Chapter 14: Probability

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Exercise 14.1 — Exercise 14.1

Q1 A die is rolled. Let E be the event 'die shows 4' and F be the event 'die shows even number'. Are E and F mutually exclusive?

SOLUTION

The sample space is S = {1, 2, 3, 4, 5, 6}.
E = {4} (die shows 4).
F = {2, 4, 6} (die shows an even number).

For two events to be mutually exclusive, their intersection must be empty.
$E \cap F = \{4\} \cap \{2, 4, 6\} = \{4\} \neq \varphi$.

Since $E \cap F \neq \varphi$, the events E and F are not mutually exclusive.

Answer: E and F are not mutually exclusive, since $E \cap F = \{4\} \neq \varphi$.

Q2 A die is thrown. Describe the following events:
(i) A: a number less than 7
(ii) B: a number greater than 7
(iii) C: a multiple of 3
(iv) D: a number less than 4
(v) E: an even number greater than 4
(vi) F: a number not less than 3
Also find $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $A - C$, $D - E$, $E \cap F'$, F'.

SOLUTION

The sample space is S = {1, 2, 3, 4, 5, 6}.

(i) A: a number less than 7. All outcomes of S are less than 7, so A = {1, 2, 3, 4, 5, 6} = S.

(ii) B: a number greater than 7. No outcome of S is greater than 7, so B = φ.

(iii) C: a multiple of 3. Multiples of 3 in S are 3 and 6. So C = {3, 6}.

(iv) D: a number less than 4. Numbers less than 4 are 1, 2, 3. So D = {1, 2, 3}.

(v) E: an even number greater than 4. Even numbers in S are 2, 4, 6; those greater than 4 give E = {6}.

(vi) F: a number not less than 3, i.e., a number ≥ 3. So F = {3, 4, 5, 6}.

Now we find the required set operations:

$A \cup B = S \cup \varphi = S = \{1, 2, 3, 4, 5, 6\}$.

$A \cap B = S \cap \varphi = \varphi$.

$B \cup C = \varphi \cup \{3, 6\} = \{3, 6\}$.

$E \cap F = \{6\} \cap \{3, 4, 5, 6\} = \{6\}$.

$D \cap E = \{1, 2, 3\} \cap \{6\} = \varphi$.

$A - C = S - \{3, 6\} = \{1, 2, 4, 5\}$.

$D - E = \{1, 2, 3\} - \{6\} = \{1, 2, 3\}$.

$F' = S - F = \{1, 2, 3, 4, 5, 6\} - \{3, 4, 5, 6\} = \{1, 2\}$.

$E \cap F' = \{6\} \cap \{1, 2\} = \varphi$.

Answer: $A = S = \{1,2,3,4,5,6\}$, $B = \varphi$, $C = \{3,6\}$, $D = \{1,2,3\}$, $E = \{6\}$, $F = \{3,4,5,6\}$. $A \cup B = S$, $A \cap B = \varphi$, $B \cup C = \{3,6\}$, $E \cap F = \{6\}$, $D \cap E = \varphi$, $A - C = \{1,2,4,5\}$, $D - E = \{1,2,3\}$, $F' = \{1,2\}$, $E \cap F' = \varphi$.

Q3 An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:
A: the sum is greater than 8, B: 2 occurs on either die, C: the sum is at least 7 and a multiple of 3.
Which pairs of these events are mutually exclusive?

SOLUTION

The sample space has 36 equally likely outcomes: $S = \{(i, j) : i, j = 1, 2, 3, 4, 5, 6\}$.

A: sum > 8.
$A = \{(3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)\}$.

B: 2 occurs on either die (i.e., first die shows 2 or second die shows 2).
$B = \{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(1,2),(3,2),(4,2),(5,2),(6,2)\}$.

C: sum ≥ 7 and sum is a multiple of 3. The multiples of 3 that are ≥ 7 are 9 and 12.
Sum = 9: (3,6),(4,5),(5,4),(6,3). Sum = 12: (6,6).
C = {(3,6), (4,5), (5,4), (6,3), (6,6)}.

Check mutual exclusivity of each pair:

A ∩ B: Every element of A has sum > 8, so both dice must be ≥ 3. Hence no element of A has 2 on either die.
A ∩ B = φ. So A and B are mutually exclusive.

A ∩ C: C = {(3,6),(4,5),(5,4),(6,3),(6,6)} and all these have sum ≥ 9 > 8, so all are in A.
A ∩ C = {(3,6),(4,5),(5,4),(6,3),(6,6)} ≠ φ. So A and C are not mutually exclusive.

B ∩ C: Elements of C are (3,6),(4,5),(5,4),(6,3),(6,6). None of these has a 2.
B ∩ C = φ. So B and C are mutually exclusive.

Answer: A = {(3,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)}, B contains 11 outcomes with 2 on either die, C = {(3,6),(4,5),(5,4),(6,3),(6,6)}. A and B are mutually exclusive; B and C are mutually exclusive; A and C are not mutually exclusive.

Q4 Three coins are tossed once. Let A denote the event 'three heads show', B denote the event 'two heads and one tail show', C denote the event 'three tails show' and D denote the event 'a head shows on the first coin'. Which events are
(i) mutually exclusive?
(ii) simple?
(iii) Compound?

SOLUTION

The sample space is S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

A = {HHH}, B = {HHT, HTH, THH}, C = {TTT}, D = {HHH, HHT, HTH, HTT}.

(i) Mutually exclusive events (A ∩ B = φ, etc.):
A ∩ B = {HHH} ∩ {HHT, HTH, THH} = φ. ✓
A ∩ C = {HHH} ∩ {TTT} = φ. ✓
A ∩ D = {HHH} ∩ {HHH, HHT, HTH, HTT} = {HHH} ≠ φ. ✗
B ∩ C = {HHT, HTH, THH} ∩ {TTT} = φ. ✓
B ∩ D = {HHT, HTH, THH} ∩ {HHH, HHT, HTH, HTT} = {HHT, HTH} ≠ φ. ✗

$C \cap D = \{TTT\} \cap \{HHH, HHT, HTH, HTT\} = \varphi$. ✓

Mutually exclusive pairs: (A, B), (A, C), (B, C), (C, D).

(ii) Simple events (containing exactly one sample point): A = {HHH} and C = {TTT} are simple events.

(iii) Compound events (containing more than one sample point): B = {HHT, HTH, THH} and D = {HHH, HHT, HTH, HTT} are compound events.

Answer: (i) Mutually exclusive pairs: (A,B), (A,C), (B,C), (C,D). (ii) Simple events: A and C. (iii) Compound events: B and D.

Q5 Three coins are tossed. Describe
(i) Two events which are mutually exclusive.
(ii) Three events which are mutually exclusive and exhaustive.
(iii) Two events, which are not mutually exclusive.
(iv) Two events which are mutually exclusive but not exhaustive.
(v) Three events which are mutually exclusive but not exhaustive.

SOLUTION

The sample space for tossing three coins is:
S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

(i) Two mutually exclusive events:
Let E1 = {HHH} (all heads) and E2 = {TTT} (all tails).
$E1 \cap E2 = \varphi$, so they are mutually exclusive.

(ii) Three mutually exclusive and exhaustive events:
Let A = {HHH, HHT, HTH, THH} (at least two heads),
B = {HTT, THT, TTH} (exactly one head),
C = {TTT} (no head).
$A \cap B = \varphi$, $A \cap C = \varphi$, $B \cap C = \varphi$ (pairwise disjoint), and $A \cup B \cup C = S$ (exhaustive).

(iii) Two events that are not mutually exclusive:
Let P = {HHH, HHT, HTH, THH} (at least two heads),
Q = {HHH, HHT, HTH, HTT} (head on first coin).
$P \cap Q = \{HHH, HHT, HTH\} \neq \varphi$, so P and Q are not mutually exclusive.

(iv) Two mutually exclusive but not exhaustive events:
Let E3 = {HHH} and E4 = {TTT}.

E3 ∩ E4 = φ (mutually exclusive), but E3 ∪ E4 = {HHH, TTT} ≠ S (not exhaustive).

(v) Three mutually exclusive but not exhaustive events:
Let F1 = {HHH}, F2 = {HHT}, F3 = {TTT}.
These are pairwise disjoint, but F1 ∪ F2 ∪ F3 = {HHH, HHT, TTT} ≠ S (not exhaustive).

Note: Any two or three correct examples satisfying the stated conditions are acceptable.

Answer: (i) E.g. {HHH} and {TTT} — mutually exclusive. (ii) E.g. 'at least 2H', 'exactly 1H', 'no H' — mutually exclusive and exhaustive. (iii) E.g. 'at least 2H' and 'head on first coin' — not mutually exclusive (share HHH, HHT, HTH). (iv) E.g. {HHH} and {TTT} — mutually exclusive but not exhaustive. (v) E.g. {HHH}, {HHT}, {TTT} — mutually exclusive but not exhaustive.

Q6 Two dice are thrown. The events A, B and C are as follows:
A: getting an even number on the first die.
B: getting an odd number on the first die.
C: getting the sum of the numbers on the dice ≤ 5.
Describe the events
(i) A′ (ii) not B (iii) A or B (iv) A and B (v) A but not C (vi) B or C (vii) B and C
(viii) A ∩ B′ ∩ C′

SOLUTION

The sample space S has 36 outcomes: S = {(i, j) : i, j = 1, 2, 3, 4, 5, 6}.

A = {(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} (first die even).

B = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)} (first die odd).

C (sum ≤ 5) = {(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)}.

(i) A′ = S – A = B = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)}.

(ii) not B = B′ = S – B = A =

{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}.

(iii) A or B = A ∪ B = S (since every outcome has an even or odd first die).

(iv) A and B = A ∩ B = φ (a number cannot be both even and odd simultaneously).

(v) A but not C = A – C = A ∩ C′.
A ∩ C = {(2,1),(2,2),(2,3),(4,1)} (elements of C with first die even).
A – C = A – {(2,1),(2,2),(2,3),(4,1)} = {(2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}.

(vi) B or C = B ∪ C = B ∪ {(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)}.
Elements of C not in B: (2,1),(2,2),(2,3),(4,1).
B ∪ C = {all 18 elements of B} ∪ {(2,1),(2,2),(2,3),(4,1)} (22 elements total).

(vii) B and C = B ∩ C = {(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)} (elements with odd first die and sum ≤ 5).

(viii) A ∩ B′ ∩ C′: B′ = A, so A ∩ B′ = A ∩ A = A. Then A ∩ A ∩ C′ = A ∩ C′ = A – C = A but not C.
A ∩ B′ ∩ C′ = {(2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}.

Answer: (i) A′ = B. (ii) not B = A. (iii) A∪B = S. (iv) A∩B = φ. (v) A–C = {(2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}. (vi) B∪C: B plus (2,1),(2,2),(2,3),(4,1). (vii) B∩C = {(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)}. (viii) A∩B′∩C′ = A–C (same as (v)).

Q7 Refer to question 6 above, state true or false: (give reason for your answer)
(i) A and B are mutually exclusive
(ii) A and B are mutually exclusive and exhaustive
(iii) A = B′
(iv) A and C are mutually exclusive
(v) A and B′ are mutually exclusive.
(vi) A′, B′, C are mutually exclusive and exhaustive.

SOLUTION

Using the sets defined in Q6: A (even first die), B (odd first die), C (sum ≤ 5).
A and B partition S: A ∪ B = S, A ∩ B = φ, so B′ = A and A′ = B.

(i) A and B are mutually exclusive — TRUE.

Reason: $A \cap B = \varphi$ since no number is both even and odd.

(ii) A and B are mutually exclusive and exhaustive — TRUE.
Reason: $A \cap B = \varphi$ (mutually exclusive) and $A \cup B = S$ (exhaustive, since every outcome has an even or odd first die).

(iii) $A = B'$ — TRUE.
Reason: $B' = S - B$ = set of outcomes where first die is NOT odd = set of outcomes where first die is even = A.

(iv) A and C are mutually exclusive — FALSE.
Reason: $A \cap C = \{(2,1),(2,2),(2,3),(4,1)\} \neq \varphi$. Both events share outcomes (e.g., the outcome (2,1) has first die even and sum = $3 \leq 5$).

(v) A and B' are mutually exclusive — FALSE.
Reason: $B' = A$ (proved in (iii)), so $A \cap B' = A \cap A = A \neq \varphi$. In fact, A and B' are the same event.

(vi) A', B', C are mutually exclusive and exhaustive — FALSE.
Reason: $A' = B$ and $B' = A$. Now $A' \cap C = B \cap C = \{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\} \neq \varphi$. So A' and C are not mutually exclusive; the three events are not pairwise disjoint.
(Though $A' \cup B' \cup C = B \cup A \cup C = S$, they fail the mutual exclusivity condition.)

Answer: (i) TRUE — $A \cap B = \varphi$. (ii) TRUE — $A \cap B = \varphi$ and $A \cup B = S$. (iii) TRUE — $B' = A$. (iv) FALSE — $A \cap C = \{(2,1),(2,2),(2,3),(4,1)\} \neq \varphi$. (v) FALSE — $B' = A$, so $A \cap B' = A \neq \varphi$. (vi) FALSE — $A' \cap C = B \cap C \neq \varphi$.

Exercise 14.2 — Exercise 14.2

Q1 Which of the following can not be valid assignment of probabilities for outcomes of sample space S = {ω_1, ω_2, ω_3, ω_4, ω_5, ω_6, ω_7}?

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(b)	1/7	1/7	1/7	1/7	1/7	1/7	1/7
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(d)	–0.1	0.2	0.3	0.4	–0.2	0.1	0.3
(e)	1/14	2/14	3/14	4/14	5/14	6/14	15/14

SOLUTION

For a valid probability assignment, two conditions must hold:
Condition I: Each P(ω) must satisfy $0 \le P(\omega) \le 1$.
Condition II: The sum of all probabilities must equal 1.

(a) Each value is between 0 and 1. Sum = 0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 = 1.00. Both conditions satisfied. Assignment (a) is VALID.

(b) Each value is 1/7, which is between 0 and 1. Sum = 7 × (1/7) = 1. Both conditions satisfied. Assignment (b) is VALID.

(c) Each value is non-negative. Sum = 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7 = 2.8 ≠ 1. Condition II fails. Assignment (c) is NOT VALID.

(d) $P(\omega_1) = -0.1 < 0$ and $P(\omega_5) = -0.2 < 0$. Condition I fails. Assignment (d) is NOT VALID.

(e) $P(\omega_7) = 15/14 > 1$. Condition I fails. Assignment (e) is NOT VALID.
(Also, sum = (1+2+3+4+5+6+15)/14 = 36/14 ≠ 1, so Condition II also fails.)

Answer: Assignments (a) and (b) are valid. Assignments (c), (d), and (e) are not valid — (c) because the sum exceeds 1, (d) because two probabilities are negative, (e) because $P(\omega_7) = 15/14 > 1$.

Q2 A coin is tossed twice, what is the probability that atleast one tail occurs?

SOLUTION

The sample space for tossing a coin twice is S = {HH, HT, TH, TT}. All 4 outcomes are equally likely.

Event A: at least one tail = {HT, TH, TT}. Number of favourable outcomes = 3.

P(A) = 3/4.

Answer: P(at least one tail) = 3/4.

Q3 A die is thrown, find the probability of following events:
(i) A prime number will appear,
(ii) A number greater than or equal to 3 will appear,
(iii) A number less than or equal to one will appear,
(iv) A number more than 6 will appear,
(v) A number less than 6 will appear.

SOLUTION

S = {1, 2, 3, 4, 5, 6}. Total outcomes = 6.

(i) Prime numbers in S: {2, 3, 5}. Favourable outcomes = 3.
P(prime) = 3/6 = 1/2.

(ii) Numbers ≥ 3: {3, 4, 5, 6}. Favourable outcomes = 4.
P(≥ 3) = 4/6 = 2/3.

(iii) Numbers ≤ 1: {1}. Favourable outcomes = 1.
P(≤ 1) = 1/6.

(iv) Numbers > 6: none (a die shows 1 to 6). Favourable outcomes = 0.
P(> 6) = 0.

(v) Numbers < 6: {1, 2, 3, 4, 5}. Favourable outcomes = 5.
P(< 6) = 5/6.

Answer: (i) 1/2 (ii) 2/3 (iii) 1/6 (iv) 0 (v) 5/6.

Q4 A card is selected from a pack of 52 cards.
(a) How many points are there in the sample space?
(b) Calculate the probability that the card is an ace of spades.
(c) Calculate the probability that the card is (i) an ace (ii) black card.

SOLUTION

(a) A standard pack has 52 cards. When one card is drawn, each card is an equally likely outcome.
The sample space has 52 points.

(b) There is exactly 1 ace of spades in the deck.
P(ace of spades) = 1/52.

(c)(i) There are 4 aces (one of each suit) in the deck.
P(ace) = 4/52 = 1/13.

(c)(ii) There are 26 black cards (13 clubs + 13 spades).
P(black card) = 26/52 = 1/2.

Answer: (a) 52. (b) 1/52. (c)(i) 1/13. (c)(ii) 1/2.

Q5 A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12.

SOLUTION

The coin shows either 1 or 6. The die shows 1, 2, 3, 4, 5, or 6.
Total number of equally likely outcomes = 2 × 6 = 12.
Sample space: {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}, where first entry is the coin face.

(i) Sum = 3: We need coin + die = 3.
If coin = 1: die = 2, giving (1, 2). ✓
If coin = 6: die = –3, not possible.
Favourable outcomes = 1. P(sum = 3) = 1/12.

(ii) Sum = 12: We need coin + die = 12.
If coin = 6: die = 6, giving (6, 6). ✓
If coin = 1: die = 11, not possible.
Favourable outcomes = 1. P(sum = 12) = 1/12.

Answer: (i) P(sum = 3) = 1/12. (ii) P(sum = 12) = 1/12.

Q6 There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

SOLUTION

Total council members = 4 + 6 = 10.
Number of women = 6.

P(woman selected) = 6/10 = 3/5.

Answer: P(a woman is selected) = 3/5.

Q7 A fair coin is tossed four times, and a person wins Re 1 for each head and loses Rs 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

SOLUTION

When a fair coin is tossed 4 times, the sample space has $2^4 = 16$ equally likely outcomes.

If a person gets h heads and t tails (h + t = 4), the net amount = h × 1 – t × 1.50 = h – (4 – h) × 1.50 = h – 6 + 1.5h = 2.5h – 6.

The possible values of h are 0, 1, 2, 3, 4:

h = 0 (0 heads, 4 tails): Amount = 0 – 4(1.50) = –6.00. Number of outcomes = C(4,0) = 1. P = 1/16.

h = 1 (1 head, 3 tails): Amount = 1 – 3(1.50) = 1 – 4.50 = –3.50. Number of outcomes = C(4,1) = 4. P = 4/16 = 1/4.

h = 2 (2 heads, 2 tails): Amount = 2 – 2(1.50) = 2 – 3 = –1.00. Number of outcomes = C(4,2) = 6. P = 6/16 = 3/8.

h = 3 (3 heads, 1 tail): Amount = 3 – 1(1.50) = 3 – 1.50 = +1.50. Number of outcomes = C(4,3) = 4. P = 4/16 = 1/4.

h = 4 (4 heads, 0 tails): Amount = 4 – 0 = +4.00. Number of outcomes = C(4,4) = 1. P = 1/16.

There are 5 different amounts possible.

Answer: 5 different amounts: –Rs 6 (P = 1/16), –Rs 3.50 (P = 1/4), –Rs 1 (P = 3/8), +Re 1.50 (P = 1/4), +Rs 4 (P = 1/16).

Q8 Three coins are tossed once. Find the probability of getting
(i) 3 heads (ii) 2 heads (iii) atleast 2 heads (iv) atmost 2 heads (v) no head (vi) 3 tails (vii) exactly two tails (viii) no tail (ix) atmost two tails.

SOLUTION

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}. Total outcomes = 8, all equally likely.

(i) 3 heads: {HHH}. Favourable = 1. P = 1/8.

(ii) 2 heads: {HHT, HTH, THH}. Favourable = 3. P = 3/8.

(iii) At least 2 heads: {HHH, HHT, HTH, THH}. Favourable = 4. P = 4/8 = 1/2.

(iv) At most 2 heads (i.e., 0, 1, or 2 heads): S – {HHH} = {HHT, HTH, THH, HTT, THT, TTH, TTT}. Favourable = 7. P = 7/8.

(v) No head (0 heads = all tails): {TTT}. Favourable = 1. P = 1/8.

(vi) 3 tails: {TTT}. Favourable = 1. P = 1/8.

(vii) Exactly two tails: {HTT, THT, TTH}. Favourable = 3. P = 3/8.

(viii) No tail (0 tails = all heads): {HHH}. Favourable = 1. P = 1/8.

(ix) At most 2 tails (0, 1, or 2 tails): S – {TTT} = {HHH, HHT, HTH, THH, HTT, THT, TTH}. Favourable = 7. P = 7/8.

Answer: (i) 1/8 (ii) 3/8 (iii) 1/2 (iv) 7/8 (v) 1/8 (vi) 1/8 (vii) 3/8 (viii) 1/8 (ix) 7/8.

Q9 If 2/11 is the probability of an event, what is the probability of the event 'not A'.

SOLUTION

P(A) = 2/11.

P(not A) = P(A′) = 1 – P(A) = 1 – 2/11 = 9/11.

Answer: P(not A) = 9/11.

Q10 A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is (i) a vowel (ii) a consonant.

SOLUTION

The word ASSASSINATION has 13 letters: A, S, S, A, S, S, I, N, A, T, I, O, N.
Total letters = 13.

Vowels (A, E, I, O, U) in ASSASSINATION: A appears 3 times, I appears 2 times, O appears 1 time.
Total vowels = 3 + 2 + 1 = 6.

Consonants: S appears 4 times, N appears 2 times, T appears 1 time.
Total consonants = 4 + 2 + 1 = 7.

(i) P(vowel) = 6/13.

(ii) P(consonant) = 7/13.

Answer: (i) P(vowel) = 6/13. (ii) P(consonant) = 7/13.

Q11 In a lottery, a person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? [Hint: order of the numbers is not important.]

SOLUTION

The total number of ways to choose 6 different numbers from 1 to 20 (order not important):
n(S) = C(20, 6) = 20! / (6! × 14!) = (20 × 19 × 18 × 17 × 16 × 15) / (6 × 5 × 4 × 3 × 2 × 1) = 38760.

There is exactly 1 winning combination (the six numbers fixed by the lottery committee).

P(winning) = 1/38760.

Answer: P(winning the prize) = 1/38760.

Q12 Check whether the following probabilities P(A) and P(B) are consistently defined:
(i) P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.6
(ii) P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.8

SOLUTION

(i) P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.6.
Since A ∩ B ⊆ A, we must have P(A ∩ B) ≤ P(A).
But P(A ∩ B) = 0.6 > P(A) = 0.5. This is a contradiction.
Therefore, the given probabilities are NOT consistently defined.

(ii) P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.8.
Using the addition formula: P(A ∩ B) = P(A) + P(B) – P(A ∪ B) = 0.5 + 0.4 – 0.8 = 0.1.
Check: 0 ≤ P(A ∩ B) = 0.1 ≤ P(A) = 0.5 and P(A ∩ B) = 0.1 ≤ P(B) = 0.4, and P(A ∪ B) = 0.8 ≤ 1.
All conditions are satisfied. The given probabilities are consistently defined.

Answer: (i) NOT consistently defined — P(A ∩ B) = 0.6 > P(A) = 0.5 is impossible. (ii) Consistently defined — P(A ∩ B) = 0.1 ≥ 0 and all values are valid.

Q13 Fill in the blanks in following table:

	P(A)	P(B)	P(A ∩ B)	P(A ∪ B)
(i)	1/3	1/5	1/15	...
(ii)	0.35	...	0.25	0.6
(iii)	0.5	0.35	...	0.7

SOLUTION

Using the formula P(A ∪ B) = P(A) + P(B) – P(A ∩ B):

(i) P(A) = 1/3, P(B) = 1/5, P(A ∩ B) = 1/15.
P(A ∪ B) = 1/3 + 1/5 – 1/15 = 5/15 + 3/15 – 1/15 = 7/15.

(ii) P(A) = 0.35, P(A ∩ B) = 0.25, P(A ∪ B) = 0.6.
Rearranging: P(B) = P(A ∪ B) – P(A) + P(A ∩ B) = 0.6 – 0.35 + 0.25 = 0.5.

(iii) P(A) = 0.5, P(B) = 0.35, P(A ∪ B) = 0.7.
P(A ∩ B) = P(A) + P(B) – P(A ∪ B) = 0.5 + 0.35 – 0.7 = 0.15.

Answer: (i) P(A ∪ B) = 7/15. (ii) P(B) = 0.5. (iii) P(A ∩ B) = 0.15.

Q14 Given P(A) = 3/5 and P(B) = 1/5. Find P(A or B), if A and B are mutually exclusive events.

SOLUTION

Since A and B are mutually exclusive, P(A ∩ B) = 0.

P(A or B) = P(A ∪ B) = P(A) + P(B) = 3/5 + 1/5 = 4/5.

Answer: P(A or B) = 4/5.

Q15 If E and F are events such that P(E) = 1/4, P(F) = 1/2 and P(E and F) = 1/8, find
(i) P(E or F), (ii) P(not E and not F).

SOLUTION

P(E) = 1/4, P(F) = 1/2, P(E ∩ F) = 1/8.

(i) P(E or F) = P(E ∪ F) = P(E) + P(F) – P(E ∩ F) = 1/4 + 1/2 – 1/8 = 2/8 + 4/8 – 1/8 = 5/8.

(ii) P(not E and not F) = P(E′ ∩ F′) = P((E ∪ F)′) [by De Morgan's Law]
= 1 – P(E ∪ F) = 1 – 5/8 = 3/8.

Answer: (i) P(E or F) = 5/8. (ii) P(not E and not F) = 3/8.

Q16 Events E and F are such that P(not E or not F) = 0.25. State whether E and F are mutually exclusive.

SOLUTION

P(not E or not F) = P(E′ ∪ F′).

By De Morgan's Law: E′ ∪ F′ = (E ∩ F)′.

So P(E′ ∪ F′) = P((E ∩ F)′) = 1 – P(E ∩ F).

Given: 1 – P(E ∩ F) = 0.25, so P(E ∩ F) = 0.75.

Since P(E ∩ F) = 0.75 ≠ 0, the events E and F are NOT mutually exclusive.

Answer: E and F are not mutually exclusive. [P(E ∩ F) = 0.75 ≠ 0.]

Q17 A and B are events such that P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16. Determine (i) P(not A), (ii) P(not B) and (iii) P(A or B).

SOLUTION

(i) P(not A) = 1 – P(A) = 1 – 0.42 = 0.58.

(ii) P(not B) = 1 – P(B) = 1 – 0.48 = 0.52.

(iii) P(A or B) = P(A ∪ B) = P(A) + P(B) – P(A ∩ B) = 0.42 + 0.48 – 0.16 = 0.74.

Answer: (i) 0.58 (ii) 0.52 (iii) 0.74.

Q18 In Class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

SOLUTION

Let M = event of studying Mathematics, B = event of studying Biology.
P(M) = 40/100 = 0.4, P(B) = 30/100 = 0.3, P(M ∩ B) = 10/100 = 0.1.

P(M or B) = P(M ∪ B) = P(M) + P(B) – P(M ∩ B) = 0.4 + 0.3 – 0.1 = 0.6.

Answer: P(studying Mathematics or Biology) = 0.6.

Q19 In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing atleast one of them is 0.95. What is the probability of passing both?

SOLUTION

Let E = event of passing the first exam, F = event of passing the second exam.
P(E) = 0.8, P(F) = 0.7, P(E ∪ F) = 0.95.

Using the addition formula:
P(E ∪ F) = P(E) + P(F) – P(E ∩ F)
0.95 = 0.8 + 0.7 – P(E ∩ F)
P(E ∩ F) = 1.5 – 0.95 = 0.55.

Answer: P(passing both examinations) = 0.55.

Q20 The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

SOLUTION

Let E = event of passing English, H = event of passing Hindi.
P(E ∩ H) = 0.5 (pass both), P(E′ ∩ H′) = 0.1 (pass neither), P(E) = 0.75.

P(E′ ∩ H′) = P((E ∪ H)′) = 1 – P(E ∪ H) = 0.1.
So P(E ∪ H) = 0.9.

Using the addition formula:
P(E ∪ H) = P(E) + P(H) – P(E ∩ H)
0.9 = 0.75 + P(H) – 0.5
P(H) = 0.9 – 0.75 + 0.5 = 0.65.

Answer: P(passing the Hindi examination) = 0.65.

Q21 In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that
(i) The student opted for NCC or NSS.
(ii) The student has opted neither NCC nor NSS.
(iii) The student has opted NSS but not NCC.

SOLUTION

Total students = 60. Let N = NCC, S = NSS.
n(N) = 30, n(S) = 32, n(N ∩ S) = 24.

n(N ∪ S) = n(N) + n(S) – n(N ∩ S) = 30 + 32 – 24 = 38.

(i) P(N or S) = P(N ∪ S) = 38/60 = 19/30.

(ii) P(neither NCC nor NSS) = P((N ∪ S)′) = 1 – 19/30 = 11/30.

(iii) Students in NSS but not NCC = n(S) – n(N ∩ S) = 32 – 24 = 8.
P(NSS but not NCC) = 8/60 = 2/15.

Answer: (i) 19/30 (ii) 11/30 (iii) 2/15.

Misc — Miscellaneous Exercise

Q1 A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that
(i) all will be blue?
(ii) atleast one will be green?

SOLUTION

Total marbles = 10 + 20 + 30 = 60.
Total ways to choose 5 marbles from 60: n(S) = C(60, 5) = 5461512.

(i) All 5 blue: choose 5 from 20 blue marbles.
Favourable outcomes = C(20, 5) = 15504.
P(all blue) = 15504/5461512 = 34/11977.

(ii) At least one green: use complement.
P(at least one green) = 1 – P(no green).
P(no green) = choosing 5 from the 30 non-green (red + blue) marbles.
= C(30, 5) / C(60, 5) = 142506 / 5461512 = 117/4484.

P(at least one green) = 1 – 117/4484 = 4367/4484.

Answer: (i) 34/11977. (ii) 4367/4484.

Q2 4 cards are drawn from a well-shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

SOLUTION

Total ways to draw 4 cards from 52: C(52, 4) = 270725.

Favourable outcomes: 3 diamonds from 13 diamonds AND 1 spade from 13 spades.
= C(13, 3) × C(13, 1) = 286 × 13 = 3718.

P(3 diamonds and 1 spade) = 3718/270725 = 286/20825.

Answer: P(3 diamonds and 1 spade) = 286/20825.

Q3 A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine
(i) P(2) (ii) P(1 or 3) (iii) P(not 3).

SOLUTION

The die has 6 faces: two show '1', three show '2', one shows '3'. All faces are equally likely.

(i) P(2) = number of faces showing 2 / total faces = 3/6 = 1/2.

(ii) P(1 or 3) = (faces showing 1 + faces showing 3) / 6 = (2 + 1)/6 = 3/6 = 1/2.

(iii) P(not 3) = 1 – P(3) = 1 – 1/6 = 5/6.

Answer: (i) P(2) = 1/2. (ii) P(1 or 3) = 1/2. (iii) P(not 3) = 5/6.

Q4 In a certain lottery 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy (a) one ticket (b) two tickets (c) 10 tickets.

SOLUTION

Total tickets = 10,000. Prize tickets = 10. Non-prize tickets = 9990.

(a) One ticket: P(no prize) = 9990/10000 = 999/1000.

(b) Two tickets: The number of ways to choose 2 tickets from 10000 is C(10000, 2).
Favourable (both non-prize): C(9990, 2).
P(no prize) = C(9990, 2) / C(10000, 2) = (9990 × 9989) / (10000 × 9999) = 99780110/99990000.
Simplified: P(no prize) = 1108779/1111000.

(c) Ten tickets: P(no prize) = C(9990, 10) / C(10000, 10).
= (9990 × 9989 × 9988 × ... × 9981) / (10000 × 9999 × 9998 × ... × 9991).

Answer: (a) 999/1000. (b) C(9990,2)/C(10000,2) = 1108779/1111000. (c) C(9990,10)/C(10000,10).

Q5 Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that
(a) you both enter the same section?
(b) you both enter the different sections?

SOLUTION

Total ways to select 2 students (you and your friend) from 100: C(100, 2) = 4950.

(a) Both in the same section:
Both in section A (40 students): C(40, 2) = 780 ways.
Both in section B (60 students): C(60, 2) = 1770 ways.
Favourable = 780 + 1770 = 2550.
P(same section) = 2550/4950 = 17/33.

(b) Both in different sections:
P(different sections) = 1 – P(same section) = 1 – 17/33 = 16/33.

Answer: (a) 17/33. (b) 16/33.

Q6 Three letters are dictated to three persons and an envelope is addressed to each of them. The letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

SOLUTION

Total ways to insert 3 letters into 3 envelopes (one each): 3! = 6.

Let A be the event that letter i goes into its correct envelope (i = 1, 2, 3).

Using inclusion-exclusion:
P(at least one correct) = $P(A_1 \cup A_2 \cup A_3)$
= $P(A_1) + P(A_2) + P(A_3) – P(A_1 \cap A_2) – P(A_1 \cap A_3) – P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$.

P(A) = 2!/3! = 2/6 = 1/3 for each i. (Fix letter i; the other 2 can go in 2! ways.)
P(A ∩ A) = 1!/3! = 1/6 for each pair. (Fix letters i and j; the remaining 1 is forced.)
$P(A_1 \cap A_2 \cap A_3)$ = 1/6. (All 3 fixed — only 1 arrangement.)

P(at least one correct) = 3(1/3) – 3(1/6) + 1/6 = 1 – 1/2 + 1/6 = 6/6 – 3/6 + 1/6 = 4/6 = 2/3.

Alternatively, enumerate: out of 6 arrangements, those with at least 1 correct are (1,2,3), (1,3,2), (2,1,3), (3,2,1) — 4 arrangements.

P = 4/6 = 2/3.

Answer: P(at least one letter in its correct envelope) = 2/3.

Q7 A and B are two events such that P(A) = 0.54, P(B) = 0.69 and P(A ∩ B) = 0.35. Find
(i) P(A ∪ B) (ii) P(A′ ∩ B′) (iii) P(A ∩ B′) (iv) P(B ∩ A′).

SOLUTION

(i) P(A ∪ B) = P(A) + P(B) – P(A ∩ B) = 0.54 + 0.69 – 0.35 = 0.88.

(ii) P(A′ ∩ B′) = P((A ∪ B)′) [De Morgan's Law] = 1 – P(A ∪ B) = 1 – 0.88 = 0.12.

(iii) P(A ∩ B′) = P(A) – P(A ∩ B) = 0.54 – 0.35 = 0.19.
(Here A = (A ∩ B) ∪ (A ∩ B′) where the two parts are disjoint.)

(iv) P(B ∩ A′) = P(B) – P(A ∩ B) = 0.69 – 0.35 = 0.34.

Answer: (i) 0.88 (ii) 0.12 (iii) 0.19 (iv) 0.34.

Q8 From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:
1. Harish — Male — 30 years
2. Rohan — Male — 33 years
3. Sheetal — Female — 46 years
4. Alis — Female — 28 years
5. Salim — Male — 41 years
A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

SOLUTION

Total persons = 5. Each person is equally likely to be selected.

Let M = event that spokesperson is male: Harish, Rohan, Salim → n(M) = 3.
Let O = event that spokesperson is over 35 years: Sheetal (46), Salim (41) → n(O) = 2.
M ∩ O (male and over 35): Salim → n(M ∩ O) = 1.

P(M ∪ O) = P(M) + P(O) – P(M ∩ O) = 3/5 + 2/5 – 1/5 = 4/5.

Answer: P(spokesperson is male or over 35) = 4/5.

Q9 If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when,
(i) the digits are repeated?
(ii) the repetition of digits is not allowed?

SOLUTION

Available digits: {0, 1, 3, 5, 7}. We form 4-digit numbers > 5000.

(i) Digits can be repeated:
For a 4-digit number > 5000, the first (thousands) digit must be 5 or 7 (2 choices).
Each of the remaining 3 positions: 5 choices each.
Total 4-digit numbers > 5000 = 2 × 5 × 5 × 5 = 250.

For divisibility by 5, the units digit must be 0 or 5.
First digit: 5 or 7 (2 choices), middle two digits: 5 choices each, last digit: 0 or 5 (2 choices).
Favourable = 2 × 5 × 5 × 2 = 100.

P(divisible by 5) = 100/250 = 2/5.

(ii) Digits not repeated (each digit used at most once):
For a 4-digit number > 5000, first digit = 5 or 7.

Case 1: First digit = 5. Remaining digits available: {0, 1, 3, 7}.
For divisibility by 5, last digit must be 0 or 5. Since 5 is used, last digit = 0 only.
Middle 2 digits: choose 2 from {1, 3, 7} in order = P(3, 2) = 6.
Favourable with first = 5: 6 numbers.
Total with first = 5: remaining 3 positions from {0,1,3,7} = 4 × 3 × 2 = 24.

Case 2: First digit = 7. Remaining digits available: {0, 1, 3, 5}.
For divisibility by 5, last digit = 0 or 5.
Last = 0: middle 2 from {1, 3, 5} = P(3, 2) = 6.
Last = 5: middle 2 from {0, 1, 3} = P(3, 2) = 6.
Favourable with first = 7: 6 + 6 = 12 numbers.
Total with first = 7: 4 × 3 × 2 = 24.

Total 4-digit numbers > 5000, no repetition = 24 + 24 = 48.
Total favourable = 6 + 12 = 18.

P(divisible by 5) = 18/48 = 3/8.

Answer: (i) 2/5. (ii) 3/8.

Q10 The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

SOLUTION

Each wheel shows digits 0 to 9. A sequence of 4 digits is chosen with no repeats.

Total number of 4-digit sequences with no repetition from {0,1,...,9}:
= 10 × 9 × 8 × 7 = 5040.

Only 1 sequence opens the lock.

P(getting the right sequence) = 1/5040.

Answer: P(getting the right sequence) = 1/5040.