Chapter 2: Motion in a Straight Line

Exercises — Exercises

Q1 In which of the following examples of motion, can the body be considered approximately a point object: (a) a railway carriage moving without jerks between two stations. (b) a monkey sitting on top of a man cycling smoothly on a circular track. (c) a spinning cricket ball that turns sharply on hitting the ground. (d) a tumbling beaker that has slipped off the edge of a table.

SOLUTION

A body can be treated as a point object when its size is much smaller than the distance it travels or the scale of the motion under study.

(a) Railway carriage between two stations: The length of the carriage (~20 m) is negligible compared to the distance between stations (several kilometres). The carriage can be treated as a point object. [YES]

(b) Monkey on top of a man cycling: The monkey's size is negligible compared to the circumference of the circular track. We are interested only in the position of the system (monkey + man) along the track, so the monkey can be treated as a point object. [YES]

(c) Spinning cricket ball that turns sharply: Here the rotation (spin) itself is the phenomenon of interest, and the orientation and size of the ball are crucial for the sharp turn. The ball's size cannot be neglected. [NO]

(d) Tumbling beaker slipping off a table: The rotation (tumbling) of the beaker is the relevant motion. Its size and orientation matter. The beaker cannot be treated as a point object. [NO]

Answer: (a) and (b) can be considered point objects; (c) and (d) cannot.

Q2 The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. 2.9. Choose the correct entries in the brackets below: (a) (A/B) lives closer to the school than (B/A) (b) (A/B) starts from the school earlier than (B/A) (c) (A/B) walks faster than (B/A) (d) A and B reach home at the (same/different) time (e) (A/B) overtakes (B/A) on the road (once/twice).

SOLUTION

Reading from the x-t graph (Fig. 2.9):

- The school O is at x = 0.
- A's home P is at a smaller x-value; B's home Q is at a larger x-value.
- A's line starts at t = 0 (earlier); B's line starts at a later time.
- B's line has a steeper slope (greater $\Delta x/\Delta t$) than A's line.
- Both lines terminate at the same value of t (same time of arrival).
- The two lines intersect once.

(a) A lives closer to the school than B.
(P is at smaller x than Q.)

(b) A starts from the school earlier than B.
(A's line begins at a smaller t.)

(c) B walks faster than A.
(B's graph has a steeper slope ⇒ greater speed.)

(d) A and B reach home at the same time.
(Both lines end at the same value of t.)

(e) B overtakes A on the road once.
(The two lines cross at exactly one point.)

Answer: (a) A lives closer to school than B. (b) A starts earlier than B. (c) B walks faster than A. (d) A and B reach home at the same time. (e) B overtakes A once.

Q3 A woman starts from her home at 9.00 am, walks with a speed of 5 km h^{-1} on a straight road up to her office 2.5 km away, stays at the office up to 5.00 pm, and returns home by an auto with a speed of 25 km h^{-1}. Choose suitable scales and plot the x-t graph of her motion.

SOLUTION

Key events and coordinates (taking home as origin x = 0, t = 0 at 9:00 AM):

Time to walk to office: t = 2.5 km / 5 km h^{-1} = 0.5 h = 30 min
=> She reaches office at 9:30 AM (t = 0.5 h, x = 2.5 km).

She stays at office until 5:00 PM: t = 8 h (from 9:00 AM), x = 2.5 km.

Time to return by auto: t = 2.5 km / 25 km h^{-1} = 0.1 h = 6 min
=> She reaches home at 5:06 PM (t = 8.1 h, x = 0 km).

Key points for the x-t graph:
(t = 0 h, x = 0 km) — home at 9:00 AM
(t = 0.5 h, x = 2.5 km) — office at 9:30 AM [straight line, slope = 5 km/h]
(t = 8.0 h, x = 2.5 km) — still at office 5:00 PM [horizontal line]
(t = 8.1 h, x = 0 km) — home at 5:06 PM [straight line, slope = −25 km/h]

The graph consists of three segments:
Segment 1 (0 to 0.5 h): rising straight line, slope = +5 km/h.
Segment 2 (0.5 h to 8 h): horizontal line at x = 2.5 km (stationary at office).
Segment 3 (8 h to 8.1 h): steep falling straight line, slope = −25 km/h back to x = 0.

Suggested scales: x-axis 0–2.5 km (1 cm = 0.5 km); t-axis 0–8.5 h (1 cm = 1 h).

Answer: Three-segment x-t graph: (0 h, 0 km) → (0.5 h, 2.5 km) [slope +5 km/h] → (8.0 h, 2.5 km) [horizontal] → (8.1 h, 0 km) [slope −25 km/h].

Q4 A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the x-t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.

SOLUTION

Each step = 1 m, 1 s. Pattern: 5 forward then 3 backward, net gain = +2 m per cycle of 8 s.

Tracking position at each second:

t = 0 s: x = 0 m
t = 1 s: x = 1 m (forward)
t = 2 s: x = 2 m (forward)
t = 3 s: x = 3 m (forward)
t = 4 s: x = 4 m (forward)
t = 5 s: x = 5 m (forward — end of 1st forward burst)
t = 6 s: x = 4 m (back)
t = 7 s: x = 3 m (back)
t = 8 s: x = 2 m (back — end of 1st backward burst)
[Cycle 2]
t = 9 s: x = 3 m t = 10 s: x = 4 m t = 11 s: x = 5 m
t = 12 s: x = 6 m t = 13 s: x = 7 m (end of 2nd forward burst)
t = 14 s: x = 6 m t = 15 s: x = 5 m t = 16 s: x = 4 m
[Cycle 3]
t = 17 s: x = 5 m t = 18 s: x = 6 m t = 19 s: x = 7 m
t = 20 s: x = 8 m t = 21 s: x = 9 m (end of 3rd forward burst)
t = 22 s: x = 8 m t = 23 s: x = 7 m t = 24 s: x = 6 m
[Cycle 4]
t = 25 s: x = 7 m t = 26 s: x = 8 m t = 27 s: x = 9 m
t = 28 s: x = 10 m t = 29 s: x = 11 m (end of 4th forward burst)
t = 30 s: x = 10 m t = 31 s: x = 9 m t = 32 s: x = 8 m
[Cycle 5]
t = 33 s: x = 9 m t = 34 s: x = 10 m t = 35 s: x = 11 m
t = 36 s: x = 12 m t = 37 s: x = 13 m ← falls into pit!

The drunkard reaches x = 13 m at t = 37 s.

Alternative (analytical): After each complete cycle of 8 s the drunkard is at 2n m (after n cycles). After 5 complete cycles (40 s) he would be at 10 m. But during the 5th cycle's forward burst he reaches 11 m (t = 35 s), 12 m (t = 36 s), 13 m (t = 37 s) — the pit is reached.

The x-t graph is a zigzag: rises for 5 s (slope +1 m/s), falls for 3 s (slope −1 m/s), repeating, with maximum x increasing by 2 m each cycle.

Answer: The drunkard falls into the pit after 37 s.

Q5 A car moving along a straight highway with speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?

SOLUTION

Given:

Initial speed: v_0 = 126 km/h = 126 × (1000/3600) m/s = 35 m/s

Final speed: v = 0 m/s

Distance: x = 200 m

Step 1 — Find retardation using $v^2 = v_0^2 + 2ax$:

$0 = (35)^2 + 2 \times a \times 200$

$2 \times 200 \times a = -(35)^2$

$a = -(1225/400) = -3.0625$ m/s²

Retardation = |a| = 3.06 m/s² (taking 3 sig figs: 3.06 m/s²)

Step 2 — Find time using $v = v_0 + at$:

$0 = 35 + (-3.0625) \times t$

$t = 35 / 3.0625 = 400/35 = 80/7 \approx 11.4$ s

Check with $x = v_0t + \frac{1}{2}at^2$:

$x = 35 \times (80/7) + \frac{1}{2} \times (-1225/400) \times (80/7)^2$

$= 400 - \frac{1}{2} \times (1225/400) \times (6400/49)$

$= 400 - 200 = 200$ m ✓

Answer: Retardation = 3.06 m/s²; time to stop = 80/7 ≈ 11.4 s.

Q6 A player throws a ball upwards with an initial speed of 29.4 m s^{-1}. (a) What is the direction of acceleration during the upward motion of the ball? (b) What are the velocity and acceleration of the ball at the highest point of its motion? (c) Choose the x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion. (d) To what height does the ball rise and after how long does the ball return to the player's hands? (Take g = 9.8 m s^{-2} and neglect air resistance).

SOLUTION

(a) During upward motion, gravity acts downward. Acceleration = g = 9.8 m/s² directed downward (towards the Earth). This is true throughout — both during upward and downward motion.

(b) At the highest point, the ball momentarily comes to rest:

Velocity = 0 m/s.

Acceleration = 9.8 m/s² downward (acceleration due to gravity is always downward; it does not become zero at the highest point).

(c) With origin at the highest point and positive x taken downward:

During upward motion (before reaching the highest point): the ball is between the player's hand and the highest point, i.e. below the origin, which is the positive-x side. It is moving upward, i.e. toward the origin in the negative-x direction.

- $x > 0$ (ball is below the highest point = displaced downward from origin)
- $v < 0$ (ball moves upward = negative x-direction)
- $a > 0$ (gravity is downward = positive direction)

During downward motion (ball falls from highest point back to player's hand):

- $x > 0$ (ball is below the highest point — same side as before)
- $v > 0$ (ball moves downward = positive direction)
- $a > 0$ (gravity = downward = positive, unchanged)

(d) Taking origin at the player's hand, positive direction upward:

$v_0 = +29.4$ m/s, $a = -9.8$ m/s², $v = 0$ at highest point.

Maximum height H:

$v^2 = v_0^2 + 2aH$

$0 = (29.4)^2 + 2 \times (-9.8) \times H$

$H = (29.4)^2 / (2 \times 9.8) = 864.36 / 19.6 = 44.1$ m

Time to reach highest point: $t_1 = v_0/g = 29.4/9.8 = 3.0$ s

By symmetry, time to return to the same height (player's hand):

$t = 2t_1 = 2 \times 3.0 = 6.0$ s

Answer: (a) Acceleration is directed downward (towards Earth) throughout. (b) At highest point: velocity = 0, acceleration = 9.8 m/s² downward. (c) Upward motion: $x > 0$, $v < 0$, $a > 0$; Downward motion: $x > 0$, $v > 0$, $a > 0$. (d) Ball rises to H = 44.1 m; returns to player's hands after t = 6.0 s.

Q7 Read each statement below carefully and state with reasons and examples, if it is true or false; A particle in one-dimensional motion (a) with zero speed at an instant may have non-zero acceleration at that instant (b) with zero speed may have non-zero velocity, (c) with constant speed must have zero acceleration, (d) with positive value of acceleration must be speeding up.

SOLUTION

(a) TRUE.

Reason: Zero speed means the magnitude of velocity is zero at that instant, but acceleration is the rate of change of velocity, which can be non-zero even when $v = 0$.

Example: A ball thrown vertically upward has zero speed at the highest point, yet has acceleration $g = 9.8\ m/s^2$ directed downward at that instant.

(b) FALSE.

Reason: Speed is the magnitude of velocity. If speed = $|v| = 0$, then $v = 0$ (velocity is also zero). Non-zero velocity would require non-zero speed.

Zero speed necessarily implies zero velocity.

(c) TRUE (for one-dimensional motion).

Reason: In one-dimensional motion with constant speed, the particle moves with constant magnitude along a straight line. Since both magnitude and direction (sign) of velocity are constant, acceleration $a = dv/dt = 0$.

(Note: This would be false for 2D/3D where constant speed with changing direction gives centripetal acceleration. For 1D, constant speed means zero acceleration.)

(d) FALSE.

Reason: 'Positive acceleration' means acceleration in the positive x-direction. Whether the particle speeds up or slows down depends on whether acceleration is in the same direction as velocity or opposite.

Example: If a particle moves in the negative x-direction ($v < 0$) with positive acceleration ($a > 0$), the speed $|v|$ is decreasing (the particle is slowing down).

Only if $v > 0$ and $a > 0$ simultaneously does the particle speed up.

Answer: (a) True — e.g. ball at topmost point. (b) False — zero speed means zero velocity. (c) True for 1D motion. (d) False — depends on direction of velocity relative to acceleration.

Q8 A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between t = 0 to 12 s.

SOLUTION

Given: height = 90 m, g = 9.8 m/s²; at each bounce, speed after = 0.9 × speed before.

Phase 1 — Free fall from 90 m (t = 0 to t_1):

Speed at first impact: $v_1 = \sqrt{(2 \times 9.8 \times 90)} = \sqrt{1764} = 42$ m/s

Time of first impact: $t_1 = v_1/g = 42/9.8 = 30/7 \approx 4.29$ s

(Speed increases linearly from 0 to 42 m/s)

Phase 2 — First bounce (t_1 to t_2):

Speed just after 1st bounce: $v_2 = 0.9 \times 42 = 37.8$ m/s

Ball goes up, decelerates to 0, then falls back to floor.

Duration of 1st bounce cycle: $\Delta t_2 = 2 \times v_2/g = 2 \times 37.8/9.8 = 75.6/9.8 = 756/98 \approx 7.71$ s

Time of 2nd impact: $t_2 = 30/7 + 756/98 = 420/98 + 756/98 = 1176/98 = 12.0$ s

So within the interval 0 to 12 s, the ball hits the floor exactly twice:

1st hit at t = 30/7 ≈ 4.29 s (speed 42 m/s → 37.8 m/s)

2nd hit at t = 12.0 s (speed 37.8 m/s → 34.02 m/s; but this is at boundary)

Speed-time graph description (speed = |velocity| ≥ 0 always):

t = 0 to 30/7 s: speed increases linearly 0 → 42 m/s (free fall)

At t = 30/7 s: abrupt drop 42 → 37.8 m/s (1st bounce)

t = 30/7 s to 8.14 s: speed decreases linearly 37.8 → 0 (ball moving up)

t = 8.14 s to 12 s: speed increases linearly 0 → 37.8 m/s (ball falling)

At t = 12 s: abrupt drop 37.8 → 34.02 m/s (2nd bounce)

The graph consists of two V-shaped sections (each a pair of straight lines with slope ±g) connected at the bounce points. The second V is lower than the first (37.8 m/s vs 42 m/s). The time for each phase increases by a factor of 0.9.

Answer: 1st hit at t = 30/7 ≈ 4.29 s (speed 42 m/s → 37.8 m/s); 2nd hit at t = 12 s (speed 37.8 m/s). The speed-time graph shows two inverted V segments: speed rises 0→42 m/s (slope = g), drops to 37.8, then forms a symmetric V from 37.8→0→37.8 m/s over t_1 to 12 s.

Q9 Explain clearly, with examples, the distinction between: (a) magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval; (b) magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval]. Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true? [For simplicity, consider one-dimensional motion only].

SOLUTION

(a) Magnitude of displacement vs total path length:

Displacement is the vector from initial position to final position; its magnitude $|\Delta x| = |x_f - x_i|$.

Total path length is the actual distance travelled along the path, regardless of direction.

Example: A person walks 4 m east, then 3 m west. Displacement = 4 − 3 = 1 m east; |displacement| = 1 m. But path length = 4 + 3 = 7 m.

Proof that path length ≥ |displacement|:

In one dimension, if the particle reverses direction at any point, the path length counts both the forward and the return distances, while displacement measures only the net change. Mathematically:

path length = $\int |dx| \geq |\int dx| = |\Delta x|$

(The modulus of an integral ≤ the integral of the modulus.)

Equality holds when the particle moves in one direction only (no reversal). In that case path length = |displacement|.

(b) Average speed vs magnitude of average velocity:

Magnitude of average velocity = $|\Delta x|/\Delta t$ = |displacement|/time.

Average speed = (total path length)/Δt.

Since path length ≥	displacement	(from part a), dividing both sides by Δt > 0:		
average speed = (path length)/Δt ≥	Δx	/Δt =	average velocity	.

Example (same as above, total time = 7 s):

|Average velocity| = 1 m / 7 s ≈ 0.14 m/s
Average speed = 7 m / 7 s = 1.0 m/s

Equality holds when the particle does not reverse direction, so path length = |displacement|.

Answer: Path length ≥ |displacement| always; equality when motion is in one direction only. Average speed ≥ |average velocity| always; equality when path length = |displacement| (no reversal of direction).

Q10 A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1}. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1}. What is the (a) magnitude of average velocity, and (b) average speed of the man over the interval of time (i) 0 to 30 min, (ii) 0 to 50 min, (iii) 0 to 40 min?

SOLUTION

Setup:
Distance to market: d = 2.5 km
Speed going: v_1 = 5 km/h => time to market: t_1 = 2.5/5 = 0.5 h = 30 min
Speed returning: v_2 = 7.5 km/h => time to return: t_2 = 2.5/7.5 = 1/3 h = 20 min
Total trip time = 30 + 20 = 50 min.

(i) Interval: 0 to 30 min (0 to 0.5 h)
The man is walking to the market the entire time.
At t = 30 min, he is just at the market (x = +2.5 km from home).
Displacement = 2.5 km; Path length = 2.5 km.
|Average velocity| = 2.5 km / 0.5 h = 5 km/h
Average speed = 2.5 km / 0.5 h = 5 km/h

(ii) Interval: 0 to 50 min (0 to 5/6 h)
He walks to market (30 min), then returns home (20 min).
At t = 50 min he is back at home => displacement = 0.
Path length = 2.5 + 2.5 = 5.0 km.
|Average velocity| = 0 km / (5/6 h) = 0 km/h
Average speed = 5.0 km / (5/6 h) = 5.0 × (6/5) = 6.0 km/h

(iii) Interval: 0 to 40 min (0 to 2/3 h)
He reaches market at t = 30 min, then walks back for 10 min.
Distance covered on return in 10 min = 7.5 × (10/60) = 7.5/6 = 1.25 km

Position at t = 40 min: x = 2.5 − 1.25 = 1.25 km from home.
Displacement = 1.25 km; Path length = 2.5 + 1.25 = 3.75 km.
|Average velocity| = 1.25 / (2/3) = 1.25 × 3/2 = 1.875 km/h
Average speed = 3.75 / (2/3) = 3.75 × 3/2 = 5.625 km/h

Answer: (i) |avg velocity| = 5 km/h, avg speed = 5 km/h. (ii) |avg velocity| = 0, avg speed = 6 km/h. (iii) |avg velocity| = 1.875 km/h, avg speed = 5.625 km/h.

Q11 In Exercises 2.9 and 2.10, we have carefully distinguished between average speed and magnitude of average velocity. No such distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?

SOLUTION

Instantaneous velocity is defined as the limit of the average velocity as the time interval $\Delta t \to 0$:

$$v = \lim_{\Delta t \to 0} \Delta x/\Delta t = dx/dt$$

In this limit, the time interval Δt becomes infinitesimally small. Over such an infinitesimally short interval, the particle travels an infinitesimally short distance. The particle cannot reverse direction within such an infinitesimal interval — any reversal would require a finite time.

Therefore, over the infinitesimal interval $\Delta t \to 0$:

$$\text{path length} = |\Delta x|$$

As a result:

$$\text{Instantaneous speed} = \lim_{\Delta t \to 0} (\text{path length}/\Delta t) = \lim_{\Delta t \to 0} |\Delta x/\Delta t| = |dx/dt| = |v|$$

Since instantaneous speed = $|dx/dt| = |v|$ = magnitude of instantaneous velocity, the two are always equal.

In contrast, over a finite time interval, the path length can exceed $|\Delta x|$ if the particle reverses direction. But in the limit $\Delta t \to 0$, this cannot happen, so the equality always holds instantaneously.

Answer: As $\Delta t \to 0$, the particle cannot reverse direction in the infinitesimal interval, so path length = $|\Delta x|$. Hence instantaneous speed = $\lim|\Delta x/\Delta t| = |dx/dt|$ = magnitude of instantaneous velocity.

Q12 Look at the graphs (a) to (d) (Fig. 2.10) carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.

SOLUTION

Graphs that cannot represent one-dimensional motion of a particle:

(a) x-t graph shaped like a bowtie / figure-of-eight closed loop (passing through both positive and negative x):

CANNOT represent 1D motion. Along the loop, a single instant t corresponds to two different values of x. A particle cannot be at two positions simultaneously. Position must be a single-valued function of time.

(b) v-t graph in the form of a closed circle:

CANNOT represent 1D motion of a single particle. On a circle, one instant t corresponds to two different velocity values. At a particular instant a particle can have only one velocity, so this is physically impossible for a single particle.

(c) Speed-time graph dipping below the time axis (negative speed):

CANNOT represent 1D motion. Speed is defined as the magnitude of velocity and is always ≥ 0. Negative speed is physically meaningless.

(d) Total path length decreasing with time:

CANNOT represent 1D motion. Total path length is a cumulative quantity; it can only increase or remain constant (when the particle is stationary). It can never decrease.

All four graphs (a), (b), (c), and (d) are impossible representations.

Answer: All four graphs (a)–(d) are impossible for 1D motion: (a) multi-valued x at one t, (b) multi-valued v at one t, (c) negative speed, (d) decreasing total path length.

Q13 Figure 2.11 shows the x-t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.

SOLUTION

It is NOT correct to interpret the x-t graph as indicating the geometric path (trajectory) of the particle.

An x-t graph plots position x on one axis and time t on the other. It shows how

position changes with time, NOT the shape of the path in space. The particle in this problem moves in one dimension (straight line) throughout — that is the given constraint.

From Fig. 2.11:

For $t < 0$: the x-t graph is a horizontal straight line at $x = 0$ (particle at rest at origin).

For $t > 0$: the graph shows a parabola starting from the origin, indicating $x \propto t^2$ (uniformly accelerated motion).

Suitable physical context:

A ball is at rest on a flat surface for $t < 0$. At $t = 0$ it is pushed or released from rest on an inclined plane and slides down with uniform acceleration. Its position along the incline (x) grows as $x = \frac{1}{2}at^2$, giving the parabolic x-t curve for $t > 0$.

The particle moves along a straight line at all times; the parabolic curve appears only in the x-t plot, not in real space.

Answer: No — the x-t graph does not show the spatial path. The particle is always on a straight line. The graph shows the particle at rest ($x = 0$) for $t < 0$, then undergoing uniform acceleration for $t > 0$ (e.g. released from rest on an inclined plane at $t = 0$).

Q14 A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 m s^{-1}, with what speed does the bullet hit the thief's car? (Note: Obtain that speed which is relevant for damaging the thief's car).

SOLUTION

Given:

Speed of police van: $v_P = 30$ km/h $= 30/3.6$ m/s $= 25/3$ m/s ≈ 8.33 m/s

Speed of thief's car: $v_T = 192$ km/h $= 192/3.6$ m/s $= 160/3$ m/s ≈ 53.33 m/s

Muzzle speed of bullet (relative to van): $v_b = 150$ m/s

Speed of bullet relative to ground (both van and bullet in same direction):

$v_{bullet} = v_b + v_P = 150 + (30/3.6) = 150 + 25/3 \approx 158.33$ m/s

The speed relevant for damaging the thief's car is the speed of the bullet relative to the thief's car:

$v_{rel} = v_{bullet} - v_T$

$= (150 + 30/3.6) - (192/3.6)$

= 150 + (30 − 192)/3.6
= 150 − 162/3.6
= 150 − 45
= 105 m/s

Answer: The bullet hits the thief's car at a relative speed of 105 m/s.

Q15 Suggest a suitable physical situation for each of the following graphs (Fig 2.12): (a) x-t graph: x rises from a negative constant value up to a positive peak A, then falls to a negative value B and stays constant there. (b) v-t graph: several parallel line segments of negative slope, giving more than one velocity at some instants. (c) a-t graph: acceleration is zero, rises to a single sharp positive pulse, then returns to zero.

SOLUTION

(a) x-t graph: x is a negative constant for a while, then increases to a positive maximum A, then decreases to a negative value B and stays constant there.

Physical situation: A ball is thrown vertically upward from the hand of a person standing on a raised platform (take upward as +x, the ground below the platform as negative x). The ball rises to a maximum height A, then falls back, passing the launch level and landing on the lower ground at B, where it comes to rest and stays. The upward-then-downward parabola-like arc with a rest level at the end fits the curve.

Alternatively: a football kicked up a slope that rolls up to A, rolls back down past the start, and stops in a ditch at B.

(b) v-t graph: several disconnected straight-line segments with negative slope, so at certain instants the graph shows more than one value of v.

Physical situation: A ball bouncing on the floor with a fixed downward acceleration (g). During each fall the velocity decreases linearly (negative slope); at each bounce the velocity jumps abruptly (the collision is nearly instantaneous), producing the several parallel segments and the apparent multi-valued velocity at the bounce instants. Each successive segment corresponds to a successive bounce.

(c) a-t graph: acceleration is zero, then a single sharp positive pulse, then zero again.

Physical situation: A ball (or a bat striking a ball) that moves freely with zero

acceleration, then experiences a brief, large impulsive force during a very short contact time (the sharp positive spike), after which the acceleration is zero again. This models an impulsive collision — e.g. a cricket ball struck by a bat.

Answer: (a) A ball thrown up from a raised platform: it rises to a maximum height A, falls back past the launch level, and comes to rest on the lower ground at B. (b) A ball bouncing on the floor: velocity falls linearly (slope −g) during each fall and jumps abruptly at each bounce, giving the parallel segments and multi-valued velocity. (c) A ball given a brief impulsive hit (e.g. by a bat): zero acceleration except for one short, large positive pulse during the contact time.

Q16 Figure 2.13 gives the x-t plot of a particle executing one-dimensional simple harmonic motion. (You will learn about this motion in more detail in Chapter 13). Give the signs of position, velocity and acceleration variables of the particle at t = 0.3 s, 1.2 s, -1.2 s.

SOLUTION

From Fig. 2.13, the x-t graph is sinusoidal with period T = 2 s and x = 0 at t = 0. Just after t = 0 the curve goes below the axis (a trough near t = 0.5 s), so t = 0 is a DESCENDING zero crossing. The curve therefore has the form $x = -A \sin(\pi t)$, with A > 0 (peak at t = −0.5 s, trough at t = +0.5 s, peak again at t = 1.5 s).

In SHM ($\omega = \pi$): $v = dx/dt = -A\pi \cos(\pi t)$, $a = -\omega^2 x = \pi^2 A \sin(\pi t)$.

At t = 0.3 s ($\pi t = 54°$):

$x = -A \sin(54°) < 0$ => position: NEGATIVE

$v = -A\pi \cos(54°) < 0$ => velocity: NEGATIVE (particle moving toward the trough, in −x direction)

$a = +\pi^2 A \sin(54°) > 0$ => acceleration: POSITIVE (restoring force pulls back toward x = 0)

At t = 1.2 s ($\pi t = 216°$):

$x = -A \sin(216°) = +A \sin(36°) > 0$ => position: POSITIVE

$v = -A\pi \cos(216°) = +A\pi \cos(36°) > 0$ => velocity: POSITIVE (moving in +x direction)

$a = +\pi^2 A \sin(216°) = -\pi^2 A \sin(36°) < 0$ => acceleration: NEGATIVE (restoring toward x = 0)

At t = −1.2 s ($\pi t = -216°$):

$x = -A \sin(-216°) = -A \sin(36°) < 0$ => position: NEGATIVE

v = −Aπ cos(−216°) = −Aπ cos(36°) < 0 … using cos(−216°) = cos(216°) = −cos(36°): v = −Aπ(−cos 36°) = +Aπ cos(36°) > 0 => velocity: POSITIVE

a = +π²A sin(−216°) = +π²A sin(36°) > 0 => acceleration: POSITIVE (restoring toward x = 0 from the negative side)

Answer: t = 0.3 s: x < 0, v < 0, a > 0. t = 1.2 s: x > 0, v > 0, a < 0. t = −1.2 s: x < 0, v > 0, a > 0.

Q17 Figure 2.14 gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval.

SOLUTION

From Fig. 2.14, the x-t curve rises from a positive value to a peak (around interval 2), then falls steeply, crosses x = 0, and goes negative to a trough. The three shaded equal time intervals are labelled 1, 2, 3 on the t-axis: interval 1 is on the rising part, interval 2 straddles the peak, interval 3 is on the steep descent.

Average speed over an interval = |Δx| / Δt (magnitude of the chord's slope on the x-t graph).

Reading the graph:

Interval 1: moderate positive slope — intermediate	Δx	. x is positive and rising.
Interval 2: near the peak the curve is almost flat, so	Δx	is smallest. Average speed: LEAST.
Interval 3: steep downward slope — largest	Δx	. Average speed: GREATEST.

Sign of average velocity (= Δx/Δt):

Interval 1: x increases => Δx > 0 => average velocity POSITIVE.

Interval 2: x rises to and just past the peak => Δx ≥ 0 => average velocity POSITIVE.

Interval 3: x falls steeply (positive to negative) => Δx < 0 => average velocity NEGATIVE.

Answer: Average speed is greatest in interval 3 and least in interval 2. Average velocity is positive in intervals 1 and 2, and negative in interval 3.

Q18 Figure 2.15 gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude? In which interval is the average speed greatest? Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. What are the accelerations at the points A, B, C and D?

SOLUTION

From Fig. 2.15, speed (always ≥ 0) versus time graph. Motion is along a constant direction (positive x). The curve has:

Point A: local minimum at start of interval 1 (low speed)
Point B: local maximum (peak) at end of interval 1 / start of interval 2
Point C: local minimum at end of interval 2 / start of interval 3
Point D: local maximum (highest peak) at end of interval 3

Average acceleration in each interval = Δ(speed)/Δt (since direction is constant, speed = v):

Interval 1 (A to B): speed increases from A to B => avg acceleration > 0
Interval 2 (B to C): speed decreases from B to C => avg acceleration < 0
Interval 3 (C to D): speed increases from C to D => avg acceleration > 0

Magnitude of average acceleration = |change in speed| / Δt over each interval. From the graph the speed drops sharply from the first peak B down to the trough C, which is the largest speed change over an equal time interval.
Greatest magnitude of average acceleration: INTERVAL 2 (steepest speed drop, B to C).

Average speed greatest: INTERVAL 3 (the speed values there are the highest — the curve rises to the tallest peak D).

Signs of v and a in each interval (positive direction = direction of motion):

Interval 1: v > 0 (moving in positive direction), a > 0 (speeding up)
Interval 2: v > 0 (moving in positive direction), a < 0 (slowing down)
Interval 3: v > 0 (moving in positive direction), a > 0 (speeding up)

Accelerations at points A, B, C, D:

B and D are local maxima of the speed curve => d(speed)/dt = 0 => a = 0 at B

and D.

C is a local minimum of the speed curve => d(speed)/dt = 0 => a = 0 at C.

A is the starting point; from the graph A appears to be a local minimum too => a = 0 at A.

Therefore, accelerations at A, B, C, and D are all zero.

Answer: Greatest average acceleration magnitude: interval 2 (B to C, steepest speed decrease). Greatest average speed: interval 3. Signs: v > 0 and a > 0 in intervals 1 and 3; v > 0 and a < 0 in interval 2. Accelerations at A, B, C, D are all zero (these are local extrema of the speed-time curve where d(speed)/dt = 0).