Chapter 5: Work, Energy and Power

Exercises — Exercises

Q1 The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:
(a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.
(b) work done by gravitational force in the above case,
(c) work done by friction on a body sliding down an inclined plane,
(d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,
(e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

SOLUTION

$W = F \cdot d = Fd \cos\theta$, where θ is the angle between the force and the displacement.

(a) The man pulls the rope upward; the bucket moves upward. Force and displacement are in the same direction ($\theta = 0°$). W = positive.

(b) Gravitational force acts downward; the bucket moves upward. Force and displacement are in opposite directions ($\theta = 180°$). W = negative.

(c) When a body slides down an inclined plane, friction acts up the slope (opposing motion), while displacement is down the slope. $\theta = 180°$. $W_{friction}$ = negative.

(d) For uniform velocity on a rough surface, the applied force is in the direction of motion ($\theta = 0°$). $W_{applied}$ = positive.

(e) The resistive force of air always opposes the motion of the pendulum bob. At every instant, the force and displacement are in opposite directions ($\theta = 180°$). W_{air} = negative.

Answer: (a) Positive (b) Negative (c) Negative (d) Positive (e) Negative

Q2 A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the
(a) work done by the applied force in 10 s,
(b) work done by friction in 10 s,
(c) work done by the net force on the body in 10 s,
(d) change in kinetic energy of the body in 10 s,
and interpret your results.

SOLUTION

Given: $m = 2$ kg, $F_{applied} = 7$ N, $\mu_k = 0.1$, $u = 0$, $t = 10$ s, $g = 9.8$ m/s²

Friction force: $f = \mu_k \times m \times g = 0.1 \times 2 \times 9.8 = 1.96$ N

Net force: $F_{net} = F - f = 7 - 1.96 = 5.04$ N

Acceleration: $a = F_{net} / m = 5.04 / 2 = 2.52$ m/s²

Displacement in 10 s (from rest): $s = \frac{1}{2}at^2 = \frac{1}{2} \times 2.52 \times 100 = 126$ m

(a) Work by applied force: $W_F = F \times s = 7 \times 126 = 882$ J

(b) Work by friction: $W_f = -f \times s = -1.96 \times 126 = -246.96$ J ≈ −247 J
(The negative sign shows friction opposes motion.)

(c) Work by net force: $W_{net} = F_{net} \times s = 5.04 \times 126 = 635.04$ J ≈ 635 J

(d) Final velocity: $v = at = 2.52 \times 10 = 25.2$ m/s
$\Delta KE = \frac{1}{2}mv^2 - 0 = \frac{1}{2} \times 2 \times (25.2)^2 = 635.04$ J ≈ 635 J

Interpretation: ΔKE equals the work done by the net force (Work-Energy theorem). $W_{net} = W_F + W_f = 882 - 247 = 635$ J.

Answer: (a) 882 J (b) −247 J (c) 635 J (d) ΔKE = 635 J [Work-Energy theorem confirmed: ΔKE = W_net]

Q3 Given in Fig. 5.11 are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.

SOLUTION

A particle cannot be in a region where $V(x) > E$ (total energy), since that would require negative kinetic energy, which is physically impossible. $K = E - V(x) \geq 0$.

(a) Graph (a): $V(x) = V_o$ (a constant positive value) for $x > 0$; $V = 0$ for $x < 0$. The total energy E is shown as a cross between 0 and V_o, i.e. $0 < E < V_o$.
Forbidden region: $x > 0$, since $V(x) = V_o > E$ there.
The particle is confined to $x < 0$.
Minimum total energy: $-\infty$ (there is no lower bound on V in this region — the particle can have any negative kinetic energy approaching zero). Practically, the minimum energy to exist in the classically accessible region is $E \to 0^+$.
Physical context: A particle approaching a potential barrier (e.g. step barrier).

(b) Graph (b): V(x) is an upward staircase. For $x < 0$ it sits at the lowest step level; at $x = 0$ it jumps to V_o, then rises through further steps at x = a, c, b and reaches its peak V_3 for $b < x < d$, before dropping to an intermediate level (above V_o) for $x > d$. The total energy E is marked on the ordinate BELOW even the lowest ($x < 0$) step.
Since E lies below the minimum value of V(x), we have $V(x) > E$ for every x. That would require $K = E - V(x) < 0$ everywhere.
Forbidden region: the entire region — the particle cannot be found anywhere for the given E.
Minimum total energy the particle must have to exist = the minimum value of V(x), i.e. the level of the lowest ($x < 0$) step.

(c) Graph (c): V(x) is a finite potential well. $V = V_o$ (constant, positive) for $x < a$; $V = -V_1$ (constant, negative) inside the well $a < x < b$; $V = V_o$ again for $x > b$. The total energy E is marked positive, between 0 and V_o.
The particle cannot be where $V(x) > E$. Since $V = V_o > E$ both for $x < a$ and for $x > b$, those regions are forbidden.
Forbidden regions: $x < a$ and $x > b$ (everywhere outside the well). Inside the well ($a < x < b$), $V = -V_1 < E$, so the particle is confined there and oscillates within the

well.
Minimum total energy: $-V_1$ (the bottom of the well, the minimum value of V).
Physical context: Particle bound in a finite potential well.

(d) Graph (d): V(x) has two symmetric humps that rise to a positive peak near $x = \pm a/2$, a well of depth $-V_1$ in the middle ($-a/2 < x < a/2$), and it decays to 0 as $x \to \pm\infty$ (crossing zero near $x = \pm b/2$). The total energy E is marked as a small POSITIVE value on the ordinate.
The particle cannot be where $V(x) > E$. The two humps rise above E, so the barrier regions on either side of the central well are forbidden.
Forbidden regions: $-b/2 < x < -a/2$ and $a/2 < x < b/2$ (the two humps where $V(x) > E$). The particle CAN be found in the central well (near $x = 0$, where $V = -V_1 < E$) and in the outer regions $|x| > b/2$ (where $V < E$).
Minimum total energy: $-V_1$ (the bottom of the central well, the minimum value of V).
Physical context: A particle scattering off / bound by a repulsive barrier with an attractive well at the centre.

Answer: (a) Forbidden region: $x > 0$; min energy = 0. (b) $V(x) > E$ everywhere (E is below the lowest step), so the particle cannot exist in any region for this E; min energy = the lowest step value of V(x). (c) Forbidden regions: $x < a$ and $x > b$ (outside the well); particle confined to $a < x < b$; min energy = $-V_1$. (d) Forbidden regions: the two humps $-b/2 < x < -a/2$ and $a/2 < x < b/2$; particle allowed in the central well and for $|x| > b/2$; min energy = $-V_1$.

Q4 The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = kx^2/2$, where k is the force constant of the oscillator. For $k = 0.5$ N m^{-1}, the graph of V(x) versus x is shown in Fig. 5.12. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches $x = \pm 2$ m.

SOLUTION

Given: $V(x) = kx^2/2$, $k = 0.5$ N m^{-1}, $E = 1$ J.

The kinetic energy at position x is: $K(x) = E - V(x) = 1 - (0.5 \times x^2/2) = 1 - x^2/4$

A particle 'turns back' (i.e., momentarily stops and reverses direction) when $K(x) = 0$.

Setting $K(x) = 0$:

$1 - x^2/4 = 0$

$x^2/4 = 1$

$x^2 = 4$

$x = \pm 2$ m

At x = ±2 m, the potential energy equals the total energy:
$V(\pm 2) = 0.5 \times (2)^2 / 2 = 0.5 \times 4 / 2 = 1$ J = E

All the energy is potential; kinetic energy is zero. The particle cannot go beyond x = ±2 m because that would require K < 0, which is impossible.
Therefore, the particle must turn back at x = ±2 m. This confirms the amplitude of oscillation is 2 m.

Answer: At x = ±2 m, $V = kx^2/2 = 0.5 \times 4/2 = 1$ J = E (total energy). K = 0 at these points, so the particle turns back. It cannot exist beyond |x| = 2 m.

Q5 Answer the following :
(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?
(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why ?
(c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth ?
(d) In Fig. 5.13(i) the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. 5.13(ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater ?

SOLUTION

(a) The heat energy for burning the rocket casing comes at the expense of the rocket's kinetic energy (and hence its fuel). The friction between the rocket and the atmosphere converts some of the rocket's kinetic energy into heat. The atmosphere does not supply the energy — it is the agent of the frictional force that acts on the rocket (i.e., the rocket does work against friction). So the energy comes from the rocket itself.

(b) Gravity is a conservative force. The work done by a conservative force depends only on the initial and final positions, not on the path taken. Over a complete orbit the comet returns to exactly the same position it started from (initial = final position). Therefore, the work done by the gravitational force over one complete orbit is zero.

(c) As the satellite spirals inward due to atmospheric drag, it loses total mechanical energy (E = KE + PE). However, for a circular orbit, the PE = −2 × KE. When the satellite moves to a lower orbit, its potential energy decreases by twice as much as its kinetic energy increases. So even as total energy decreases (becomes more negative), the kinetic energy increases and hence speed increases. The satellite 'falls' into a lower, faster orbit — the energy lost to drag comes from the (larger) decrease in gravitational potential energy.

(d) In case (i), the man carries the 15 kg mass horizontally (2 m walk). The force he exerts is vertical (upward, to support the weight mg), but the displacement is horizontal. These are perpendicular, so the work done against gravity is zero. (No work is done in the physics sense.)
In case (ii), the man pulls the rope horizontally while the mass on the other side of the pulley is lifted vertically. As he pulls the rope 2 m horizontally, the mass rises by 2 m vertically. Work done = mgh = 15 × 9.8 × 2 = 294 J.
Therefore, work done is greater in case (ii).

Answer: (a) At the expense of the rocket's kinetic energy (its fuel). (b) Gravity is conservative; work over a closed path is zero. (c) As orbit lowers, PE decreases more than KE increases — net loss goes to drag, but speed still rises. (d) Case (ii): work done = mgh = 15 × 9.8 × 2 = 294 J; case (i): zero work by gravity.

Q6 Underline the correct alternative :
(a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.
(b) Work done by a body against friction always results in a loss of its kinetic/potential energy.
(c) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system.
(d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.

SOLUTION

(a) For a conservative force, $\Delta V = -W$ (the change in potential energy equals negative of work done by the force). If the force does positive work ($W > 0$), then $\Delta V = -W < 0$. The potential energy decreases.
Answer: decreases.

(b) Friction is a non-conservative force. The work done against friction converts kinetic energy into heat/internal energy. It is the kinetic energy that decreases directly (the body slows down). Potential energy is associated with conservative forces.
Answer: kinetic energy.

(c) By Newton's Third Law, all internal forces between particles in a system form action-reaction pairs that cancel. Only external forces change the total momentum. Newton's Second Law for a system: $F_{ext} = dp_{total}/dt$.
Answer: external force.

(d) In an inelastic collision, kinetic energy is not conserved (some is converted to heat/deformation). Total energy is conserved (1st law of thermodynamics: energy transforms but total is conserved). Total linear momentum is always conserved in any collision (no net external force).
Answer: total linear momentum.

Answer: (a) decreases (b) kinetic energy (c) external force (d) total linear momentum

Q7 State if each of the following statements is true or false. Give reasons for your answer.
(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.
(b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.
(c) Work done in the motion of a body over a closed loop is zero for every force in nature.
(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

SOLUTION

(a) FALSE. In an elastic collision, the total momentum of the system and the total kinetic energy of the system are conserved. However, the momentum and energy of each individual body change during the collision (each body experiences a force and hence a change in momentum and energy). What is conserved is the sum.

(b) FALSE. The total energy of a system is conserved only when no external forces do work on the system (i.e., the system is isolated). If external forces act on the system, they can transfer energy into or out of the system, and the total mechanical energy of the system changes. The total energy of the universe is conserved, but not necessarily that of a sub-system with external forces acting on it.

(c) FALSE. This is true only for conservative forces (like gravity, spring force, electrostatic force). For non-conservative forces (like friction, viscosity), the work done over a closed loop is not zero — energy is dissipated as heat.

(d) TRUE. In an inelastic collision, some kinetic energy is converted into other forms (heat, sound, deformation). Therefore, the final kinetic energy is always less than the initial kinetic energy. (In a perfectly inelastic collision, the loss is maximum; in a partially inelastic collision, some KE is lost.)

Answer: (a) False — individual momenta/energies change; only totals are conserved. (b) False — total energy is conserved only for an isolated system. (c) False — only true for conservative forces. (d) True — kinetic energy is lost to heat/deformation.

Q8 Answer carefully, with reasons :
(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e. when they are in contact) ?
(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls ?
(c) What are the answers to (a) and (b) for an inelastic collision ?
(d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic ? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

SOLUTION

(a) NO. During the contact time, the balls deform and compress. Kinetic energy is converted into elastic potential energy of deformation (like a compressed spring). The total kinetic energy is NOT conserved during the collision — it is temporarily stored as potential energy. After the collision is over and the balls separate, the potential energy converts back to kinetic energy, and the total kinetic energy is again equal to the initial value. Kinetic energy conservation applies only before and after, not during.

(b) YES. Total linear momentum is conserved at every instant of the collision, including during the contact phase. By Newton's Third Law, the mutual forces between the balls are equal and opposite at every instant, so the total impulse on the system is zero. Hence total linear momentum is conserved throughout.

(c) For an inelastic collision:
(a) Kinetic energy is NOT conserved during the collision (same reasoning as above — additionally, some energy is permanently converted to heat/deformation and is not recovered).
(b) Total linear momentum IS still conserved during the collision (Newton's Third Law applies regardless of elasticity).

(d) If the potential energy depends only on the separation distance r between the centres, then the force $F = -dV/dr$ is a central conservative force (depends only on r). For a conservative force, the total mechanical energy (KE + PE) is conserved. At the start and end of the collision (when balls are far apart and PE = 0), total KE is conserved. This means the collision is ELASTIC.

Answer: (a) No — KE converts to elastic PE during contact. (b) Yes — conserved at every instant (Newton's 3rd Law). (c) Inelastic: KE not conserved during or after; momentum conserved at all times. (d) Elastic — the force is conservative, so total mechanical energy (and hence KE when far apart) is conserved.

Q9 A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to
(i) t^(1/2) (ii) t (iii) t^(3/2) (iv) t^2

SOLUTION

Given: body starts from rest, constant acceleration a.

Velocity at time t: v = u + at = 0 + at = at

Force on body (constant): F = ma

Power delivered: P = F × v = (ma) × (at) = ma^2t

Since m and a are constants, P ∝ t.

Answer: (ii) t

Answer: Power P = F·v = (ma)(at) = $ma^2t \propto t$. Correct option: (ii) t

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Q10 A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to
(i) t^(1/2) (ii) t (iii) t^(3/2) (iv) t^2

SOLUTION

Given: constant power P applied to a body of mass m.

Power = Force × velocity = mav, so: mav = P (constant)
=> mv(dv/dt) = P
=> v dv = (P/m) dt

Integrating from 0 to t (initial velocity = 0):
$v^2/2 = (P/m) \times t$
=> $v = \sqrt{(2Pt/m)} \propto t^{(1/2)}$

Now, dx/dt = v ∝ t^(1/2)
Integrating:
$x = \int_0 v\,dt \propto \int_0 t^{(1/2)}\,dt = (2/3)\,t^{(3/2)}$

Therefore, displacement x ∝ t^(3/2).

Answer: (iii) t^(3/2)

Answer: Displacement x ∝ t^(3/2). Correct option: (iii) t^(3/2)

Q11 A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by
F = −î + 2ĵ + 3k̂ N
where î, ĵ, k̂ are unit vectors along the x-, y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis ?

SOLUTION

Given: F = (−1 î + 2 ĵ + 3 k̂) N
Displacement: d = 4 k̂ m (along z-axis only)

Work done: W = F · d
= (−1 î + 2 ĵ + 3 k̂) · (4 k̂)
= (−1)(0) + (2)(0) + (3)(4)
= 0 + 0 + 12
= 12 J

Note: The x- and y-components of the force do no work since there is no displacement in those directions. Only the z-component (3 N) does work.

Answer: W = F · d = 3 × 4 = 12 J

Q12 An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton ? Obtain the ratio of their speeds. (electron mass = 9.11 × 10^{-31} kg, proton mass = 1.67 × 10^{-27} kg, 1 eV = 1.60 × 10^{-19} J).

SOLUTION

Given: K_e = 10 keV = 10 × 10^3 × 1.60 × 10^{-19} J = 1.60 × 10^{-15} J
K_p = 100 keV = 100 × 10^3 × 1.60 × 10^{-19} J = 1.60 × 10^{-14} J
m_e = 9.11 × 10^{-31} kg, m_p = 1.67 × 10^{-27} kg

Using K = ½mv²: v = √(2K/m)

Speed of electron:
v_e = √(2 × 1.60 × 10^{-15} / 9.11 × 10^{-31})

$= \sqrt{(3.20 \times 10^{-15} / 9.11 \times 10^{-31})}$
$= \sqrt{(3.513 \times 10^{15})}$
$= 5.93 \times 10^{7}$ m/s

Speed of proton:
$v_p = \sqrt{(2 \times 1.60 \times 10^{-14} / 1.67 \times 10^{-27})}$
$= \sqrt{(3.20 \times 10^{-14} / 1.67 \times 10^{-27})}$
$= \sqrt{(1.916 \times 10^{13})}$
$= 4.38 \times 10^{6}$ m/s

The electron is faster.

Ratio of speeds:
$v_e / v_p = 5.93 \times 10^{7} / 4.38 \times 10^{6} = 13.54 \approx 13.5$

Alternatively: $v_e/v_p = \sqrt{(K_e \times m_p / (K_p \times m_e))}$
$= \sqrt{(10 \times 1.67 \times 10^{-27} / (100 \times 9.11 \times 10^{-31}))}$
$= \sqrt{(1.67 \times 10^{-26} / 9.11 \times 10^{-29})}$
$= \sqrt{(183.3)} = 13.5$

Answer: The electron is faster. $v_e/v_p = \sqrt{(K_e \times m_p / K_p \times m_e)} \approx 13.5$

Q13 A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey ? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 m s^{-1} ?

SOLUTION

Given: $r = 2$ mm $= 2 \times 10^{-3}$ m, $h = 500$ m, $v_{final} = 10$ m/s, $\rho_{water} = 1000$ kg/m^3, $g = 9.8$ m/s^2

Mass of the drop:
$m = \rho \times (4/3)\pi r^3 = 1000 \times (4/3)\pi \times (2 \times 10^{-3})^3$
$= 1000 \times (4/3)\pi \times 8 \times 10^{-9}$
$= 1000 \times 3.351 \times 10^{-8}$
$= 3.35 \times 10^{-5}$ kg

Work done by gravity depends only on the vertical displacement, not on speed or acceleration:

First half (250 m): $W_{g1} = mgh_1 = 3.35 \times 10^{-5} \times 9.8 \times 250 = 0.082$ J

Second half (250 m): $W_{g2} = mgh_2 = 3.35 \times 10^{-5} \times 9.8 \times 250 = 0.082$ J

Work by gravitational force is the same in both halves (same vertical drop).

For the work done by the resistive force over the entire journey:
Using the work-energy theorem: $W_{total} = \Delta KE$
$W_{gravity} + W_{resistance} = KE_{final} - KE_{initial}$

$W_{gravity}$ (total) $= mgh = 3.35 \times 10^{-5} \times 9.8 \times 500 = 0.164$ J
$KE_{final} = \frac{1}{2}mv^2 = \frac{1}{2} \times 3.35 \times 10^{-5} \times (10)^2 = 1.675 \times 10^{-3}$ J
$KE_{initial} = 0$ (starts from rest)

$W_{resistance} = \Delta KE - W_{gravity} = 1.675 \times 10^{-3} - 0.164 = -0.162$ J

The resistive force does negative work (opposes motion).

Answer: Work by gravity: first half = 0.082 J, second half = 0.082 J. Work by resistive force (full journey) = −0.162 J

Q14 A molecule in a gas container hits a horizontal wall with speed 200 m s^{-1} and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision ? Is the collision elastic or inelastic ?

SOLUTION

The molecule hits the wall at 30° to the normal and rebounds at 30° to the normal with the same speed (200 m/s).

Components of velocity (let normal be x-direction, wall surface be y-direction):
Before: $v_x = -200 \cos 30°$ (towards wall), $v_y = 200 \sin 30°$ (along wall)
After: $v_x = +200 \cos 30°$ (away from wall), $v_y = 200 \sin 30°$ (along wall, unchanged)

Change in momentum of molecule:
$\Delta p_y = 0$ (tangential component unchanged)
$\Delta p_x = m \times [200 \cos 30° - (-200 \cos 30°)] = 2m \times 200 \cos 30° \neq 0$

The momentum of the molecule alone is NOT conserved — it changes in the normal direction.
However, the total momentum of the system (molecule + wall/Earth) IS conserved, because the wall exerts a force on the molecule (and by Newton's Third Law, the molecule exerts an equal and opposite force on the wall/Earth).

Is the collision elastic?
Initial KE = ½m(200)²
Final KE = ½m(200)² (same speed, hence same KE)
Since kinetic energy is conserved, the collision is ELASTIC.

Answer: Momentum of the molecule alone is not conserved (wall exerts impulse); total momentum of molecule + wall system is conserved. The collision is elastic (same speed before and after, so KE is conserved).

Q15 A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m³ in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump ?

SOLUTION

Given: V = 30 m³, t = 15 min = 900 s, H = 40 m, η = 30% = 0.30
ρ_water = 1000 kg/m³, g = 9.8 m/s²

Mass of water: m = ρV = 1000 × 30 = 3 × 10⁴ kg

Useful work done (against gravity): W = mgh = 3 × 10⁴ × 9.8 × 40 = 1.176 × 10⁷ J

Useful power output:
P_useful = W/t = 1.176 × 10⁷ / 900 = 1.307 × 10⁴ W ≈ 13.07 kW

Since efficiency η = P_useful / P_consumed:
P_consumed = P_useful / η = 1.307 × 10⁴ / 0.30 = 4.356 × 10⁴ W ≈ 43.56 kW

Answer: Electric power consumed = P_useful / η = (mgh/t) / 0.30 = 4.36 × 10⁴ W ≈ 43.6 kW

Q16 Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V. If the collision is elastic, which of the following (Fig. 5.14) is a possible result after collision ?
(i) The first ball stops; balls 2 and 3 move together at V/2.
(ii) The first ball stops; ball 2 remains at rest; ball 3 moves at V.
(iii) All three balls move.

SOLUTION

Let each ball have mass m. Initial state: ball 1 moves at V, balls 2 and 3 at rest.

Check case (i): balls 2 and 3 move together at V/2.

Momentum: mV = m(V/2) + m(V/2) = mV ✓ conserved
KE initial = ½mV²
KE final = ½m(V/2)² + ½m(V/2)² = mV²/4
KE is NOT conserved. This violates the elastic collision condition. Case (i) is NOT possible.

Check case (ii): ball 1 stops, ball 2 at rest, ball 3 moves at V.
Momentum: mV = m(0) + m(0) + m(V) = mV ✓ conserved
KE initial = ½mV²
KE final = ½mV² ✓ conserved
Both momentum and kinetic energy are conserved. Case (ii) IS possible.

Physical explanation: Ball 1 undergoes elastic collision with ball 2 (equal masses, head-on): ball 1 stops, ball 2 gets speed V. Ball 2 immediately undergoes elastic collision with ball 3 (equal masses, head-on): ball 2 stops, ball 3 gets speed V. Net result: ball 1 stops, ball 2 stops, ball 3 moves at V.

Answer: Case (ii) is the correct result.

Answer: Case (ii): ball 1 stops, ball 2 remains at rest, ball 3 moves at V. This conserves both momentum (mV = mV) and kinetic energy (½mV² = ½mV²).

Q17 The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. 5.15. How high does the bob A rise after the collision ? Neglect the size of the bobs and assume the collision to be elastic.

SOLUTION

Let m be the mass of each bob, L be the length of the pendulum.

Speed of bob A just before collision:
Using energy conservation from release point (height h = L − L cos 30° = L(1 − cos 30°)):
$v_A = \sqrt{(2g \times L(1 - \cos 30°))}$

Elastic collision between equal masses (bob A moving at v_A, bob B at rest):
In an elastic collision between two equal masses where one is at rest, the moving mass comes to a complete stop and the stationary mass moves off with the initial speed.

$v_{A_after} = 0$, $v_{B_after} = v_A$

Since bob A comes to rest immediately after the collision, it has zero kinetic energy.
Bob A therefore does not rise at all — it remains at rest at the table level.

Height risen by bob A after collision = 0.

Answer: Bob A comes to rest after the elastic collision (equal masses, one initially at rest). Bob A does not rise — height = 0.

Q18 The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance ?

SOLUTION

Given: L = 1.5 m, bob released from horizontal (height h = L above lowest point), 5% energy dissipated.
g = 9.8 m/s²

Initial potential energy (taking lowest point as reference): PE = mgL

Energy available as KE at the lowest point (after 5% loss):
KE = 0.95 × mgL

½mv² = 0.95 × mgL
v² = 2 × 0.95 × gL = 1.90 × 9.8 × 1.5 = 27.93 m²/s²
v = √27.93 = 5.28 m/s

Answer: v = √(2 × 0.95 × gL) = √(1.90 × 9.8 × 1.5) = √27.93 ≈ 5.28 m/s

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Q19 A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the floor of the trolley at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty ?

SOLUTION

Given: m_trolley = 300 kg, m_sand = 25 kg, v_initial = 27 km/h = 7.5 m/s, frictionless track.

The sand leaks out through a hole in the floor of the trolley — it falls vertically downward (in the ground frame it falls with the horizontal speed of the trolley).

At the moment any sand grain leaves the trolley, it has the same horizontal velocity as the trolley (since it is on the moving trolley). The leaked sand carries away horizontal momentum proportional to its own mass × speed, but there is no external horizontal force on the system.

The key point: the sand leaks out, not pushed backward. Each grain leaves with the current speed of the trolley (not relative to it). There is no horizontal reaction force on the trolley.

By conservation of horizontal momentum of the system (no external horizontal force, frictionless track):
The speed of the trolley does not change.

Speed after sandbag is empty = 27 km/h = 7.5 m/s.

Answer: Speed of trolley remains 27 km/h (7.5 m/s). The sand leaks out with the same horizontal velocity as the trolley, so there is no net horizontal force; momentum and speed of the trolley are unchanged.

Q20 A body of mass 0.5 kg travels in a straight line with velocity v = ax^(3/2) where a = 5 m^(−1/2) s^(−1). What is the work done by the net force during its displacement from x = 0 to x = 2 m ?

SOLUTION

Given: m = 0.5 kg, v = ax^(3/2) where a = 5 m^(−1/2) s^(−1)

Using the Work-Energy Theorem: $W_{net} = \Delta KE = KE_f - KE_i$

At x = 0: $v_0 = a \times 0^{(3/2)} = 0$, so $KE_i = 0$

At x = 2 m: $v_f = 5 \times (2)^{(3/2)} = 5 \times 2\sqrt{2} = 10\sqrt{2}$ m/s
$v_f^2 = (10\sqrt{2})^2 = 200$ m²/s²

$KE_f = \frac{1}{2}mv_f^2 = \frac{1}{2} \times 0.5 \times 200 = 50$ J

Work done = ΔKE = 50 − 0 = 50 J

Answer: $W = \Delta KE = \frac{1}{2}m \times v_f^2 - 0 = \frac{1}{2} \times 0.5 \times (10\sqrt{2})^2 = \frac{1}{2} \times 0.5 \times 200 = 50$ J

Q21 The blades of a windmill sweep out a circle of area A. (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t ? (b) What is the kinetic energy of the air ? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that A = 30 m^2, v = 36 km/h and the density of air is 1.2 kg m^{-3}. What is the electrical power produced ?

SOLUTION

(a) Volume of air swept in time t: Volume = A × v × t
Mass: m = ρ × A × v × t
where ρ = density of air

(b) Kinetic energy of air:
KE = $\frac{1}{2}mv^2 = \frac{1}{2} \times (\rho Avt) \times v^2 = \frac{1}{2}\rho Av^3t$

(c) Given: A = 30 m^2, v = 36 km/h = 10 m/s, ρ = 1.2 kg/m^3

KE per unit time (power in wind): $P_{wind} = \frac{1}{2}\rho Av^3$
$= \frac{1}{2} \times 1.2 \times 30 \times (10)^3$
$= \frac{1}{2} \times 1.2 \times 30 \times 1000$
= 18000 W

Electrical power = 25% × P_{wind} = 0.25 × 18000 = 4500 W = 4.5 kW

Answer: (a) m = ρAvt (b) KE = $\frac{1}{2}\rho Av^3t$ (c) Electrical power = 0.25 × $\frac{1}{2}\rho Av^3$ = 0.25 × 18000 = 4500 W = 4.5 kW

Q22 A person trying to lose weight (dieter) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each time. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force ? (b) Fat supplies 3.8×10^7 J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up ?

SOLUTION

Given: m = 10 kg, h = 0.5 m, n = 1000 lifts, g = 9.8 m/s^2
Fat energy = 3.8×10^7 J/kg, efficiency = 20% = 0.20

(a) Work done against gravity per lift = mgh = 10 × 9.8 × 0.5 = 49 J
Total work = n × mgh = 1000 × 49 = 49000 J = 49 kJ

(Note: when lowering, the PE is 'dissipated' — the dieter does not recover this energy. But work done against gravity on the way up is what we count.)

(b) The mechanical energy output is 49000 J.
Since efficiency = 20%, the energy from fat consumed:
$E_{fat} = W_{useful} / \text{efficiency} = 49000 / 0.20 = 245000 \text{ J} = 2.45 \times 10^5 \text{ J}$

Fat consumed: $m_{fat} = E_{fat} / (3.8 \times 10^7) = 2.45 \times 10^5 / 3.8 \times 10^7$
$= 6.45 \times 10^{-3}$ kg ≈ 6.45 g

Answer: (a) Work done against gravity = 49 kJ. (b) Fat consumed = 49000 / (0.20 × 3.8×10^7) ≈ 6.45×10^{-3} kg ≈ 6.45 g

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Q23 A family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW? (b) Compare this area to that of the roof of a typical house.

SOLUTION

(a) Given: $P_{required} = 8 \text{ kW} = 8000 \text{ W}$
Solar incident power per $m^2 = 200 \text{ W/m}^2$
Efficiency = 20% = 0.20

Useful electrical power per $m^2 = 0.20 \times 200 = 40 \text{ W/m}^2$

Area required: $A = P_{required} / (\text{useful power per } m^2) = 8000 / 40 = 200 \text{ m}^2$

(b) A typical house roof has an area of about 50 to 200 m^2.
The required area of 200 m^2 is comparable to the roof area of a large house.
This shows that solar panels covering the entire roof of a reasonably sized house could supply the family's power needs — underlining the practical viability of rooftop solar energy.

Answer: (a) Area required = 200 m^2. (b) This is comparable to the roof area of a large house (~50–200 m^2), showing rooftop solar is a viable option.