Chapter 3: Motion in a Plane

Exercises — Exercises

Q1 State, for each of the following physical quantities, if it is a scalar or a vector: volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

SOLUTION

A scalar has magnitude only; a vector has both magnitude and direction.

Volume — scalar (no direction associated with it)
Mass — scalar
Speed — scalar (magnitude of velocity, no direction)
Acceleration — vector (has both magnitude and direction)
Density — scalar
Number of moles — scalar
Velocity — vector
Angular frequency (ω) — scalar (it is the magnitude of angular velocity)
Displacement — vector
Angular velocity — vector (direction given by the right-hand rule along the axis of rotation)

Answer: Scalars: volume, mass, speed, density, number of moles, angular frequency. Vectors: acceleration, velocity, displacement, angular velocity.

Q2 Pick out the two scalar quantities in the following list:
force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

SOLUTION

Scalars have magnitude only; vectors have magnitude and direction.

Force — vector
Angular momentum — vector
Work — scalar ($W = F \cdot d$; a dot product gives a scalar)
Current — scalar (its direction is specified by the direction of conventional current, but current itself obeys scalar addition rules in circuits; it is treated as a scalar in general physics)
Linear momentum — vector

Electric field — vector
Average velocity — vector
Magnetic moment — vector
Relative velocity — vector

The two scalar quantities are work and current.

Answer: Work and current are the two scalar quantities.

Q3 Pick out the only vector quantity in the following list:
Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

SOLUTION

Temperature — scalar
Pressure — scalar (force per unit area; acts equally in all directions for a fluid, so it is a scalar)
Impulse — vector (impulse = force × time; since force is a vector, impulse is a vector)
Time — scalar
Power — scalar
Total path length — scalar
Energy — scalar
Gravitational potential — scalar
Coefficient of friction — scalar (a dimensionless ratio)
Charge — scalar

Impulse is the only vector in the list.

Answer: Impulse is the only vector quantity.

Q4 State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:
(a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions, (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

SOLUTION

(a) Adding any two scalars: Meaningful ONLY if both scalars represent the same physical quantity (same dimensions). Example: adding two masses, or two energies. Adding a mass to a temperature is physically meaningless even though both are scalars.

(b) Adding a scalar to a vector of the same dimensions: NOT meaningful. A scalar and a vector are fundamentally different mathematical objects; they cannot be added regardless of dimensions.

(c) Multiplying any vector by any scalar: Meaningful. The product is a vector whose magnitude is |scalar| × |vector| and whose direction is the same as (or opposite to) the original vector depending on the sign of the scalar. Example: multiplying velocity (vector) by mass (scalar) gives momentum (vector).

(d) Multiplying any two scalars: Meaningful. The product is a scalar. Scalars of different dimensions can be multiplied. Example: mass × acceleration² gives energy (per unit mass), or time × velocity gives distance.

(e) Adding any two vectors: Meaningful ONLY if both vectors represent the same physical quantity (same dimensions). Example: adding two displacement vectors or two force vectors. Adding displacement to force is not meaningful.

(f) Adding a component of a vector to the same vector: NOT meaningful in the strict vector sense. A component (e.g., Ax) is a scalar, while the original vector A is a vector. You cannot add a scalar to a vector.

Answer: (a) Meaningful only if same physical quantity. (b) Not meaningful. (c) Meaningful. (d) Meaningful. (e) Meaningful only if same physical quantity. (f) Not meaningful (scalar cannot be added to a vector).

Q5 Read each statement below carefully and state with reasons, if it is true or false:
(a) The magnitude of a vector is always a scalar.
(b) Each component of a vector is always a scalar.
(c) The total path length is always equal to the magnitude of the displacement vector of a particle.
(d) The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.
(e) Three vectors not lying in a plane can never add up to give a null vector.

SOLUTION

(a) TRUE. The magnitude of a vector is a non-negative number (a scalar). For example, |v| = v is a scalar representing speed.

(b) FALSE. Each component of a vector (e.g., Ax, Ay) is a scalar number, but the component vectors (Ax î, Ay ĵ) are vectors. Strictly, the statement is ambiguous: if 'component' means the scalar value, it is true; but generally 'components' include the directional component vectors. The NCERT answer treats this as FALSE — each component of a vector is itself a vector (e.g., Ax î is a vector along x-axis).

(c) FALSE. The total path length is equal to the magnitude of displacement only when the particle moves in a straight line without reversing direction. In general, the path length ≥ displacement. For a curved or back-and-forth path, path length > displacement.

(d) TRUE. Average speed = total path length / time; average velocity magnitude = displacement / time. Since path length ≥ displacement (the shortest path between two points is a straight line), average speed ≥ |average velocity|. Equality holds only when path length equals displacement (straight-line motion in one direction).

(e) TRUE. For three vectors to give a null (zero) resultant, they must close a triangle. Three vectors that close a triangle must be coplanar (since two of them define a plane and the third must lie in that same plane to return to the origin). Therefore, three vectors NOT lying in a plane can never sum to zero.

Answer: (a) True. (b) False — component vectors are vectors. (c) False — path length ≥ displacement. (d) True. (e) True.

Q6 Establish the following vector inequalities geometrically or otherwise:
(a) |a + b| ≤ |a| + |b|
(b) |a + b| ≥ ||a| − |b||
(c) |a − b| ≤ |a| + |b|
(d) |a − b| ≥ ||a| − |b||
When does the equality sign above apply?

SOLUTION

(a)	a + b	≤	a	+	b	(Triangle Inequality)		
By the triangle law, vectors a and b form two sides of a triangle, and their resultant (a + b) is the third side. In any triangle, the length of one side is less than or equal to the sum of the other two sides. Hence	a + b	≤	a	+	b	.		

Equality holds when a and b are parallel and in the same direction (angle between them = 0°).

(b) |a + b| ≥ ||a| − |b||
From the triangle inequality applied to vector b and resultant (a + b):

	a	=	(a + b) + (−b)	≤	a + b	+	b	
=>	a + b	≥	a	−	b			
Similarly,	a + b	≥	b	−	a	.		
Combining:	a + b	≥		a	−	b		.

Equality holds when a and b are antiparallel (angle = 180°).

(c)	a − b	≤	a	+	b							
Write a − b = a + (−b). Since	−b	=	b	, by the triangle inequality (part a):								

(c)	$\vec{a} - \vec{b}$	≤	$\vec{a}$	+	$\vec{b}$							
	$\vec{a} - \vec{b}$	=	$\vec{a} + (-\vec{b})$	≤	$\vec{a}$	+	$-\vec{b}$	=	$\vec{a}$	+	$\vec{b}$	.

Equality holds when $\vec{a}$ and $-\vec{b}$ are in the same direction, i.e., $\vec{a}$ and $\vec{b}$ are antiparallel (angle = 180°).

(d) $|\vec{a} - \vec{b}| \geq ||\vec{a}| - |\vec{b}||$

Write $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$ and apply part (b):

	$\vec{a} + (-\vec{b})$	≥		$\vec{a}$	−	$-\vec{b}$		=		$\vec{a}$	−	$\vec{b}$		.
Hence	$\vec{a} - \vec{b}$	≥		$\vec{a}$	−	$\vec{b}$		.						

Equality holds when $\vec{a}$ and $\vec{b}$ are parallel (in the same direction, angle = 0°).

Answer: (a) Equality when $\vec{a} \parallel \vec{b}$ (same direction). (b) Equality when $\vec{a}$ and $\vec{b}$ are antiparallel. (c) Equality when $\vec{a}$ and $\vec{b}$ are antiparallel. (d) Equality when $\vec{a} \parallel \vec{b}$ (same direction).

Q7 Given $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$, which of the following statements are correct:
(a) $\vec{a}$, $\vec{b}$, $\vec{c}$, and $\vec{d}$ must each be a null vector.
(b) The magnitude of $(\vec{a} + \vec{c})$ equals the magnitude of $(\vec{b} + \vec{d})$.
(c) The magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b}$, $\vec{c}$, and $\vec{d}$.
(d) $\vec{b} + \vec{c}$ must lie in the plane of $\vec{a}$ and $\vec{d}$ if $\vec{a}$ and $\vec{d}$ are not collinear, and in the line of $\vec{a}$ and $\vec{d}$, if they are collinear.

SOLUTION

Given: $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$

(a) FALSE. Individual vectors need not be null vectors. For example, $\vec{a} = \hat{i}$, $\vec{b} = \hat{j}$, $\vec{c} = -\hat{i}$, $\vec{d} = -\hat{j}$ satisfy $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$, yet none is a null vector.

(b) TRUE. From $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$:

=> a + c = −(b + d)
=> |a + c| = |−(b + d)| = |b + d|.
The magnitudes are equal.

(c) TRUE. From a + b + c + d = 0:
=> a = −(b + c + d)
=> |a| = |b + c + d| ≤ |b| + |c| + |d| (by the triangle inequality applied repeatedly).
So the magnitude of a can never exceed the sum of magnitudes of b, c, and d.

(d) TRUE. From a + b + c + d = 0:
=> b + c = −(a + d)
The vector (b + c) must equal −(a + d). If a and d are not collinear, they define a plane, and (a + d) lies in that plane; hence −(a + d) = (b + c) also lies in that plane. If a and d are collinear, their sum lies along the same line, and (b + c) must also lie along that line.

Answer: Correct statements: (b), (c), and (d). Statement (a) is incorrect.

Q8 Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. 3.19. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skate?

SOLUTION

Displacement is the straight-line distance from the initial point P to the final point Q, regardless of the path taken.

P and Q are diametrically opposite on a circle of radius R = 200 m.
Distance PQ = diameter = 2R = 2 × 200 = 400 m.

For all three girls, the displacement magnitude is the same:
|PQ| = 400 m

From Fig. 3.19, the three girls follow paths A, B, and C:
• Girl A follows a curved path from P to Q (longer than the diameter).
• Girl B follows the straight diameter from P to Q (path length = 2R = 400 m).
• Girl C follows a longer, wavy (zig-zag) path from P to Q.

The displacement equals the actual path length only when the path is a straight line from P to Q.
This is true for Girl B (who skates along the straight diameter).

Answer: Displacement = 400 m for each girl. The displacement equals the path length only for girl B, who travels along the straight diameter.

Q9 A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. 3.20. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?

SOLUTION

Given: Radius R = 1 km = 1000 m, time = 10 min = 600 s.

From Fig. 3.20: O is the centre; P lies on the edge to the east and Q on the edge to the north, so the arc P → Q is a quarter of the circle (90°). The cyclist goes O → P (radial path, 1 km), then along the quarter-circle arc P → Q (quarter circumference = (1/4)(2πR) = πR/2 = π/2 km), then Q → O (radial path, 1 km).

(a) Net displacement:

The cyclist starts and ends at centre O.

Net displacement = 0.

(b) Average velocity:

Average velocity = net displacement / time = 0 / 600 = 0 m/s.

(c) Average speed:

Total path length = OP + arc PQ + QO

= 1 + (π/2) × 1 + 1 = (2 + π/2) km

= (2 + 1.571) km ≈ 3.571 km = 3571 m

Average speed = total path / time = 3571 / 600 ≈ 5.95 m/s

Or in km/h: (2 + π/2) km / (10/60 h) = 6(2 + π/2) ≈ 21.4 km/h

Answer: (a) Net displacement = 0. (b) Average velocity = 0. (c) Average speed ≈ 21.4 km/h (≈ 5.95 m/s).

Q10 On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

SOLUTION

The motorist turns 60° to the left after every 500 m leg. This traces a regular hexagon (since 6 × 60° = 360°, the path closes after 6 legs).

Set up coordinates: start at origin O, initial direction along +x axis.
After each 500 m leg, direction rotates 60° CCW.

Positions after each turn:
- Turn 1: (500, 0) m
- Turn 2: (750, 433) m
- Turn 3: (500, 866) m
- Turn 4: (0, 866) m
- Turn 5: (−250, 433) m
- Turn 6: (0, 0) m ← back to start
- Turn 7: (500, 0) m ← same as Turn 1
- Turn 8: (750, 433) m ← same as Turn 2

At the 3rd turn:
- Position = (500, 866) m
- Displacement = $\sqrt{(500^2 + 866^2)} = \sqrt{(250000 + 750000)} = \sqrt{1000000} = 1000$ m
- Direction: 60° from the initial direction (north-east of start)
- Path length = 3 × 500 = 1500 m
- Ratio: path/displacement = 1500/1000 = 1.5

At the 6th turn:
- Position = (0, 0) — motorist returns to starting point
- Displacement = 0 m
- Path length = 6 × 500 = 3000 m
- Ratio: path/displacement is not defined (displacement = 0)

At the 8th turn:
- Position = (750, 433) m
- Displacement = $\sqrt{(750^2 + 433^2)} = \sqrt{(562500 + 187489)} \approx 866$ m $= 500\sqrt{3}$ m
- Direction: 30° from the initial direction
- Path length = 8 × 500 = 4000 m

Ratio: path/displacement = 4000/866 ≈ 4.6

Answer: 3rd turn: displacement = 1000 m at 60° from start direction, path = 1500 m (ratio 1.5). 6th turn: displacement = 0, path = 3000 m. 8th turn: displacement = 500√3 ≈ 866 m at 30° from start direction, path = 4000 m (ratio ≈ 4.6).

Q11 A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal?

SOLUTION

Given:

Total path length = 23 km

Displacement (station to hotel, straight road) = 10 km

Time = 28 min = 28/60 h

(a) Average speed:

Average speed = total path length / time

= 23 km ÷ (28/60 h)

= 23 × 60/28

≈ 49.3 km/h

(b) Magnitude of average velocity:

|Average velocity| = displacement / time

= 10 km ÷ (28/60 h)

= 10 × 60/28

≈ 21.4 km/h

The two are NOT equal because the path taken (23 km) is longer than the displacement (10 km). Average speed > |average velocity| whenever the actual path length exceeds the displacement.

Answer: (a) Average speed ≈ 49.3 km/h. (b) Magnitude of average velocity ≈ 21.4 km/h. They are not equal.

Q12 The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall?

SOLUTION

Given: ceiling height h = 25 m, initial speed v_0 = 40 m/s, g = 9.8 m/s^2.

For the ball to just reach (but not exceed) the ceiling, the maximum height h_m = 25 m.

$h_m = (v_0 \sin\theta)^2 / (2g)$

$25 = (40 \sin\theta)^2 / (2 \times 9.8)$

$(40 \sin\theta)^2 = 490$

$\sin\theta = \sqrt{490} / 40 = 22.136 / 40 = 0.5534$

$\theta \approx 33.6°$

Note: Throwing at 45° would give $h_m = (40 \times \sin 45°)^2/(19.6) \approx 40.8$ m > 25 m, so 45° is not allowed. The optimum angle is the largest angle that keeps h_m = 25 m.

Range at this angle:

$\cos\theta = \sqrt{(1 - 0.5534^2)} = \sqrt{(1 - 0.3063)} = \sqrt{0.6937} = 0.8329$

$\sin 2\theta = 2 \sin\theta \cos\theta = 2 \times 0.5534 \times 0.8329 = 0.9222$

$R = v_0^2 \sin 2\theta / g = (40)^2 \times 0.9222 / 9.8$

$R = 1600 \times 0.9222 / 9.8 = 1475.5 / 9.8 \approx 150.5$ m

Answer: Maximum horizontal distance ≈ 150.5 m (achieved when launch angle θ ≈ 33.6°, just grazing the ceiling).

Q13 A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

SOLUTION

Given: Maximum horizontal range R_{max} = 100 m.

Maximum range occurs at θ = 45°:

$R_{max} = v_0^2 / g$

$100 = v_0^2 / 9.8$

$v_0^2 = 980$ m^2 s^{-2}

To throw the ball to maximum height, the cricketer throws it vertically (θ = 90°). Maximum height achieved (using $v^2 = v_0^2 - 2g\,h$, with v = 0 at top):

$h_{max} = v_0^2 / (2g)$
$h_{max} = 980 / (2 \times 9.8)$
$h_{max} = 980 / 19.6$
$h_{max} = 50$ m

Answer: Maximum height = 50 m.

Q14 A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

SOLUTION

Given: radius r = 80 cm = 0.80 m, 14 revolutions in 25 s.

Frequency: f = 14/25 = 0.56 Hz
Time period: T = 25/14 = 1.786 s
Angular speed: $\omega = 2\pi f = 2\pi \times 0.56 = 3.52$ rad/s

Centripetal acceleration:
$a_c = \omega^2 r = (3.52)^2 \times 0.80$
$a_c = 12.39 \times 0.80 \approx 9.91$ m/s²

Alternatively: $a_c = 4\pi^2 f^2 r = 4\pi^2 \times (0.56)^2 \times 0.80 \approx 9.90$ m/s²

Direction: The centripetal acceleration is always directed towards the centre of the circular path (centripetal — 'centre-seeking').
Since the stone moves in a horizontal circle, the acceleration is horizontal, pointing towards the centre at every instant.

Answer: Acceleration ≈ 9.9 m/s², directed towards the centre of the circular path.

Q15 An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

SOLUTION

Given: R = 1.00 km = 1000 m, v = 900 km/h = 900 × (1000/3600) = 250 m/s.

Centripetal acceleration:
$a_c = v^2 / R = (250)^2 / 1000 = 62500 / 1000 = 62.5$ m/s²

Comparing with g = 9.8 m/s²:

$a_c / g = 62.5 / 9.8 \approx 6.38$

The centripetal acceleration is approximately 6.4 times the acceleration due to gravity.

Answer: $a_c = 62.5$ m/s², and $a_c/g \approx 6.4$ (the centripetal acceleration is about 6.4g).

Q16 Read each statement below carefully and state, with reasons, if it is true or false:
(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.
(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.
(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

SOLUTION

(a) FALSE.
In UNIFORM circular motion (constant speed), the net acceleration is entirely centripetal — directed towards the centre along the radius. However, in NON-UNIFORM circular motion (changing speed), there is also a tangential component of acceleration (along the tangent, responsible for changing speed). In that case, the net acceleration is NOT purely along the radius.

(b) TRUE.
Velocity is defined as the rate of change of position vector (dr/dt). As the time interval $\Delta t \to 0$, the displacement Δr becomes tangent to the path. Therefore, the instantaneous velocity vector is always directed along the tangent to the path at that point, in the direction of motion.

(c) TRUE.
In uniform circular motion, the magnitude of centripetal acceleration is constant ($a_c = v^2/R$), but its direction continuously changes, always pointing towards the centre. Over one complete cycle, the acceleration vector sweeps through all directions uniformly. By symmetry, its vector average over one full revolution is zero — the components in every direction cancel.
Mathematically, if $a = -a_c (\cos \omega t\, \hat{i} + \sin \omega t\, \hat{j})$, then the average over one period T is zero since $\int_0 \cos \omega t\, dt = 0$ and $\int_0 \sin \omega t\, dt = 0$.

Answer: (a) False — true only for uniform circular motion; not in general. (b) True. (c) True.

Q17 The position of a particle is given by

$\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k}$ m

where t is in seconds and the coefficients have the proper units for r to be in metres.

(a) Find the v and a of the particle.

(b) What is the magnitude and direction of velocity of the particle at t = 2.0 s?

SOLUTION

Given: $\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k}$ m

(a) Velocity $\vec{v} = d\vec{r}/dt$:

$\vec{v} = \frac{d}{dt}(3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k})$

$\vec{v} = 3.0\,\hat{i} - 4.0t\,\hat{j}$ m/s

Acceleration $\vec{a} = d\vec{v}/dt$:

$\vec{a} = \frac{d}{dt}(3.0\,\hat{i} - 4.0t\,\hat{j})$

$\vec{a} = -4.0\,\hat{j}$ m/s²

The acceleration is constant: 4.0 m/s² in the negative y-direction.

(b) At t = 2.0 s:

$\vec{v} = 3.0\,\hat{i} - 4.0 \times 2.0\,\hat{j} = 3.0\,\hat{i} - 8.0\,\hat{j}$ m/s

Magnitude: $|\vec{v}| = \sqrt{(3.0^2 + 8.0^2)} = \sqrt{(9 + 64)} = \sqrt{73} \approx 8.54$ m/s

Direction: The velocity has components $v_x = 3.0$ m/s and $v_y = -8.0$ m/s.

The angle below the positive x-axis:

$\theta = \arctan(|v_y|/v_x) = \arctan(8.0/3.0) = \arctan(2.667) \approx 69.4°$

The velocity is directed at 69.4° below the positive x-axis (into the fourth quadrant of the x-y plane).

Answer: (a) $\vec{v} = (3.0\,\hat{i} - 4.0t\,\hat{j})$ m/s; $\vec{a} = -4.0\,\hat{j}$ m/s². (b) At t = 2 s: $|\vec{v}| \approx 8.54$ m/s, directed at 69.4° below the x-axis.

Q18 A particle starts from the origin at t = 0 s with a velocity of 10.0 ĵ m/s and moves in the x-y plane with a constant acceleration of (8.0 î + 2.0 ĵ) m s^{-2}.
(a) At what time is the x-coordinate of the particle 16 m? What is the y-coordinate of the particle at that time?
(b) What is the speed of the particle at that time?

SOLUTION

Given: $r_o = 0$, $v_o = 10.0$ ĵ m/s, $a = (8.0\ î + 2.0\ ĵ)$ m/s^2, starts at t = 0.
$a_x = 8.0$ m/s^2, $a_y = 2.0$ m/s^2
$v_{ox} = 0$, $v_{oy} = 10.0$ m/s

Position equations:
$x(t) = v_{ox}\, t + \frac{1}{2}\, a_x\, t^2 = 0 + \frac{1}{2} \times 8.0 \times t^2 = 4.0\, t^2$
$y(t) = v_{oy}\, t + \frac{1}{2}\, a_y\, t^2 = 10.0t + \frac{1}{2} \times 2.0 \times t^2 = 10.0t + t^2$

(a) When x = 16 m:
$4.0\, t^2 = 16$
$t^2 = 4 \Rightarrow t = 2.0$ s

y-coordinate at t = 2.0 s:
$y = 10.0 \times 2.0 + (2.0)^2 = 20 + 4 = 24$ m

(b) Speed at t = 2.0 s:
$v_x = v_{ox} + a_x\, t = 0 + 8.0 \times 2.0 = 16.0$ m/s
$v_y = v_{oy} + a_y\, t = 10.0 + 2.0 \times 2.0 = 14.0$ m/s
Speed $= \sqrt{(v_x^2 + v_y^2)} = \sqrt{(16^2 + 14^2)} = \sqrt{(256 + 196)} = \sqrt{452} \approx 21.3$ m/s

Answer: (a) t = 2.0 s; y = 24 m. (b) Speed ≈ 21.3 m/s.

Q19 î and ĵ are unit vectors along x- and y-axis respectively. What is the magnitude and direction of the vectors î + ĵ and î − ĵ? What are the components of a vector A = 2î + 3ĵ along the directions of î + ĵ and î − ĵ? [You may use graphical method]

SOLUTION

Vector (î + ĵ):
Magnitude $= \sqrt{(1^2 + 1^2)} = \sqrt{2} \approx 1.414$
Direction: angle with x-axis = arctan(1/1) = 45° (in the first quadrant, i.e., NE direction)

Vector (î − ĵ):

Magnitude = $\sqrt{(1^2 + (-1)^2)} = \sqrt{2} \approx 1.414$

Direction: angle with x-axis = $\arctan(-1/1) = -45°$ (i.e., 45° below x-axis, SE direction)

Unit vector along $(\hat{i} + \hat{j})$: $\hat{n}_1 = (\hat{i} + \hat{j})/\sqrt{2}$

Unit vector along $(\hat{i} - \hat{j})$: $\hat{n}_2 = (\hat{i} - \hat{j})/\sqrt{2}$

Given $A = 2\hat{i} + 3\hat{j}$

Component of A along $(\hat{i} + \hat{j})$:

$= A \cdot \hat{n}_1 = (2\hat{i} + 3\hat{j}) \cdot (\hat{i} + \hat{j})/\sqrt{2} = (2 \times 1 + 3 \times 1)/\sqrt{2} = 5/\sqrt{2} \approx 3.54$

Component of A along $(\hat{i} - \hat{j})$:

$= A \cdot \hat{n}_2 = (2\hat{i} + 3\hat{j}) \cdot (\hat{i} - \hat{j})/\sqrt{2} = (2 \times 1 + 3 \times (-1))/\sqrt{2} = (2 - 3)/\sqrt{2} = -1/\sqrt{2} \approx -0.707$

The negative sign means the component is in the direction opposite to $(\hat{i} - \hat{j})$, i.e., along $(\hat{j} - \hat{i})$.

Answer: $|\hat{i} + \hat{j}| = \sqrt{2}$ at 45° from x-axis; $|\hat{i} - \hat{j}| = \sqrt{2}$ at −45° from x-axis. Component of A along $(\hat{i} + \hat{j}) = 5/\sqrt{2} \approx 3.54$; component along $(\hat{i} - \hat{j}) = -1/\sqrt{2} \approx -0.707$.

Q20 For any arbitrary motion in space, which of the following relations are true:

(a) $v_{average} = (1/2)(v(t_1) + v(t_2))$

(b) $v_{average} = [r(t_2) - r(t_1)]/(t_2 - t_1)$

(c) $v(t) = v(0) + at$

(d) $r(t) = r(0) + v(0)t + (1/2)at^2$

(e) $a_{average} = [v(t_2) - v(t_1)]/(t_2 - t_1)$

(The 'average' stands for average of the quantity over the time interval t_1 to t_2)

SOLUTION

(a) FALSE in general.

$v_{average} = (1/2)(v(t_1) + v(t_2))$ holds only when acceleration is constant (uniform). For arbitrary motion (variable acceleration), the average velocity is not simply the arithmetic mean of initial and final velocities.

(b) TRUE (always).

By definition, the average velocity over any time interval is the total displacement divided by the time interval:

$v_{average} = [r(t_2) - r(t_1)] / (t_2 - t_1)$

This is the definition and is universally valid for any motion.

(c) FALSE in general.
$v(t) = v(0) + at$ is valid only when acceleration a is constant. For arbitrary motion, acceleration can vary with time, and this equation does not hold.

(d) FALSE in general.
$r(t) = r(0) + v(0)t + (1/2)at^2$ is valid only for constant acceleration. For arbitrary motion, this equation does not apply.

(e) TRUE (always).
The average acceleration is defined as the change in velocity divided by the time interval:
$a_{average} = [v(t_2) - v(t_1)] / (t_2 - t_1)$
This definition holds for any motion, uniform or non-uniform.

Answer: True: (b) and (e). False (only valid for constant acceleration): (a), (c), (d).

Q21 Read each statement below carefully and state, with reasons and examples, if it is true or false:
A scalar quantity is one that
(a) is conserved in a process
(b) can never take negative values
(c) must be dimensionless
(d) does not vary from one point to another in space
(e) has the same value for observers with different orientations of axes.

SOLUTION

(a) FALSE.
Being a scalar has nothing to do with conservation. Kinetic energy (scalar) is not conserved in inelastic collisions. Temperature (scalar) changes when a body is heated. Scalars can increase, decrease, or remain constant.

(b) FALSE.
Scalars can be negative. Examples: temperature can be −20°C; potential energy can be negative; electric charge can be negative (−e for electron). A scalar only lacks direction; it can have any sign.

(c) FALSE.
Most scalars have dimensions. Mass (kg), energy (J), temperature (K), speed (m/s) are all scalars with dimensions. Being dimensionless is not a requirement for a scalar.

(d) FALSE.
Scalar quantities can vary from point to point in space. Temperature, pressure, density, and gravitational potential energy all vary spatially — they are scalar fields.

(e) TRUE.
This is in fact the correct characteristic of a scalar. When the coordinate axes are rotated, a scalar quantity (like mass, speed, or temperature) does not change its value. A vector, by contrast, has components that change upon rotation, though its magnitude remains the same. The invariance of value under rotation of axes is what fundamentally distinguishes a scalar from a vector.

Answer: (a) False. (b) False. (c) False. (d) False. (e) True — invariance under rotation of axes is the defining property of a scalar.

Q22 An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30°, what is the speed of the aircraft?

SOLUTION

Given: height h = 3400 m, time interval Δt = 10.0 s, angle subtended at observer O = 30°.

Assume the aircraft flies in a straight horizontal line at constant height h. Let O be the ground observer directly below the midpoint of the flight path (symmetric configuration).

Let the aircraft be at position A at t = 0 and at position B at t = 10 s.
Distance AB = v × 10 (where v is the aircraft speed).

Since O is below the midpoint, the geometry gives:
Half the angle = 15°
$\tan(15°) = (AB/2) / h = (v \times 5) / 3400$

Solving for v:
$v \times 5 = 3400 \times \tan(15°)$
$\tan(15°) = \tan(45° - 30°) = (1 - 1/\sqrt{3})/(1 + 1/\sqrt{3}) = (\sqrt{3} - 1)/(\sqrt{3} + 1) \approx 0.2679$
$v = 3400 \times 0.2679 / 5 = 910.9 / 5 \approx 182.2$ m/s

Converting: v ≈ 182 m/s ≈ 655 km/h

Answer: Speed of aircraft ≈ 182 m/s (≈ 655 km/h).