Chapter 1: Units and Measurement

Exercises — Exercises

Q1 Fill in the blanks

(a) The volume of a cube of side 1 cm is equal to ..... m^3

(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to ... $(mm)^2$

(c) A vehicle moving with a speed of 18 km h^{-1} covers .... m in 1 s

(d) The relative density of lead is 11.3. Its density is .... g cm^{-3} or .... kg m^{-3}.

SOLUTION

(a) Side of cube = 1 cm = 1×10^{-2} m

Volume = $(1 \times 10^{-2})^3 = 1 \times 10^{-6}$ m^3

(b) Radius r = 2.0 cm = 20.0 mm, Height h = 10.0 cm = 100.0 mm

Surface area of solid cylinder = $2\pi r(r + h)$

$= 2\pi \times 20.0 \times (20.0 + 100.0)$

$= 2\pi \times 20.0 \times 120.0$

$= 2 \times 3.14159 \times 2400$

$= 15079.6$ mm^2

Rounded to 2 significant figures (r = 2.0 cm has 2 sf): 1.5×10^4 mm^2

(c) Speed = 18 km h^{-1} = 18 × (1000 m) / (3600 s) = 5.0 m s^{-1}

Distance covered in 1 s = 5.0 × 1 = 5.0 m

(d) Relative density (specific gravity) = density of substance / density of water

Density of water = 1.0 g cm^{-3} = 1000 kg m^{-3}

Density of lead = 11.3 × 1.0 g cm^{-3} = 11.3 g cm^{-3}

$= 11.3 \times 1000$ kg $m^{-3} = 1.13 \times 10^4$ kg m^{-3}

Answer: (a) 1×10^{-6} m^3 (b) 1.5×10^4 mm^2 (c) 5.0 m (d) 11.3 g cm^{-3} or 1.13×10^4 kg m^{-3}

Q2 Fill in the blanks by suitable conversion of units

(a) 1 kg $m^2 s^{-2}$ = ... g $cm^2 s^{-2}$

(b) 1 m = ... ly

(c) 3.0 m s^{-2} = ... km h^{-2}

(d) G = 6.67×10^{-11} N m^2 $(kg)^{-2}$ = ... $(cm)^3$ s^{-2} g^{-1}.

SOLUTION

(a) 1 kg $m^2 s^{-2}$

1 kg = 10^3 g, 1 m = 10^2 cm => 1 m^2 = 10^4 cm^2

1 kg m^2 s^{-2} = 10^3 g × 10^4 cm^2 × s^{-2} = 10^7 g cm^2 s^{-2}

(b) Speed of light c = 3×10^8 m s^{-1}

1 year = $365.25 \times 24 \times 3600 = 3.156 \times 10^7$ s

1 light year (ly) = $3 \times 10^8 \times 3.156 \times 10^7 = 9.47 \times 10^{15}$ m

1 m = $1 / (9.47 \times 10^{15})$ ly = 1.057×10^{-16} ly ≈ 1.1×10^{-16} ly

(c) 3.0 m s^{-2}

1 m = 10^{-3} km, 1 s = (1/3600) h => 1 s^2 = $(1/3600)^2$ h^2

1 m s^{-2} = 10^{-3} km / $(1/3600)^2$ h^2 = $10^{-3} \times 3600^2$ km h^{-2} = 1.296×10^4 km h^{-2}

3.0 m s^{-2} = $3.0 \times 1.296 \times 10^4 = 3.888 \times 10^4$ km h^{-2} ≈ 3.9×10^4 km h^{-2}

(d) G = 6.67×10^{-11} N m^2 kg^{-2}

Dimensions of G: $[M^{-1} L^3 T^{-2}]$ (since N = kg m s^{-2} => N m^2 kg^{-2} = m^3 s^{-2} kg^{-1})

1 m^3 = $(10^2 \text{ cm})^3 = 10^6$ cm^3, 1 kg^{-1} = $(10^3 \text{ g})^{-1} = 10^{-3}$ g^{-1}

G = $6.67 \times 10^{-11} \times 10^6$ $cm^3 \times 10^{-3}$ g^{-1} s^{-2}

= $6.67 \times 10^{-11} \times 10^3$ cm^3 s^{-2} g^{-1}

= 6.67×10^{-8} cm^3 s^{-2} g^{-1}

Answer: (a) 10^7 g cm^2 s^{-2} (b) 1.1×10^{-16} ly (c) 3.9×10^4 km h^{-2} (d) 6.67×10^{-8} cm^3 s^{-2} g^{-1}

Q3 A calorie is a unit of heat (energy in transit) and it equals about 4.2 J where 1 J = 1 kg m^2 s^{-2}. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude 4.2 $\alpha^{-1} \beta^{-2} \gamma^2$ in terms of the new units.

SOLUTION

Energy has dimensions $[M L^2 T^{-2}]$.

In the new system: unit of mass = α kg, unit of length = β m, unit of time = γ s.

The new unit of energy = (α kg)(β m)^2(γ s)$^{-2}$ = $\alpha \beta^2 \gamma^{-2}$ joules

Number of new energy units in 4.2 J:

$n = 4.2\ J / (\alpha\ \beta^2\ \gamma^{-2}\ J \text{ per unit})$

$= 4.2 / (\alpha\ \beta^2\ \gamma^{-2})$

$= 4.2 \times \alpha^{-1} \times \beta^{-2} \times \gamma^2$

Therefore, 1 calorie = $4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2$ in the new system of units. (Proved)

Answer: 1 calorie = $4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2$ new units (derived from dimensional analysis of $[M\ L^2\ T^{-2}]$)

Q4 Explain this statement clearly: 'To call a dimensional quantity large or small is meaningless without specifying a standard for comparison'. In view of this, reframe the following statements wherever necessary:
(a) atoms are very small objects
(b) a jet plane moves with great speed
(c) the mass of Jupiter is very large
(d) the air inside this room contains a large number of molecules
(e) a proton is much more massive than an electron
(f) the speed of sound is much smaller than the speed of light.

SOLUTION

A dimensional quantity (one with units) has a numerical value that depends entirely on the chosen unit. For example, the length 1 m = 100 cm = 1000 mm — the number changes with the unit. Saying a quantity is 'large' or 'small' is only meaningful relative to some reference standard of the same kind.

(a) Reframed: Atoms are very small objects compared to everyday objects around us (e.g., a grain of sand or a human hair).

(b) Reframed: A jet plane moves with great speed compared to a car or ordinary vehicle (but much slower than the speed of light).

(c) Reframed: The mass of Jupiter is very large compared to the mass of the Earth (Jupiter's mass ≈ 318 times Earth's mass).

(d) Reframed: The air inside this room contains a large number of molecules compared to the number of molecules in, say, one mole of gas ($\sim 6 \times 10^{23}$).
Note: This statement is already implicitly comparative — large relative to any humanly countable number. It can stand as is.

(e) This statement is already meaningful: it is a comparison between two

quantities of the same kind (mass). A proton is about 1836 times more massive than an electron. No reframing needed.

(f) This statement is already meaningful: it compares two speeds. The speed of sound (~340 m/s) is much smaller than the speed of light (3×10^8 m/s). No reframing needed.

Answer: Statements (e) and (f) are already correctly framed as comparisons. Statements (a), (b), (c), (d) need a reference standard to be meaningful.

Q5 A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?

SOLUTION

In the new unit system, c = 1 new-length-unit per new-time-unit.
Let the new unit of time be 1 s (standard second).
Then c = 1 new-length-unit / s, meaning 1 new length unit = distance light travels in 1 s.

Time taken by light to travel from Sun to Earth:
t = 8 min 20 s = 8 × 60 + 20 = 480 + 20 = 500 s

Distance = speed × time = c × t = 1 × 500 = 500 new units of length

Answer: 500 new units of length (since light covers exactly 1 new unit per second and takes 500 s)

Q6 Which of the following is the most precise device for measuring length:
(a) a vernier callipers with 20 divisions on the sliding scale
(b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale
(c) an optical instrument that can measure length to within a wavelength of light?

SOLUTION

The precision of a measuring instrument is indicated by its least count (LC) — the smallest length it can measure.

(a) Vernier callipers with 20 divisions on the sliding scale:
LC = 1 MSD / 20 = 1 mm / 20 = 0.05 mm

(b) Screw gauge with pitch 1 mm and 100 divisions on circular scale:

LC = pitch / number of divisions = 1 mm / 100 = 0.01 mm

(c) Optical instrument measuring to within a wavelength of light:
Wavelength of visible light $\lambda \approx 400–700$ nm ≈ 600 nm $= 6 \times 10^{-4}$ mm $= 0.0006$ mm

Comparing least counts:
(a) 0.05 mm > (b) 0.01 mm > (c) 0.0006 mm
The smaller the LC, the more precise the instrument.

Answer: (c) The optical instrument is the most precise, with a least count of about 6×10^{-4} mm (one wavelength of light), far smaller than the vernier (0.05 mm) or screw gauge (0.01 mm).

Q7 A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate of the thickness of hair?

SOLUTION

The microscope magnifies the actual width by a factor of 100.
Width of hair as seen under microscope = 3.5 mm

Actual thickness of hair = observed width / magnification
$= 3.5$ mm $/ 100$
$= 0.035$ mm
$= 3.5 \times 10^{-5}$ m

Answer: Thickness of hair = 0.035 mm $= 3.5 \times 10^{-5}$ m

Q8 Answer the following:
(a) You are given a thread and a metre scale. How will you estimate the diameter of the thread?
(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale?
(c) The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only?

SOLUTION

(a) Wind the thread closely over the metre scale (ruler) for a known number of turns, say N turns, so that the turns are tightly packed with no gaps and no overlaps. Measure the total length L occupied by the N turns using the scale. The diameter of the thread is:
$d = L / N$
This method amplifies the small diameter N-fold, reducing the relative error in measurement.

(b) The least count of a screw gauge = pitch / number of circular divisions.
Increasing the number of divisions decreases the least count mathematically. However, there is a practical limit: manufacturing tolerances and mechanical backlash mean that the screw cannot be read to an accuracy finer than the precision with which the pitch itself is machined. Beyond a certain point, more divisions give a false sense of accuracy without improving actual precision. So no, accuracy cannot be increased arbitrarily by increasing divisions alone.

(c) In any measurement, random errors arise from unpredictable variations in the measurement process (vibration, line-of-sight error, slight variations in the position of the instrument, etc.). These random errors tend to cancel when a large number of readings are averaged, because they are equally likely to be positive or negative.
With N measurements, the random error in the mean is proportional to $1/\sqrt{N}$. For N = 100, the error is reduced to $1/\sqrt{100} = 1/10$ of the error in a single measurement, while for N = 5 it is only $1/\sqrt{5} \approx 1/2.2$. Thus, 100 measurements yield a far more reliable (precise) estimate of the true mean diameter than 5 measurements.

Answer: (a) Wind N turns tightly, measure length L; diameter d = L/N. (b) No — manufacturing tolerances set a practical lower limit; arbitrary increase of divisions does not help. (c) Random errors reduce as 1/√N; 100 measurements reduce random error to 1/10, making the mean far more reliable.

Q9 The photograph of a house occupies an area of 1.75 cm² on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m². What is the linear magnification of the projector-screen arrangement?

SOLUTION

Given: Area on slide = 1.75 cm² = 1.75 × 10^{-4} m²
Area on screen = 1.55 m²

Area magnification = Area on screen / Area on slide
= 1.55 / (1.75 × 10^{-4})
= 8857

Linear magnification = √(Area magnification)
= √8857
= 94.1

Answer: Linear magnification = 94.1

Q10 State the number of significant figures in the following:
(a) 0.007 m²
(b) 2.64 × 10^{24} kg
(c) 0.2370 g cm^{-3}
(d) 6.320 J
(e) 6.032 N m^{-2}
(f) 0.0006032 m²

SOLUTION

(a) 0.007 m²:
The leading zeros (after the decimal, before the 7) are not significant. Only the digit 7 is significant.
Significant figures = 1

(b) 2.64 × 10^{24} kg:
In scientific notation, all digits in the mantissa are significant. Digits: 2, 6, 4.
Significant figures = 3

(c) 0.2370 g cm^{-3}:
Leading zero is not significant. Digits 2, 3, 7, 0 are significant (the trailing zero after decimal point is significant).
Significant figures = 4

(d) 6.320 J:
All digits including the trailing zero after decimal are significant. Digits: 6, 3, 2, 0.
Significant figures = 4

(e) 6.032 N m^{-2}:
All digits are significant; the zero between two non-zero digits is always significant. Digits: 6, 0, 3, 2.
Significant figures = 4

(f) 0.0006032 m^2:
Leading zeros (before 6) are not significant. Digits 6, 0, 3, 2 are significant (zero between non-zero digits is significant).
Significant figures = 4

Answer: (a) 1 (b) 3 (c) 4 (d) 4 (e) 4 (f) 4

Q11 The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

SOLUTION

Given: length l = 4.234 m (4 sf), breadth b = 1.005 m (4 sf), thickness t = 2.01 cm = 0.0201 m (3 sf)

Area of the sheet = l × b
= 4.234 × 1.005
= 4.255170 m^2
Both l and b have 4 significant figures, so area is rounded to 4 sf:
Area = 4.255 m^2

Volume = l × b × t
= 4.234 × 1.005 × 0.0201
= 0.085529 m^3
The thickness t has 3 significant figures (the limiting quantity), so volume is rounded to 3 sf:

Volume = 0.0855 m³

Answer: Area = 4.255 m², Volume = 0.0855 m³

Q12 The mass of a box measured by a grocer's balance is 2.30 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures?

SOLUTION

(a) Total mass:
Mass of box = 2.30 kg = 2300 g (precise to 10 g, i.e., 2 decimal places in kg)
Gold piece 1 = 20.15 g = 0.02015 kg
Gold piece 2 = 20.17 g = 0.02017 kg

Total = 2.30 + 0.02015 + 0.02017 = 2.34032 kg

For addition, retain the least number of decimal places. The box mass 2.30 kg is precise to 2 decimal places (0.01 kg level), while the gold pieces contribute 0.04032 kg (known to 5 decimal places).
Total rounded to 2 decimal places = 2.34 kg

(b) Difference in masses of the two gold pieces:
$\Delta m = 20.17 - 20.15 = 0.02$ g
Both have 2 decimal places in grams, so the difference also has 2 decimal places.
Difference = 0.02 g

Answer: (a) Total mass = 2.34 kg (b) Difference = 0.02 g

Q13 A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light, c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes:
$m = m_0 / (1 - v^2)^{(1/2)}$
Guess where to put the missing c.

SOLUTION

For the formula to be dimensionally correct, every term must have consistent dimensions.

The term $(1 - v^2)$ is inside a square root. For the expression to be dimensionless (so it can be subtracted from the dimensionless number 1), v^2 must also be dimensionless.

But $[v^2] = [L^2 T^{-2}]$, which is NOT dimensionless.

To make v^2 dimensionless, it must be divided by c^2 (which has the same dimensions as v^2):
$[v^2/c^2] = [L^2 T^{-2}] / [L^2 T^{-2}]$ = dimensionless ✓

Therefore, the correct relation is:
$m = m_0 / (1 - v^2/c^2)^{(1/2)}$

This is exactly the relativistic mass formula from Einstein's special theory of relativity.

Answer: $m = m_0 / (1 - v^2/c^2)^{(1/2)}$ — the c^2 belongs in the denominator inside the bracket, making (v^2/c^2) dimensionless.

Q14 The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å: 1 Å = 10^{-10} m. The size of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms?

SOLUTION

Given: size (radius) of hydrogen atom r = 0.5 Å = 0.5 × 10^{-10} m = 5 × 10^{-11} m
Avogadro number $N_A = 6.023 \times 10^{23}$ mol^{-1}

Volume of one hydrogen atom (sphere):
$V_{atom} = (4/3) \pi r^3$
$= (4/3) \times 3.14159 \times (5 \times 10^{-11})^3$
$= (4/3) \times 3.14159 \times 1.25 \times 10^{-31}$
$= 5.24 \times 10^{-31}$ m^3

Total atomic volume of one mole of hydrogen atoms:
$V = N_A \times V_{atom}$
$= 6.023 \times 10^{23} \times 5.24 \times 10^{-31}$
$= 3.154 \times 10^{-7}$ m^3
$\approx 3.2 \times 10^{-7}$ m^3

Answer: Total atomic volume of 1 mole of H atoms ≈ 3.2 × 10^{-7} m^3

Q15 One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large?

SOLUTION

Molar volume of ideal gas at STP:
$V_{molar} = 22.4\ L = 22.4 \times 10^{-3}\ m^3$

Size (diameter) of hydrogen molecule = 1 Å = 10^{-10} m
Radius r = 0.5 Å = 0.5×10^{-10} m = 5×10^{-11} m

Volume of one H_2 molecule:
$V_{molecule} = (4/3)\ \pi\ r^3 = (4/3) \times \pi \times (5 \times 10^{-11})^3 = 5.24 \times 10^{-31}\ m^3$

Atomic volume of one mole of hydrogen molecules:
$V_{atomic} = N_A \times V_{molecule} = 6.023 \times 10^{23} \times 5.24 \times 10^{-31} = 3.15 \times 10^{-7}\ m^3$

Ratio = V_{molar} / V_{atomic}
$= (22.4 \times 10^{-3}) / (3.15 \times 10^{-7})$
$= 7.1 \times 10^4$

Why is the ratio so large?
In the gaseous state at STP, molecules are far apart from each other — the average intermolecular distance is much larger than the molecular size itself. The actual volume occupied by the molecules is only a tiny fraction of the total volume of the gas. The ratio (~10^4) reflects the fact that most of the 'volume' of a gas is empty space between molecules. This is a fundamental difference between solids/liquids (where molecules are closely packed, ratio ≈ 1) and gases (where molecules are widely separated).

Answer: Ratio of molar volume to atomic volume ≈ 7.1×10^4. This large ratio reflects the fact that gas molecules are widely separated — most of the gas volume is empty space.

Q16 Explain this common observation clearly: If you look out of the window of a fast moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).

SOLUTION

This phenomenon is explained by the concept of parallax (or angular displacement per unit time as seen from the moving observer).

The angular speed ω at which an object appears to move, as seen from the train window, depends on its distance d from the observer and the train's speed v:
$\omega = v / d$ (approximately, for small angular changes)

For nearby objects (trees, houses): d is small => ω is large.
These objects subtend a rapidly changing angle to the observer and appear to whiz past in the opposite direction of the train's motion.

For distant objects (hills, mountains): d is very large => $\omega = v/d \approx 0$.
These objects appear nearly stationary because their angular position changes negligibly as the train moves.

For extremely distant objects (Moon, stars): d is astronomically large.
The angular displacement is essentially zero. Since the observer knows they are moving, the brain interprets these fixed-angle objects as moving along with the train.

In summary: the apparent angular speed of objects is inversely proportional to their distance from the observer. This is the principle of parallax.

Answer: Nearby objects have large angular speed ($\propto 1/d$) relative to the observer and appear to move rapidly backward; distant objects have negligible angular speed and appear stationary (or seem to follow the train). This is a consequence of the parallax effect.

Q17 The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: mass of the Sun = 2.0×10^{30} kg, radius of the Sun = 7.0×10^8 m.

SOLUTION

Initial guess: One might expect the Sun (a plasma) to have a density more like a gas than a solid or liquid. However, the enormous gravitational force acting inward compresses the solar material to high density. The correct expectation should account for this — the Sun could be denser than a normal gas.

Calculation of the Sun's density:
Mass of Sun M = 2.0×10^{30} kg
Radius of Sun R = 7.0×10^8 m

Volume of Sun (sphere):
$V = (4/3)\ \pi\ R^3$
$= (4/3) \times 3.14159 \times (7.0 \times 10^8)^3$
$= (4/3) \times 3.14159 \times 3.43 \times 10^{26}$
$= 1.437 \times 10^{27}\ m^3$

Density:
$\rho = M / V$
$= (2.0 \times 10^{30}) / (1.437 \times 10^{27})$
$= 1.39 \times 10^3\ kg\ m^{-3}$
$\approx 1.4 \times 10^3\ kg\ m^{-3}$

For comparison:
Density of gases (at STP): ~1 kg m^{-3}
Density of solids and liquids: ~10^3 kg m^{-3} (water = 1000 kg m^{-3}, iron = 7800 kg m^{-3})

The Sun's average density (~1400 kg m^{-3}) is in the range of solids and liquids — close to the density of water — despite being entirely in the plasma state. This is because the immense gravitational self-compression of the Sun's matter greatly exceeds the thermal tendency to expand.

Answer: Density of Sun ≈ 1.4 × 10^3 kg m^{-3} — in the range of solids and liquids (not gases). Gravity compresses the solar plasma to this density despite the extreme temperatures.