Chapter 7: Gravitation

Exercises — Exercises

Q1 Answer the following:
(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?
(b) An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?
(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (you can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why?

SOLUTION

(a) No. Gravitational shielding is not possible.
A hollow metallic conductor shields the interior from external electric fields because free charges on the conductor rearrange to cancel the field inside. Gravitational fields, however, cannot be screened: there is no such thing as a negative gravitational mass that could cancel the field. Every shell of matter contributes a gravitational attraction — it does not neutralise the field of matter outside it. Hence no arrangement of matter can shield a body from external gravitational influences.

(b) Yes, if the space station is large enough.
In a small orbiting space station the astronaut and every point of the station are in essentially the same gravitational field and have the same free-fall acceleration. The net apparent gravity is zero everywhere inside — weightlessness. In a very large station, the gravitational field of the Earth varies measurably from one end of the station to the other. The differential (tidal) acceleration between two points separated by a distance d in the station is approximately $2GM_E\, d / R^3$, where R is the orbital radius. For a sufficiently large d this difference is detectable, so the astronaut can sense the variation and deduce that gravity is present.

(c) The tidal effect depends on the gradient (rate of change) of gravitational field, not just its magnitude.

The tidal acceleration across the Earth due to a body of mass M at distance r is proportional to M/r^3, whereas the gravitational force on the Earth is proportional to M/r^2.

For the Sun: $M_{sun} >> M_{moon}$, but $r_{sun} >> r_{moon}$.

Tidal effect ratio: $(M_{moon} / r_{moon}^3) / (M_{sun} / r_{sun}^3)$.

$M_{moon} \approx 7.3 \times 10^{22}$ kg, $r_{moon} \approx 3.84 \times 10^8$ m.

$M_{sun} \approx 2 \times 10^{30}$ kg, $r_{sun} \approx 1.5 \times 10^{11}$ m.

Tidal ratio = $(M_{moon} \times r_{sun}^3) / (M_{sun} \times r_{moon}^3)$

$= (7.3\times10^{22} \times (1.5\times10^{11})^3) / (2\times10^{30} \times (3.84\times10^8)^3)$

$\approx (7.3\times10^{22} \times 3.375\times10^{33}) / (2\times10^{30} \times 5.66\times10^{25})$

$\approx (2.46\times10^{56}) / (1.13\times10^{56}) \approx 2.2$.

The Moon's tidal effect is about twice that of the Sun because the Moon is much closer. The $1/r^3$ dependence on distance makes the Moon's proximity dominant over the Sun's greater mass for tidal purposes.

Answer: (a) No — gravitational shielding is impossible. (b) Yes — tidal (differential) gravity becomes detectable in a sufficiently large station. (c) Tidal effect $\propto M/r^3$; though $M_{sun} >> M_{moon}$, $r_{sun} >> r_{moon}$, making the Moon's tidal effect about twice the Sun's.

Q2 Choose the correct alternative:

(a) Acceleration due to gravity increases/decreases with increasing altitude.

(b) Acceleration due to gravity increases/decreases with increasing depth (assume the earth to be a sphere of uniform density).

(c) Acceleration due to gravity is independent of mass of the earth/mass of the body.

(d) The formula $-G\,M\,m\,(1/r_2 - 1/r_1)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth.

SOLUTION

(a) Decreases with increasing altitude.

$g(h) = GM_E / (R_E + h)^2$. As h increases, $(R_E + h)^2$ increases, so g(h) decreases.

(b) Decreases with increasing depth.

$g(d) = g(1 - d/R_E)$. As depth d increases, g(d) decreases linearly. g is maximum at the surface and decreases in both directions — going up or going down.

(c) Independent of mass of the body.
From Newton's 2nd law applied to gravitation: $mg = GM_E m / R_E^2$, so $g = GM_E / R_E^2$. The mass m of the body cancels out. Acceleration due to gravity depends on the mass of the Earth (and its radius), not on the mass of the body.

(d) More accurate.
The formula $-GMm(1/r_2 - 1/r_1)$ is derived directly from the inverse-square law without approximation. It is valid for any r_1 and r_2 outside the Earth.
The formula $mg(r_2 - r_1)$ assumes g is constant and the change in height is small compared to R_E. This is an approximation valid only when $(r_2 - r_1) << R_E$.
Therefore $-GMm(1/r_2 - 1/r_1)$ is more accurate for large separations.

Answer: (a) Decreases. (b) Decreases. (c) Mass of the body. (d) More accurate.

Q3 Suppose there existed a planet that went around the Sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?

SOLUTION

Given: The planet's orbital speed is twice the Earth's, i.e., the planet completes one revolution in half the time of the Earth.
So $T_{planet} = T_{earth} / 2$.

By Kepler's Third Law: $T^2 \propto a^3$ (where a = semi-major axis).
$(T_p / T_e)^2 = (a_p / a_e)^3$
$(1/2)^2 = (a_p / a_e)^3$
$1/4 = (a_p / a_e)^3$
$a_p / a_e = (1/4)^{(1/3)} = (1/2)^{(2/3)}$

$a_p / a_e = (0.5)^{(2/3)} = 0.63$

The orbital size of the planet is 0.63 times (about 63%) that of Earth's orbit.

Answer: $a_p / a_e = (1/2)^{(2/3)} \approx 0.63$. The planet's orbital radius is about 0.63 times Earth's orbital radius.

Q4 Io, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth that of the sun.

SOLUTION

Given: $T_{Io} = 1.769 \text{ days} = 1.769 \times 24 \times 3600 \text{ s} = 1.529 \times 10^5$ s
$R_{Io} = 4.22 \times 10^8$ m
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

From Kepler's Third Law applied to Io's orbit around Jupiter:
$T^2 = (4\pi^2 / GM_J) \times R^3$
$M_J = 4\pi^2 R^3 / (G T^2)$

$M_J = (4 \times \pi^2 \times (4.22 \times 10^8)^3) / (6.67 \times 10^{-11} \times (1.529 \times 10^5)^2)$

Numerator: $4\pi^2 \times (4.22)^3 \times 10^{24} = 4 \times 9.870 \times 75.15 \times 10^{24}$
$= 39.48 \times 75.15 \times 10^{24} = 2967 \times 10^{24} = 2.967 \times 10^{27}$

Denominator: $6.67 \times 10^{-11} \times (1.529)^2 \times 10^{10}$
$= 6.67 \times 2.338 \times 10^{-1} = 1.559$

$M_J = 2.967 \times 10^{27} / 1.559 = 1.90 \times 10^{27}$ kg

Mass of the Sun $M_{sun} = 2 \times 10^{30}$ kg
$M_J / M_{sun} = 1.90 \times 10^{27} / 2 \times 10^{30} = 9.5 \times 10^{-4} \approx 1/1000$.

Hence the mass of Jupiter is about one-thousandth that of the Sun. (Proved)

Answer: $M_{Jupiter} \approx 1.90 \times 10^{27}$ kg $\approx M_{sun} / 1000$.

Q5 Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be 10^5 ly.

SOLUTION

Given: Number of stars $N = 2.5 \times 10^{11}$, each of mass $M_{sun} = 2 \times 10^{30}$ kg
Total galactic mass $M = N \times M_{sun} = 2.5 \times 10^{11} \times 2 \times 10^{30} = 5.0 \times 10^{41}$ kg
Orbital radius R = 50,000 ly $= 5 \times 10^4 \times 9.46 \times 10^{15}$ m $= 4.73 \times 10^{20}$ m
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Assuming all galactic mass is concentrated at the centre (Kepler's analogy):
$T^2 = 4\pi^2 R^3 / (GM)$
$T = 2\pi \sqrt{(R^3 / GM)}$

$R^3 = (4.73 \times 10^{20})^3 = 1.058 \times 10^{62}\ m^3$
$GM = 6.67 \times 10^{-11} \times 5.0 \times 10^{41} = 3.335 \times 10^{31}\ m^3\ s^{-2}$

$R^3 / (GM) = 1.058 \times 10^{62} / 3.335 \times 10^{31} = 3.173 \times 10^{30}\ s^2$
$\sqrt{(R^3 / GM)} = 1.781 \times 10^{15}\ s$
$T = 2\pi \times 1.781 \times 10^{15} = 1.12 \times 10^{16}\ s$

Converting to years: 1 year ≈ 3.156×10^7 s
$T = 1.12 \times 10^{16} / 3.156 \times 10^7 \approx 3.55 \times 10^8$ years

Answer: $T \approx 1.12 \times 10^{16}$ s ≈ 3.55×10^8 years.

Q6 Choose the correct alternative:
(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.
(b) The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

SOLUTION

(a) The total energy of an orbiting satellite is the negative of its kinetic energy.
For a satellite in circular orbit at radius r:
KE = GMm / (2r) (positive)
PE = −GMm / r (negative)
Total E = KE + PE = GMm/(2r) − GMm/r = −GMm/(2r)
So E = −KE. The total energy is the negative of the kinetic energy.
(Also note: E = PE/2, so total energy is also half the potential energy — but the question asks which one it is the negative of.)
Answer: kinetic energy.

(b) Less.

An orbiting satellite already has kinetic energy (it is moving in orbit). To escape, it needs additional energy equal to	E_{total}	GMm/(2r).
A stationary object at the same height r has zero KE. Its total energy is just its PE = −GMm/r. To escape from infinity it needs energy equal to	PE	= GMm/r.

Since GMm/(2r) < GMm/r, the energy required for the satellite is less than for the stationary object.

Answer: less.

Answer: (a) Kinetic energy (total energy = −KE). (b) Less.

Q7 Does the escape speed of a body from the earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, (d) the height of the location from where the body is launched?

SOLUTION

The escape speed from a point at distance r from Earth's centre is $v_e = \sqrt{(2GM_E / r)}$.

(a) Mass of the body: No.
The expression $v_e = \sqrt{(2GM_E / r)}$ does not contain the mass of the projectile. The escape speed is the same for all bodies regardless of their mass.

(b) Location from where it is projected: Yes (partially).
The escape speed depends on r, the distance from Earth's centre. If projected from the surface $r = R_E$, giving $v_e = \sqrt{(2gR_E)} \approx 11.2$ km/s. From a different point on the surface the gravitational potential is slightly different if the Earth is non-spherical; for a perfectly spherical Earth, the surface escape speed is the same everywhere on the surface.

(c) Direction of projection: No.
The escape speed is derived from energy conservation: KE must overcome PE. Energy is a scalar — it does not depend on direction. The body will escape provided it has the minimum kinetic energy corresponding to v_e, regardless of the direction of launch (ignoring atmospheric drag).

(d) Height of the location from where the body is launched: Yes.
$v_e = \sqrt{(2GM_E / r)} = \sqrt{(2GM_E / (R_E + h))}$. As height h increases, r increases and v_e decreases. A body launched from a greater height requires a smaller escape speed.

Answer: (a) No. (b) No, for a spherical Earth (same surface potential); Yes, if launched from different altitudes. (c) No. (d) Yes — escape speed decreases with increasing launch height.

Q8 A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

SOLUTION

A comet in an elliptical orbit moves under the central gravitational force of the Sun.

(a) Linear speed: Not constant.
By Kepler's Second Law (conservation of angular momentum), the comet moves faster near perihelion (closest approach) and slower near aphelion. The speed varies continuously.

(b) Angular speed: Not constant.
Angular speed $\omega = v/r$. Both v and r change with position in the orbit, and they change in such a way that the areal velocity ($\propto r^2 \times \omega$) remains constant — but ω itself is not constant.

(c) Angular momentum: Constant.
Gravity is a central force (directed along the radius vector). For any central force, the torque about the centre is zero, so angular momentum $L = r \times p$ is conserved throughout the orbit.

(d) Kinetic energy: Not constant.
$KE = \frac{1}{2}mv^2$. Since the linear speed varies, the kinetic energy also varies — it is maximum at perihelion and minimum at aphelion.

(e) Potential energy: Not constant.
$PE = -GMm/r$. Since the distance r from the Sun varies along the elliptical orbit, the potential energy also varies continuously.

(f) Total energy: Constant.
The total mechanical energy $E = KE + PE$ is conserved because gravity is a conservative force. KE and PE individually vary, but their sum remains constant throughout the orbit. (Total energy is negative for a bound orbit.)

Answer: (a) No. (b) No. (c) Yes — angular momentum is constant (central force). (d) No. (e) No. (f) Yes — total energy is constant.

Q9 Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem.

SOLUTION

In a freely orbiting spacecraft, the astronaut experiences weightlessness — there is no effective gravity to pull fluids downward in the body.

(a) Swollen feet: Unlikely.
On Earth, feet swell partly due to gravity pulling blood and fluid toward the lower extremities. In weightlessness, there is no such preferential downward pooling, so feet do not swell.

(b) Swollen face: Likely.
In weightlessness, body fluids redistribute uniformly. Fluid that would normally pool in the legs due to gravity instead shifts toward the upper body and face, causing facial puffiness. This is well documented in astronauts — the 'puffy face' effect.

(c) Headache: Possible.
The upward fluid shift increases intracranial pressure, which can cause headaches. This is observed in some astronauts, though it is not universal.

(d) Orientational problem: Likely.
On Earth, the vestibular system (inner ear) and gravity together provide a sense of 'up' and 'down'. In weightlessness there is no gravitational reference, so astronauts can experience disorientation and space sickness, especially initially.

Most likely symptoms: (b) swollen face and (d) orientational problem.

Answer: (b) Swollen face and (d) orientational problem are the most likely. Fluid redistribution causes facial puffiness; absence of gravitational reference causes disorientation.

Q10 In the following two exercises, choose the correct answer from among the given ones: The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig 7.11) (i) a, (ii) b, (iii) c, (iv) 0.

SOLUTION

Consider a hemispherical shell (like a bowl) of uniform surface mass density, with the flat circular face horizontal (open at the top) and the curved surface below.

The gravitational field at the centre of the flat face (the geometric centre of the hemisphere's base):
By symmetry, all horizontal components of the gravitational field from the curved surface cancel in pairs across the axis.
Each small mass element of the curved surface attracts the test point toward that element. The vertical (downward) components all add up, while the horizontal components cancel.
The net gravitational intensity at the centre therefore points downward — toward the curved surface, along the axis of symmetry.

From Fig 7.11, arrow 'c' points downward (toward the interior of the curved shell).

Answer: (iii) c.

Answer: (iii) c — the gravitational intensity points toward the curved surface (downward along the axis of symmetry).

Q11 For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

SOLUTION

For an arbitrary point P on the flat face of the hemispherical shell (not at the centre):

Unlike the centre, P does not lie on the axis of symmetry. The contributions from different parts of the curved surface no longer cancel by simple left-right symmetry.
The closest part of the curved surface exerts a stronger pull than the far part, so the net field is not directed straight downward.
The net gravitational intensity is directed toward the interior of the curved shell but tilted toward the nearer portion of the curved surface.

From Fig 7.11, arrow 'e' represents a direction that is angled — pointing generally toward the interior of the hemispherical shell but not straight down (not along the axis). This matches the expected tilted direction for an off-axis point.

Answer: (ii) e.

Answer: (ii) e — the gravitational intensity at an arbitrary point P is directed toward the curved interior but tilted (not along the symmetry axis).

Q12 A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

SOLUTION

Given: $M_{sun} = 2 \times 10^{30}$ kg, $M_{earth} = 6 \times 10^{24}$ kg
Earth-Sun distance $d = 1.5 \times 10^{11}$ m
Let r = distance from Earth's centre to the neutral point.
Distance from Sun's centre = $d - r$.

At the neutral point, the gravitational pull from Earth equals that from the Sun:
$G M_{earth} / r^2 = G M_{sun} / (d - r)^2$

$M_{earth} / r^2 = M_{sun} / (d - r)^2$
$(d - r)^2 / r^2 = M_{sun} / M_{earth}$
$(d - r) / r = \sqrt{(M_{sun} / M_{earth})}$

$\sqrt{(M_{sun} / M_{earth})} = \sqrt{(2 \times 10^{30} / 6 \times 10^{24})} = \sqrt{(3.33 \times 10^5)} = 577.4$

$d/r - 1 = 577.4$
$d/r = 578.4$
$r = d / 578.4 = 1.5 \times 10^{11} / 578.4$
$r = 2.59 \times 10^8$ m

Answer: $r \approx 2.59 \times 10^8$ m from Earth's centre (about 2.59×10^5 km from Earth).

Q13 How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8 km.

SOLUTION

Method: Use the Earth's orbital motion around the Sun, applying Kepler's Third Law.

Given: Orbital radius $R = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m
Orbital period $T = 1$ year $= 365 \times 24 \times 3600 = 3.156 \times 10^7$ s
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

The gravitational force provides centripetal force for Earth's orbit:
$GM_{sun} m_E / R^2 = m_E \times 4\pi^2 R / T^2$
$GM_{sun} = 4\pi^2 R^3 / T^2$
$M_{sun} = 4\pi^2 R^3 / (GT^2)$

$M_{sun} = (4 \times \pi^2 \times (1.5 \times 10^{11})^3) / (6.67 \times 10^{-11} \times (3.156 \times 10^7)^2)$

Numerator: $4\pi^2 \times 3.375 \times 10^{33} = 39.48 \times 3.375 \times 10^{33} = 1.332 \times 10^{35}$
Denominator: $6.67 \times 10^{-11} \times 9.96 \times 10^{14} = 6.64 \times 10^4$

$M_{sun} = 1.332 \times 10^{35} / 6.64 \times 10^4 = 2.01 \times 10^{30}$ kg

Answer: $M_{sun} = 4\pi^2 R^3 / (GT^2) \approx 2 \times 10^{30}$ kg.

Q14 A saturn year is 29.5 times the earth year. How far is the saturn from the sun if the earth is 1.50×10^8 km away from the sun?

SOLUTION

Given: $T_{saturn} = 29.5\ T_{earth}$
$R_{earth} = 1.50 \times 10^8$ km

By Kepler's Third Law: $T^2 \propto R^3$
$(T_s / T_e)^2 = (R_s / R_e)^3$
$(29.5)^2 = (R_s / R_e)^3$
$870.25 = (R_s / R_e)^3$
$R_s / R_e = (870.25)^{(1/3)} = (29.5)^{(2/3)}$

$(29.5)^{(2/3)}$: $29.5^{(1/3)} \approx 3.089$, so $29.5^{(2/3)} \approx (3.089)^2 \approx 9.54$

$R_s = 9.54 \times 1.50 \times 10^8$ km $= 14.32 \times 10^8$ km $\approx 1.43 \times 10^9$ km

Answer: $R_{saturn} \approx 1.43 \times 10^9$ km from the Sun.

Q15 A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

SOLUTION

Given: Weight on surface $W = 63$ N, height $h = R_E / 2$

Gravitational force at height h:
$F(h) = GM_E m / (R_E + h)^2$

At the surface: $W = GM_E m / R_E^2 = 63$ N

$F(h) / W = R_E^2 / (R_E + h)^2 = 1 / (1 + h/R_E)^2$

With $h = R_E / 2$:
$F(h) / W = 1 / (1 + 1/2)^2 = 1 / (3/2)^2 = 1 / (9/4) = 4/9$

$F(h) = 63 \times 4/9 = 252/9 = 28$ N

Answer: Gravitational force at $h = R_E/2$ is 28 N.

Q16 Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface?

SOLUTION

Given: Weight on surface $W = 250$ N, depth $d = R_E / 2$

For a uniform-density Earth, the acceleration due to gravity at depth d is:
$g(d) = g(0) \times (1 - d / R_E)$

The gravitational force (weight) at depth d:
$F(d) = m \times g(d) = W \times (1 - d / R_E)$

With $d = R_E / 2$:
$F(d) = 250 \times (1 - 1/2) = 250 \times 1/2 = 125$ N

Answer: Weight at depth $R_E/2$ is 125 N.

Q17 A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0×10^{24} kg; mean radius of the earth = 6.4×10^{6} m; G = 6.67×10^{-11} N m^2 kg^{-2}.

SOLUTION

Given: $v_i = 5$ km/s = 5×10^3 m/s (upward from surface)
$M_E = 6.0 \times 10^{24}$ kg, $R_E = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

First check: Escape speed $v_e = \sqrt{(2GM_E / R_E)}$
$= \sqrt{(2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24} / 6.4 \times 10^6)}$
$= \sqrt{(1.25 \times 10^8)} \approx 11.2 \times 10^3$ m/s
Since $v_i = 5$ km/s < $v_e = 11.2$ km/s, the rocket will return.

Applying conservation of energy (per unit mass):
At surface: $E = \frac{1}{2} v_i^2 - GM_E / R_E$
At maximum height r_{max}: $E = 0 - GM_E / r_{max}$ (KE = 0 at max height)

$\frac{1}{2} v_i^2 - GM_E / R_E = - GM_E / r_{max}$
$GM_E / r_{max} = GM_E / R_E - \frac{1}{2} v_i^2$

$r_{max} = GM_E / (GM_E / R_E - \frac{1}{2} v_i^2)$
$r_{max} = 2GM_E R_E / (2GM_E - v_i^2 R_E)$

$2GM_E = 2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24} = 8.004 \times 10^{14}$ m^3 s^{-2}
$v_i^2 R_E = (5 \times 10^3)^2 \times 6.4 \times 10^6 = 25 \times 10^6 \times 6.4 \times 10^6 = 1.6 \times 10^{14}$ m^3 s^{-2}

$2GM_E - v_i^2 R_E = 8.004 \times 10^{14} - 1.6 \times 10^{14} = 6.404 \times 10^{14}$

$r_{max} = (8.004 \times 10^{14} \times 6.4 \times 10^6) / 6.404 \times 10^{14}$
$= 8.004 \times 6.4 \times 10^6 / 6.404$
$= 51.23 \times 10^6 / 6.404$
$= 8.00 \times 10^6$ m

Height above surface: $h = r_{max} - R_E = 8.00 \times 10^6 - 6.4 \times 10^6 = 1.6 \times 10^6$ m = 1600 km

Answer: $r_{max} \approx 8.0 \times 10^6$ m from Earth's centre; height above surface $\approx 1.6 \times 10^6$ m (1600 km).

Q18 The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

SOLUTION

Given: $v_e = 11.2$ km/s (escape speed), $v_i = 3\,v_e = 3 \times 11.2 = 33.6$ km/s

Conservation of energy (per unit mass):
$\frac{1}{2} v_i^2 - GM_E / R_E = \frac{1}{2} v_f^2 - 0$ (PE = 0 at infinity)

Note: $GM_E / R_E = v_e^2 / 2$ (from definition of escape speed: $\frac{1}{2} m v_e^2 = GMm/R_E$)

$\frac{1}{2} v_i^2 - v_e^2 / 2 = \frac{1}{2} v_f^2$
$v_f^2 = v_i^2 - v_e^2$
$v_f^2 = (3 v_e)^2 - v_e^2 = 9 v_e^2 - v_e^2 = 8 v_e^2$
$v_f = v_e \times \sqrt{8} = 11.2 \times 2\sqrt{2} = 11.2 \times 2.828$
$v_f = 31.68$ km/s ≈ 31.7 km/s

Answer: $v_f = \sqrt{8} \times v_e = 2\sqrt{2} \times 11.2 \approx 31.7$ km/s.

Q19 A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0×10^{24} kg; radius of the earth = 6.4×10^6 m; G = 6.67×10^{-11} N m^2 kg^{-2}.

SOLUTION

Given: $h = 400$ km $= 4 \times 10^5$ m, $m = 200$ kg
$M_E = 6.0 \times 10^{24}$ kg, $R_E = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Orbital radius: $r = R_E + h = 6.4 \times 10^6 + 4 \times 10^5 = 6.8 \times 10^6$ m

Total energy of the orbiting satellite:
$E_{orbit} = - GM_E m / (2r)$ (for circular orbit)

$E_{orbit} = - (6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200) / (2 \times 6.8 \times 10^6)$
$= - (6.67 \times 6.0 \times 200 \times 10^{\wedge}(-11+24)) / (13.6 \times 10^6)$
$= - (8004 \times 10^{13}) / (13.6 \times 10^6)$
$= - 8.004 \times 10^{16} / 1.36 \times 10^7$
$= - 5.885 \times 10^9$ J

To escape to infinity: final energy = 0.
Energy required = $0 - E_{orbit} = - E_{orbit} = 5.885 \times 10^9$ J

≈ 5.88 × 10^9 J ≈ 5.9 × 10^9 J

Answer: Energy required ≈ 5.88 × 10^9 J ≈ 5.9 × 10^9 J.

Q20 Two stars each of one solar mass (= 2 × 10^{30} kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

SOLUTION

Given: M = 2 × 10^{30} kg (each star), G = 6.67 × 10^{-11} N m^2 kg^{-2}
Initial separation: d_i = 10^9 km = 10^{12} m
At collision: surfaces touch, so centre-to-centre distance d_f = 2 × radius = 2 × 10^4 km = 2 × 10^7 m
Initial speeds: v_i ≈ 0 (negligible).

By symmetry, both stars have equal speed v at collision.

Conservation of total mechanical energy:
Initial energy = $KE_i + PE_i = 0 + (-GM^2 / d_i) = -GM^2 / d_i$

Final energy = $KE_f + PE_f = 2 \times (\frac{1}{2} M v^2) + (-GM^2 / d_f) = Mv^2 - GM^2 / d_f$

Setting equal:
$-GM^2 / d_i = Mv^2 - GM^2 / d_f$
$Mv^2 = GM^2 / d_f - GM^2 / d_i = GM^2 (1/d_f - 1/d_i)$
$v^2 = GM (1/d_f - 1/d_i)$

v^2 = 6.67 × 10^{-11} × 2 × 10^{30} × (1/(2 × 10^7) − 1/10^{12})
= 1.334 × 10^{20} × (5 × 10^{-8} − 10^{-12})
≈ 1.334 × 10^{20} × 5 × 10^{-8} (since 10^{-12} << 5 × 10^{-8})
= 6.67 × 10^{12} m^2 s^{-2}

v = √(6.67 × 10^{12}) = 2.58 × 10^6 m/s ≈ 2.58 × 10^3 km/s

Answer: v ≈ 2.58 × 10^6 m/s ≈ 2.58 × 10^3 km/s.

Q21 Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

SOLUTION

Given: $m_1 = m_2 = 100$ kg, centre-to-centre separation = 1.0 m
Midpoint is at 0.5 m from each centre. $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Gravitational force at the midpoint on a test mass m_t:
Force from left sphere (toward left): $F_1 = G \times 100 \times m_t / (0.5)^2 = 4G \times 100\ m_t$
Force from right sphere (toward right): $F_2 = G \times 100 \times m_t / (0.5)^2 = 4G \times 100\ m_t$
These forces are equal in magnitude and opposite in direction.
Net gravitational force = $F_1 - F_2 = 0$.

Gravitational potential at the midpoint:
$V = -G\, m_1 / r_1 - G\, m_2 / r_2$
$= -G \times 100 / 0.5 - G \times 100 / 0.5$
$= -200G / 0.5 = -400G$
$= -400 \times 6.67 \times 10^{-11}$
$= -2.668 \times 10^{-8}$ J/kg
$V \approx -2.67 \times 10^{-8}$ J/kg

Equilibrium: Yes — the net force on an object at the midpoint is zero, so it is in equilibrium.

Stability: The equilibrium is UNSTABLE.
If the object is displaced slightly toward one sphere along the line joining the centres, the gravitational pull from the nearer sphere becomes stronger than from the farther sphere. There is no restoring force — the net force acts in the direction of displacement. The object will continue to accelerate toward the nearer sphere. This is characteristic of unstable equilibrium.
(For a displacement perpendicular to the line, the forces from both spheres would have a resultant pulling the object back — so it is stable against transverse displacements. Overall, the saddle-point nature makes the equilibrium unstable.)

Answer: Gravitational force at midpoint = 0; gravitational potential = -2.67×10^{-8} J/kg. Object is in equilibrium, but the equilibrium is unstable (along the axis joining the centres).