x̄| = 160 + 480 + 0 + 320 + 320 = 1280.

M.D.(x̄) = 1280/80 = 16.

Answer: M.D.(x̄) = 16

Q7 Find the mean deviation about the median for the data:

xi : 5 7 9 10 12 15

fi : 8 6 2 2 2 6

SOLUTION

Step 1 – Compute cumulative frequencies.

N = 8 + 6 + 2 + 2 + 2 + 6 = 26

Cumulative frequencies: 8, 14, 16, 18, 20, 26

Step 2 – Find median. N/2 = 13. The cumulative frequency that first reaches or exceeds 13 is 14, which corresponds to xi = 7.

Therefore Median M = 7.

| Step 3 – Weighted absolute deviations | xi − 7 | : | | |
|---|---|---|---|---|
| xi = 5: | 5 − 7 | = 2, fi· | xi − M | = 8 × 2 = 16 |
| xi = 7: | 7 − 7 | = 0, fi· | xi − M | = 6 × 0 = 0 |
| xi = 9: | 9 − 7 | = 2, fi· | xi − M | = 2 × 2 = 4 |
| xi = 10: | 10 − 7 | = 3, fi· | xi − M | = 2 × 3 = 6 |
| xi = 12: | 12 − 7 | = 5, fi· | xi − M | = 2 × 5 = 10 |
| xi = 15: | 15 − 7 | = 8, fi· | xi − M | = 6 × 8 = 48 |

Step 4 – Σfi·|xi − M| = 16 + 0 + 4 + 6 + 10 + 48 = 84.

M.D.(M) = 84/26 = 42/13 ≈ 3.23.

Answer: M.D.(M) = 42/13 ≈ 3.23

Q8 Find the mean deviation about the median for the data:

xi : 15 21 27 30 35

fi : 3 5 6 7 8

SOLUTION

Step 1 – Cumulative frequencies.

N = 3 + 5 + 6 + 7 + 8 = 29

Cumulative frequencies: 3, 8, 14, 21, 29

Step 2 – Find median. N/2 = 14.5. The first cumulative frequency ≥ 14.5 is 21, corresponding to xi = 27.

Therefore Median M = 27.

| Step 3 – Weighted absolute deviations | xi − 27 | : | | |
|---|---|---|---|---|
| xi = 15: | 15 − 27 | = 12, fi· | xi − M | = 3 × 12 = 36 |
| xi = 21: | 21 − 27 | = 6, fi· | xi − M | = 5 × 6 = 30 |
| xi = 27: | 27 − 27 | = 0, fi· | xi − M | = 6 × 0 = 0 |
| xi = 30: | 30 − 27 | = 3, fi· | xi − M | = 7 × 3 = 21 |
| xi = 35: | 35 − 27 | = 8, fi· | xi − M | = 8 × 8 = 64 |

Step 4 – Σfi·|xi − M| = 36 + 30 + 0 + 21 + 64 = 151.

M.D.(M) = 151/29 ≈ 5.21.

Answer: M.D.(M) = 151/29 ≈ 5.21

Q9 Find the mean deviation about the mean for the data:

Income per day (in ): 0-100, 100-200, 200-300, 300-400, 400-500, 500-600, 600-700, 700-800

Number of persons: 4, 8, 9, 10, 7, 5, 4, 3

SOLUTION

Step 1 – Mid-points xi: 50, 150, 250, 350, 450, 550, 650, 750.

N = 4 + 8 + 9 + 10 + 7 + 5 + 4 + 3 = 50

Step 2 – Compute Σfi·xi.

4×50 + 8×150 + 9×250 + 10×350 + 7×450 + 5×550 + 4×650 + 3×750
= 200 + 1200 + 2250 + 3500 + 3150 + 2750 + 2600 + 2250 = 17900
Mean x̄ = 17900/50 = 358

| Step 3 – Absolute deviations | xi − 358 | and fi· | xi − 358 | : |
|---|---|---|---|---|
| xi = 50: | 50 − 358 | = 308, fi·dev = 4 × 308 = 1232 | | |
| xi = 150: | 150 − 358 | = 208, fi·dev = 8 × 208 = 1664 | | |
| xi = 250: | 250 − 358 | = 108, fi·dev = 9 × 108 = 972 | | |
| xi = 350: | 350 − 358 | = 8, fi·dev = 10 × 8 = 80 | | |
| xi = 450: | 450 − 358 | = 92, fi·dev = 7 × 92 = 644 | | |
| xi = 550: | 550 − 358 | = 192, fi·dev = 5 × 192 = 960 | | |
| xi = 650: | 650 − 358 | = 292, fi·dev = 4 × 292 = 1168 | | |
| xi = 750: | 750 − 358 | = 392, fi·dev = 3 × 392 = 1176 | | |

Step 4 – Σfi·|xi − x̄| = 1232 + 1664 + 972 + 80 + 644 + 960 + 1168 + 1176 = 7896.

M.D.(x̄) = 7896/50 = 157.92.

Answer: Mean x̄ = 358; M.D.(x̄) = 157.92

Q10 Find the mean deviation about the mean for the data:

| Height in cm: | 95-105 | 105-115 | 115-125 | 125-135 | 135-145 | 145-155 |
|---|---|---|---|---|---|---|
| Number of boys: | 9 | 13 | 26 | 30 | 12 | 10 |

SOLUTION

Step 1 – Mid-points xi: 100, 110, 120, 130, 140, 150.
N = 9 + 13 + 26 + 30 + 12 + 10 = 100

Step 2 – Σfi·xi:
9×100 + 13×110 + 26×120 + 30×130 + 12×140 + 10×150
= 900 + 1430 + 3120 + 3900 + 1680 + 1500 = 12530
Mean x̄ = 12530/100 = 125.3 cm

| Step 3 – Absolute deviations and fi· | xi − 125.3 | : |
|---|---|---|
| xi = 100: | 100 − 125.3 | = 25.3, fi·dev = 9 × 25.3 = 227.7 |

| Step 3 – Absolute deviations and fi· | xi − 125.3 | : |
|---|---|---|
| xi = 110: | 110 − 125.3 | = 15.3, fi·dev = 13 × 15.3 = 198.9 |
| xi = 120: | 120 − 125.3 | = 5.3, fi·dev = 26 × 5.3 = 137.8 |
| xi = 130: | 130 − 125.3 | = 4.7, fi·dev = 30 × 4.7 = 141.0 |
| xi = 140: | 140 − 125.3 | = 14.7, fi·dev = 12 × 14.7 = 176.4 |
| xi = 150: | 150 − 125.3 | = 24.7, fi·dev = 10 × 24.7 = 247.0 |

Step 4 – $\Sigma f_i \cdot |x_i - \bar{x}| = 227.7 + 198.9 + 137.8 + 141.0 + 176.4 + 247.0 = 1128.8$.

M.D.($\bar{x}$) = 1128.8/100 = 11.28 cm.

Answer: Mean $\bar{x}$ = 125.3 cm; M.D.($\bar{x}$) = 11.28 cm

Q11 Find the mean deviation about median for the following data:

| Marks: | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|---|---|---|---|---|---|---|
| Number of Girls: | 6 | 8 | 14 | 16 | 4 | 2 |

SOLUTION

Step 1 – Cumulative frequencies.

N = 6 + 8 + 14 + 16 + 4 + 2 = 50

c.f.: 6, 14, 28, 44, 48, 50

Step 2 – Find the median class. N/2 = 25. The cumulative frequency just exceeding 25 is 28, which belongs to the class 20-30.

So median class is 20-30: l = 20, C = 14 (preceding c.f.), f = 14, h = 10.

Median M = l + ((N/2 − C)/f) × h = 20 + ((25 − 14)/14) × 10 = 20 + 110/14 = 20 + 55/7 ≈ 27.86

Step 3 – Mid-points xi: 5, 15, 25, 35, 45, 55.

| fi· | xi − M | : |
|---|---|---|
| xi = 5: | 5 − 195/7 | = 160/7, fi·dev = 6 × 160/7 = 960/7 ≈ 137.14 |
| xi = 15: | 15 − 195/7 | = 90/7, fi·dev = 8 × 90/7 = 720/7 ≈ 102.86 |
| xi = 25: | 25 − 195/7 | = 20/7, fi·dev = 14 × 20/7 = 40 |
| xi = 35: | 35 − 195/7 | = 50/7, fi·dev = 16 × 50/7 = 800/7 ≈ 114.29 |

| fi· | xi − M | : |
|---|---|---|
| xi = 45: | 45 − 195/7 | = 120/7, fi·dev = 4 × 120/7 = 480/7 ≈ 68.57 |
| xi = 55: | 55 − 195/7 | = 190/7, fi·dev = 2 × 190/7 = 380/7 ≈ 54.29 |

Step 4 – Σfi·|xi − M| = 3620/7 ≈ 517.14.

M.D.(M) = (3620/7)/50 = 362/35 ≈ 10.34.

Answer: Median ≈ 27.86; M.D.(M) = 362/35 ≈ 10.34

Q12 Calculate the mean deviation about median age for the age distribution of 100 persons given below:

| Age (years): | 16-20 | 21-25 | 26-30 | 31-35 | 36-40 | 41-45 | 46-50 | 51-55 |
|---|---|---|---|---|---|---|---|---|
| Number: | 5 | 6 | 12 | 14 | 26 | 12 | 16 | 9 |

[Hint: Convert to continuous distribution by subtracting 0.5 from lower limit and adding 0.5 to upper limit.]

SOLUTION

Step 1 – Convert to continuous classes: 15.5-20.5, 20.5-25.5, 25.5-30.5, 30.5-35.5, 35.5-40.5, 40.5-45.5, 45.5-50.5, 50.5-55.5.
Mid-points xi: 18, 23, 28, 33, 38, 43, 48, 53.
N = 5 + 6 + 12 + 14 + 26 + 12 + 16 + 9 = 100

Step 2 – Cumulative frequencies.
c.f.: 5, 11, 23, 37, 63, 75, 91, 100

Step 3 – Find median class. N/2 = 50. First c.f. ≥ 50 is 63, belonging to class 35.5-40.5.
l = 35.5, C = 37, f = 26, h = 5.
Median M = 35.5 + ((50 − 37)/26) × 5 = 35.5 + 65/26 = 35.5 + 2.5 = 38.

| Step 4 – fi· | xi − 38 | : |
|---|---|---|
| xi = 18: | 18 − 38 | = 20, fi·dev = 5 × 20 = 100 |
| xi = 23: | 23 − 38 | = 15, fi·dev = 6 × 15 = 90 |
| xi = 28: | 28 − 38 | = 10, fi·dev = 12 × 10 = 120 |
| xi = 33: | 33 − 38 | = 5, fi·dev = 14 × 5 = 70 |

| Step 4 – fi· | xi − 38 | : |
|---|---|---|
| xi = 38: | 38 − 38 | = 0, fi·dev = 26 × 0 = 0 |
| xi = 43: | 43 − 38 | = 5, fi·dev = 12 × 5 = 60 |
| xi = 48: | 48 − 38 | = 10, fi·dev = 16 × 10 = 160 |
| xi = 53: | 53 − 38 | = 15, fi·dev = 9 × 15 = 135 |

Step 5 – Σfi·|xi − M| = 100 + 90 + 120 + 70 + 0 + 60 + 160 + 135 = 735.

M.D.(M) = 735/100 = 7.35 years.

Answer: Median = 38 years; M.D.(M) = 7.35 years

## Exercise 13.2 — Exercise 13.2

### Q1 Find the mean and variance for the data: 6, 7, 10, 12, 13, 4, 8, 12.

SOLUTION

n = 8, Sum = 6 + 7 + 10 + 12 + 13 + 4 + 8 + 12 = 72
Mean $\bar{x} = 72/8 = 9$

Deviations $(x_i - \bar{x})$ and $(x_i - \bar{x})^2$:
$6 - 9 = -3,\ (-3)^2 = 9$
$7 - 9 = -2,\ (-2)^2 = 4$
$10 - 9 = 1,\ 1^2 = 1$
$12 - 9 = 3,\ 3^2 = 9$
$13 - 9 = 4,\ 4^2 = 16$
$4 - 9 = -5,\ (-5)^2 = 25$
$8 - 9 = -1,\ (-1)^2 = 1$
$12 - 9 = 3,\ 3^2 = 9$

$\Sigma(x_i - \bar{x})^2 = 9 + 4 + 1 + 9 + 16 + 25 + 1 + 9 = 74$

Variance $\sigma^2 = \Sigma(x_i - \bar{x})^2/n = 74/8 = 9.25$.
Standard deviation $\sigma = \sqrt{9.25} \approx 3.04$.

**Answer: Mean = 9; Variance = 9.25**

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### Q2 Find the mean and variance for the first n natural numbers.

SOLUTION

The first n natural numbers are 1, 2, 3, …, n.

Mean $\bar{x} = (1 + 2 + \ldots + n)/n = [n(n+1)/2]/n = (n+1)/2$.

To find the variance, use:
$\Sigma x_i^2 = 1^2 + 2^2 + \ldots + n^2 = n(n+1)(2n+1)/6$

Variance $\sigma^2 = \Sigma x_i^2/n - (\bar{x})^2$
$= [n(n+1)(2n+1)/6]/n - [(n+1)/2]^2$
$= (n+1)(2n+1)/6 - (n+1)^2/4$

Factor out (n+1):
$= (n+1)[(2n+1)/6 - (n+1)/4]$
$= (n+1)[2(2n+1) - 3(n+1)] / 12$
$= (n+1)[4n+2 - 3n - 3] / 12$

= (n+1)(n−1)/12
= (n² − 1)/12.

Answer: Mean = (n+1)/2; Variance = (n² − 1)/12

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Q3 Find the mean and variance for the first 10 multiples of 3.

SOLUTION

The first 10 multiples of 3 are: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.
n = 10

Sum = 3(1 + 2 + … + 10) = 3 × 55 = 165
Mean $\bar{x}$ = 165/10 = 16.5

Deviations $(x_i - \bar{x})$ and $(x_i - \bar{x})^2$:
3 − 16.5 = −13.5 → 182.25
6 − 16.5 = −10.5 → 110.25
9 − 16.5 = −7.5 → 56.25
12 − 16.5 = −4.5 → 20.25
15 − 16.5 = −1.5 → 2.25
18 − 16.5 = 1.5 → 2.25
21 − 16.5 = 4.5 → 20.25
24 − 16.5 = 7.5 → 56.25
27 − 16.5 = 10.5 → 110.25
30 − 16.5 = 13.5 → 182.25

$\Sigma(x_i - \bar{x})^2 = 742.5$

Variance $\sigma^2$ = 742.5/10 = 74.25.

(Equivalently, since these are 3 × {1,2,…,10}, variance = 9 × (10² − 1)/12 = 9 × 8.25 = 74.25.)

Answer: Mean = 16.5; Variance = 74.25

**Q4** Find the mean and variance for the following frequency distribution:

| $x_i$ | 6 | 10 | 14 | 18 | 24 | 28 | 30 |
|---|---|---|---|---|---|---|---|
| $f_i$ | 2 | 4 | 7 | 12 | 8 | 4 | 3 |

SOLUTION

$N = 2 + 4 + 7 + 12 + 8 + 4 + 3 = 40$

$\Sigma f_i x_i = 6\times2 + 10\times4 + 14\times7 + 18\times12 + 24\times8 + 28\times4 + 30\times3$

$= 12 + 40 + 98 + 216 + 192 + 112 + 90 = 760$

Mean $\bar{x} = 760/40 = 19$

$f_i(x_i - \bar{x})^2$:

$x_i = 6$: $(6 - 19)^2 = 169$, $f_i(x_i-\bar{x})^2 = 2 \times 169 = 338$

$x_i = 10$: $(10 - 19)^2 = 81$, $f_i(x_i-\bar{x})^2 = 4 \times 81 = 324$

$x_i = 14$: $(14 - 19)^2 = 25$, $f_i(x_i-\bar{x})^2 = 7 \times 25 = 175$

$x_i = 18$: $(18 - 19)^2 = 1$, $f_i(x_i-\bar{x})^2 = 12 \times 1 = 12$

$x_i = 24$: $(24 - 19)^2 = 25$, $f_i(x_i-\bar{x})^2 = 8 \times 25 = 200$

$x_i = 28$: $(28 - 19)^2 = 81$, $f_i(x_i-\bar{x})^2 = 4 \times 81 = 324$

$x_i = 30$: $(30 - 19)^2 = 121$, $f_i(x_i-\bar{x})^2 = 3 \times 121 = 363$

$\Sigma f_i(x_i - \bar{x})^2 = 338 + 324 + 175 + 12 + 200 + 324 + 363 = 1736$

Variance $\sigma^2 = 1736/40 = 43.4$.

Standard deviation $\sigma = \sqrt{43.4} \approx 6.59$.

Answer: Mean = 19; Variance = 43.4

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**Q5** Find the mean and variance for the following frequency distribution:

| $x_i$ | 92 | 93 | 97 | 98 | 102 | 104 | 109 |
|---|---|---|---|---|---|---|---|
| $f_i$ | 3 | 2 | 3 | 2 | 6 | 3 | 3 |

SOLUTION

$N = 3 + 2 + 3 + 2 + 6 + 3 + 3 = 22$

$\Sigma f_i x_i = 92\times3 + 93\times2 + 97\times3 + 98\times2 + 102\times6 + 104\times3 + 109\times3$

$= 276 + 186 + 291 + 196 + 612 + 312 + 327 = 2200$

Mean $\bar{x} = 2200/22 = 100$

$f_i(x_i - 100)^2$:

$x_i = 92$: $(-8)^2 = 64$, $f_i \cdot dev^2 = 3 \times 64 = 192$

$x_i = 93$: $(-7)^2 = 49$, $f_i \cdot dev^2 = 2 \times 49 = 98$

$x_i = 97$: $(-3)^2 = 9$, $f_i \cdot dev^2 = 3 \times 9 = 27$

xi = 98: $(-2)^2 = 4$, $f_i \cdot dev^2 = 2 \times 4 = 8$
xi = 102: $(2)^2 = 4$, $f_i \cdot dev^2 = 6 \times 4 = 24$
xi = 104: $(4)^2 = 16$, $f_i \cdot dev^2 = 3 \times 16 = 48$
xi = 109: $(9)^2 = 81$, $f_i \cdot dev^2 = 3 \times 81 = 243$

$\Sigma f_i \cdot (x_i - \bar{x})^2 = 192 + 98 + 27 + 8 + 24 + 48 + 243 = 640$

Variance $\sigma^2 = 640/22 = 320/11 \approx 29.09$.
Standard deviation $\sigma = \sqrt{(320/11)} \approx 5.39$.

Answer: Mean = 100; Variance = 320/11 ≈ 29.09

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Q6 Find the mean and standard deviation using the short-cut method.

| xi : | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 |
|---|---|---|---|---|---|---|---|---|---|
| fi : | 2 | 1 | 12 | 29 | 25 | 12 | 10 | 4 | 5 |

SOLUTION

Take assumed mean A = 64, h = 1.
$y_i = (x_i - 64)/1 = x_i - 64$
$N = 2 + 1 + 12 + 29 + 25 + 12 + 10 + 4 + 5 = 100$

Compute $y_i$, $f_i \cdot y_i$, $f_i \cdot y_i^2$:
xi=60: $y_i=-4$, $f_i \cdot y_i=-8$, $f_i \cdot y_i^2=32$
xi=61: $y_i=-3$, $f_i \cdot y_i=-3$, $f_i \cdot y_i^2=9$
xi=62: $y_i=-2$, $f_i \cdot y_i=-24$, $f_i \cdot y_i^2=48$
xi=63: $y_i=-1$, $f_i \cdot y_i=-29$, $f_i \cdot y_i^2=29$
xi=64: $y_i= 0$, $f_i \cdot y_i= 0$, $f_i \cdot y_i^2=0$
xi=65: $y_i= 1$, $f_i \cdot y_i= 12$, $f_i \cdot y_i^2=12$
xi=66: $y_i= 2$, $f_i \cdot y_i= 20$, $f_i \cdot y_i^2=40$
xi=67: $y_i= 3$, $f_i \cdot y_i= 12$, $f_i \cdot y_i^2=36$
xi=68: $y_i= 4$, $f_i \cdot y_i= 20$, $f_i \cdot y_i^2=80$

$\Sigma f_i \cdot y_i = -8 - 3 - 24 - 29 + 0 + 12 + 20 + 12 + 20 = 0$
$\Sigma f_i \cdot y_i^2 = 32 + 9 + 48 + 29 + 0 + 12 + 40 + 36 + 80 = 286$

Mean $\bar{x} = A + (\Sigma f_i \cdot y_i/N) \times h = 64 + (0/100) \times 1 = 64$.

Variance $\sigma^2 = (h^2/N^2)[N \cdot \Sigma f_i \cdot y_i^2 - (\Sigma f_i \cdot y_i)^2]$
$= (1/10000)[100 \times 286 - 0^2]$
$= 28600/10000 = 2.86$.

Standard deviation $\sigma = \sqrt{2.86} \approx 1.69$.

Answer: Mean = 64; Standard deviation ≈ 1.69

Q7 Find the mean and variance for the following frequency distribution:

| Classes: | 0-30 | 30-60 | 60-90 | 90-120 | 120-150 | 150-180 | 180-210 |
|---|---|---|---|---|---|---|---|
| Frequencies: | 2 | 3 | 5 | 10 | 3 | 5 | 2 |

SOLUTION

Mid-points xi: 15, 45, 75, 105, 135, 165, 195.

N = 2 + 3 + 5 + 10 + 3 + 5 + 2 = 30

$\Sigma f_i x_i = 2\times15 + 3\times45 + 5\times75 + 10\times105 + 3\times135 + 5\times165 + 2\times195$

$= 30 + 135 + 375 + 1050 + 405 + 825 + 390 = 3210$

Mean $\bar{x} = 3210/30 = 107$

$f_i(x_i - 107)^2$:

$x_i=15$: $(15-107)^2=8464$, $f_i\cdot dev^2=2\times8464=16928$

$x_i=45$: $(45-107)^2=3844$, $f_i\cdot dev^2=3\times3844=11532$

$x_i=75$: $(75-107)^2=1024$, $f_i\cdot dev^2=5\times1024=5120$

$x_i=105$: $(105-107)^2=4$, $f_i\cdot dev^2=10\times4=40$

$x_i=135$: $(135-107)^2=784$, $f_i\cdot dev^2=3\times784=2352$

$x_i=165$: $(165-107)^2=3364$, $f_i\cdot dev^2=5\times3364=16820$

$x_i=195$: $(195-107)^2=7744$, $f_i\cdot dev^2=2\times7744=15488$

$\Sigma f_i(x_i - \bar{x})^2 = 16928 + 11532 + 5120 + 40 + 2352 + 16820 + 15488 = 68280$

Variance $\sigma^2 = 68280/30 = 2276$.

Standard deviation $\sigma = \sqrt{2276} \approx 47.71$.

Answer: Mean = 107; Variance = 2276

Q8 Find the mean and variance for the following frequency distribution:

| Classes: | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|---|---|---|---|---|---|
| Frequencies: | 5 | 8 | 15 | 16 | 6 |

SOLUTION

Mid-points xi: 5, 15, 25, 35, 45.

N = 5 + 8 + 15 + 16 + 6 = 50

$\Sigma f_i x_i = 5\times5 + 8\times15 + 15\times25 + 16\times35 + 6\times45$

$= 25 + 120 + 375 + 560 + 270 = 1350$

Mean $\bar{x} = 1350/50 = 27$

$f_i\cdot(x_i - 27)^2$:

$x_i=5$: $(5-27)^2=484$, $f_i\cdot dev^2=5\times484=2420$

$x_i=15$: $(15-27)^2=144$, $f_i\cdot dev^2=8\times144=1152$

$x_i=25$: $(25-27)^2=4$, $f_i\cdot dev^2=15\times4=60$

$x_i=35$: $(35-27)^2=64$, $f_i\cdot dev^2=16\times64=1024$

$x_i=45$: $(45-27)^2=324$, $f_i\cdot dev^2=6\times324=1944$

$\Sigma f_i\cdot(x_i - \bar{x})^2 = 2420 + 1152 + 60 + 1024 + 1944 = 6600$

Variance $\sigma^2 = 6600/50 = 132$.

Standard deviation $\sigma = \sqrt{132} \approx 11.49$.

Answer: Mean = 27; Variance = 132

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Q9 Find the mean, variance and standard deviation using the short-cut method.

| Height (cm) | 70-75 | 75-80 | 80-85 | 85-90 | 90-95 | 95-100 | 100-105 | 105-110 | 110-115 |
|---|---|---|---|---|---|---|---|---|---|
| No. of children | 3 | 4 | 7 | 7 | 15 | 9 | 6 | 6 | 3 |

SOLUTION

Mid-points $x_i$: 72.5, 77.5, 82.5, 87.5, 92.5, 97.5, 102.5, 107.5, 112.5.

$N = 3+4+7+7+15+9+6+6+3 = 60$

Take assumed mean $A = 92.5$, $h = 5$.

$y_i = (x_i - 92.5)/5$

$y_i$ values: $-4, -3, -2, -1, 0, 1, 2, 3, 4$

$f_i\cdot y_i$:

$3\times(-4)=-12$, $4\times(-3)=-12$, $7\times(-2)=-14$, $7\times(-1)=-7$, $15\times0=0$,

$9\times1=9$, $6\times2=12$, $6\times3=18$, $3\times4=12$

$\Sigma f_i\cdot y_i = -12-12-14-7+0+9+12+18+12 = 6$

$f_i\cdot y_i^2$:

$3\times16=48$, $4\times9=36$, $7\times4=28$, $7\times1=7$, $15\times0=0$,

$9\times1=9$, $6\times4=24$, $6\times9=54$, $3\times16=48$

$\Sigma f_i\cdot y_i^2 = 48+36+28+7+0+9+24+54+48 = 254$

Mean $\bar{x} = A + (\Sigma f_i\cdot y_i/N)\times h = 92.5 + (6/60)\times5 = 92.5 + 0.5 = 93$ cm.

Variance $\sigma^2 = (h^2/N^2)[N\cdot\Sigma f_i\cdot y_i^2 - (\Sigma f_i\cdot y_i)^2]$

$= (25/3600)[60\times254 - 36]$

$= (25/3600)[15240 - 36]$

= (25/3600) × 15204
= 380100/3600 ≈ 105.58.

Standard deviation σ = √105.58 ≈ 10.28 cm.

Answer: Mean = 93 cm; Variance ≈ 105.58; Standard deviation ≈ 10.28 cm

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Q10 The diameters of circles (in mm) drawn in a design are given below:

| Diameters: | 33-36 | 37-40 | 41-44 | 45-48 | 49-52 |
|---|---|---|---|---|---|
| No. of circles: | 15 | 17 | 21 | 22 | 25 |

Calculate the standard deviation and mean diameter of the circles.

SOLUTION

First make the data continuous:
Classes: 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5-48.5, 48.5-52.5.
Mid-points xi: 34.5, 38.5, 42.5, 46.5, 50.5.
fi: 15, 17, 21, 22, 25.
N = 15+17+21+22+25 = 100.

Take assumed mean A = 42.5, h = 4.
yi = (xi − 42.5)/4: −2, −1, 0, 1, 2.

fi·yi: 15×(−2)=−30, 17×(−1)=−17, 21×0=0, 22×1=22, 25×2=50
Σfi·yi = −30−17+0+22+50 = 25

fi·yi²: 15×4=60, 17×1=17, 21×0=0, 22×1=22, 25×4=100
Σfi·yi² = 60+17+0+22+100 = 199

Mean $\bar{x}$ = A + (Σfi·yi/N)×h = 42.5 + (25/100)×4 = 42.5 + 1 = 43.5 mm.

Variance σ² = (h²/N²)[N·Σfi·yi² − (Σfi·yi)²]
= (16/10000)[100×199 − 625]
= (16/10000)[19900 − 625]
= (16/10000) × 19275
= 308400/10000 = 30.84.

Standard deviation σ = √30.84 ≈ 5.55 mm.

Answer: Mean diameter = 43.5 mm; Standard deviation ≈ 5.55 mm

## Misc — Miscellaneous Exercise

**Q1** The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

### SOLUTION

Let the two unknown observations be x and y.

Using the mean:
$(6 + 7 + 10 + 12 + 12 + 13 + x + y)/8 = 9$
$60 + x + y = 72$
$x + y = 12 \quad \ldots (1)$

Using the variance:
$\sigma^2 = (1/n)\Sigma xi^2 - \bar{x}^2 \Rightarrow \Sigma xi^2 = n(\sigma^2 + \bar{x}^2) = 8(9.25 + 81) = 8 \times 90.25 = 722$
Sum of squares of known six observations = $36 + 49 + 100 + 144 + 144 + 169 = 642$
$x^2 + y^2 = 722 - 642 = 80 \quad \ldots (2)$

From (1): $(x + y)^2 = 144 \Rightarrow x^2 + y^2 + 2xy = 144$
$\Rightarrow 2xy = 144 - 80 = 64 \Rightarrow xy = 32 \quad \ldots (3)$

$(x - y)^2 = (x + y)^2 - 4xy = 144 - 128 = 16 \Rightarrow x - y = \pm 4 \quad \ldots (4)$

Solving (1) and (4):
If $x - y = 4$: $x = 8, y = 4$.
If $x - y = -4$: $x = 4, y = 8$.

The two remaining observations are 4 and 8.

**Answer: The remaining two observations are 4 and 8.**

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**Q2** The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12, 14, find the remaining two observations.

### SOLUTION

Let the two unknown observations be x and y.

Using the mean:
$(2 + 4 + 10 + 12 + 14 + x + y)/7 = 8$
$42 + x + y = 56$
$x + y = 14 \quad \ldots (1)$

Using the variance:
$\Sigma x_i^2 = n(\sigma^2 + \bar{x}^2) = 7(16 + 64) = 7 \times 80 = 560$
Sum of squares of known five = 4 + 16 + 100 + 144 + 196 = 460
$x^2 + y^2 = 560 - 460 = 100$ … (2)

From (1): $2xy = (x+y)^2 - (x^2+y^2) = 196 - 100 = 96 \Rightarrow xy = 48$ … (3)

$(x - y)^2 = (x+y)^2 - 4xy = 196 - 192 = 4 \Rightarrow x - y = \pm 2$ … (4)

Solving (1) and (4):
If $x - y = 2$: $x = 8, y = 6$.
If $x - y = -2$: $x = 6, y = 8$.

The two remaining observations are 6 and 8.

Answer: The remaining two observations are 6 and 8.

---

Q3 The mean and standard deviation of six observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

SOLUTION

Let the original observations be $x_1, x_2, \ldots, x_6$ with mean $\bar{x} = 8$ and $\sigma = 4$.
New observations: $y_i = 3x_i$.

New mean $= (1/6)\Sigma y_i = (1/6)\Sigma 3x_i = 3 \times (1/6)\Sigma x_i = 3 \times 8 = 24$.

New variance:
$\sigma^2_y = (1/6)\Sigma(y_i - )^2 = (1/6)\Sigma(3x_i - 3\bar{x})^2 = (1/6) \times 9 \times \Sigma(x_i - \bar{x})^2 = 9\sigma^2_x = 9 \times 16 = 144$.

New standard deviation $\sigma_y = \sqrt{144} = 12$.

Answer: New mean = 24; New standard deviation = 12.

---

Q4 Given that $\bar{x}$ is the mean and $\sigma^2$ is the variance of n observations $x_1, x_2, \ldots, x_n$. Prove that the mean and variance of the observations $ax_1, ax_2, ax_3, \ldots, ax_n$ are $a\bar{x}$ and $a^2\sigma^2$, respectively ($a \neq 0$).

SOLUTION

Let $y_i = ax_i$ for each $i = 1, 2, \ldots, n$.

Mean of new observations:

$= (1/n)\Sigma yi = (1/n)\Sigma axi = a \times (1/n)\Sigma xi = a \times \bar{x}$.

So the new mean is $a\bar{x}$.

Variance of new observations:

$\sigma^2_y = (1/n)\Sigma(yi - )^2$

$= (1/n)\Sigma(axi - a\bar{x})^2$

$= (1/n)\Sigma a^2(xi - \bar{x})^2$

$= a^2(1/n)\Sigma(xi - \bar{x})^2$

$= a^2\sigma^2$.

Therefore the mean of ax1, ax2, …, axn is $a\bar{x}$ and the variance is $a^2\sigma^2$.

Answer: Mean of new observations = $a\bar{x}$; Variance = $a^2\sigma^2$. (Proved)

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Q5 The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:
(i) If the wrong item is omitted.
(ii) If it is replaced by 12.

SOLUTION

Given: $n = 20$, $\bar{x} = 10$, $\sigma = 2$.

Incorrect $\Sigma xi = 20 \times 10 = 200$.

Incorrect $\Sigma xi^2 = n(\sigma^2 + \bar{x}^2) = 20(4 + 100) = 2080$.

(i) Wrong item 8 is omitted (n becomes 19):

Correct $\Sigma xi = 200 - 8 = 192$.

Correct mean $= 192/19 \approx 10.105$.

Correct $\Sigma xi^2 = 2080 - 64 = 2016$.

Correct variance $= 2016/19 - (192/19)^2$

$= (2016 \times 19 - 192^2)/19^2$

$= (38304 - 36864)/361$

$= 1440/361 \approx 3.989$.

Correct standard deviation $\sigma = \sqrt{(1440/361)} = (12\sqrt{10})/19 \approx 1.997$.

(ii) Wrong item 8 is replaced by 12 (n remains 20):

Correct $\Sigma xi = 200 - 8 + 12 = 204$.

Correct mean $= 204/20 = 10.2$.

Correct $\Sigma xi^2 = 2080 - 64 + 144 = 2160$.

Correct variance = 2160/20 − (10.2)²
= 108 − 104.04 = 3.96.
Correct standard deviation σ = √3.96 ≈ 1.99.

Answer: (i) Correct mean ≈ 10.105, correct standard deviation ≈ 1.997. (ii) Correct mean = 10.2, correct standard deviation ≈ 1.99.

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Q6 The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

SOLUTION

Given: n = 100, $\bar{x}$ = 20, σ = 3.
Incorrect Σxi = 100 × 20 = 2000.
Incorrect Σxi² = n(σ² + $\bar{x}$²) = 100(9 + 400) = 40900.

Omit 21, 21, 18 (n becomes 97):
Correct Σxi = 2000 − 21 − 21 − 18 = 1940.
Correct mean = 1940/97 = 20.

Correct Σxi² = 40900 − 21² − 21² − 18²
= 40900 − 441 − 441 − 324 = 39694.

Correct variance = 39694/97 − 20²
= 39694/97 − 400
= (39694 − 38800)/97
= 894/97 ≈ 9.22.

Correct standard deviation σ = √(894/97) ≈ 3.04.

Answer: Correct mean = 20; Correct standard deviation ≈ 3.04.