Chapter 8: Mechanical Properties of Solids

Exercises — Exercises

Q1 A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m² stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0×10^{-5} m² under a given load. What is the ratio of the Young's modulus of steel to that of copper?

SOLUTION

Given:
Steel wire: $L_s = 4.7$ m, $A_s = 3.0 \times 10^{-5}$ m²
Copper wire: $L_c = 3.5$ m, $A_c = 4.0 \times 10^{-5}$ m²
Both wires stretch by the same amount ΔL under the same load F.

Young's modulus: $Y = (F \times L) / (A \times \Delta L)$

For steel: $Y_s = (F \times L_s) / (A_s \times \Delta L)$
For copper: $Y_c = (F \times L_c) / (A_c \times \Delta L)$

Dividing: $Y_s / Y_c = (L_s \times A_c) / (L_c \times A_s)$
$= (4.7 \times 4.0 \times 10^{-5}) / (3.5 \times 3.0 \times 10^{-5})$
$= (4.7 \times 4.0) / (3.5 \times 3.0)$
$= 18.8 / 10.5$
$= 1.79$

Answer: $Y_s / Y_c \approx 1.79$

Q2 Figure 8.9 shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?
[Fig. 8.9: y-axis — Stress (×10⁶ N m⁻²), gridlines 0 to 300; x-axis — Strain, 0 to 0.004. The curve is a straight line from the origin (passing through ≈150 × 10⁶ N m⁻² at strain ≈ 0.002) up to about 300 × 10⁶ N m⁻² near strain ≈ 0.004, where it bends over, peaks and starts to fall.]

SOLUTION

Reading Fig. 8.9:
The curve is a straight line (Hooke's law region) from the origin up to the yield point near the top.
Pick a point on the linear region, e.g.:
Stress ≈ 150×10^6 N m⁻² at Strain ≈ 0.002

(a) Young's modulus = slope of the linear region

Y = Stress / Strain = $(150 \times 10^6\ \text{N m}^{-2}) / 0.002$

$Y = 7.5 \times 10^{10}\ \text{N m}^{-2}$ $(= 0.75 \times 10^{11}\ \text{N m}^{-2})$

(b) The yield point is where the curve stops being linear and stress reaches its maximum before levelling/falling.

From the graph, this occurs at approximately:

Stress $\approx 300 \times 10^6\ \text{N m}^{-2} = 3 \times 10^8\ \text{N m}^{-2}$

Answer: (a) $Y \approx 7.5 \times 10^{10}\ \text{N m}^{-2}$ $(0.75 \times 10^{11}\ \text{N m}^{-2})$; (b) Yield strength $\approx 3 \times 10^8\ \text{N m}^{-2}$

Q3 The stress-strain graphs for materials A and B are shown in Fig. 8.10. The graphs are drawn to the same scale.

(a) Which of the materials has the greater Young's modulus?

(b) Which of the two is the stronger material?

[Fig. 8.10: Curves A and B on separate stress–strain axes drawn to the same scale. Curve A has the steeper slope in its linear region and extends to a large strain, reaching a high maximum stress. Curve B rises to a slightly lower maximum stress and fractures at a smaller strain.]

SOLUTION

(a) Young's modulus is the slope of the linear (initial) portion of the stress-strain curve.

Material A has a steeper slope in the linear region.

=> Material A has the greater Young's modulus.

(b) The strength of a material is measured by the maximum stress it can withstand (the ultimate tensile strength — the stress at fracture).

The two graphs are drawn to the same scale. From Fig. 8.10, material A reaches the higher maximum (ultimate) stress before it breaks; material B fractures at a lower maximum stress (and at a smaller strain).

=> Material A is the stronger material.

Answer: (a) Material A has the greater Young's modulus (steeper slope in the linear region — more stress for a given strain). (b) Material A is stronger (it withstands the greater maximum/ultimate stress before fracture).

Q4 Read the following two statements below carefully and state, with reasons, if it is true or false.
(a) The Young's modulus of rubber is greater than that of steel;
(b) The stretching of a coil is determined by its shear modulus.

SOLUTION

(a) FALSE.
Young's modulus measures the stiffness of a material — it is the ratio of stress to longitudinal strain in the elastic region.
Steel requires a very large force to produce even a small elongation ($Y_{steel} \approx 2 \times 10^{11}$ N m^{-2}).
Rubber can be stretched to several times its original length by a comparatively small force, so its Y is very small ($Y_{rubber} \approx 0.01 \times 10^{9}$ N m^{-2} or less).
A larger Young's modulus means the material is harder to deform, not easier.
Therefore, Young's modulus of rubber is much LESS than that of steel.

(b) TRUE.
When a coil spring is stretched or compressed, each small element of the coil wire is twisted — the wire undergoes torsion (angular shear deformation), not simple elongation.
Shear modulus (G) governs the resistance to this type of shape-changing (shear) deformation, so the extension of a coil spring under a load is determined by the shear modulus of the material, not the Young's modulus.

Answer: (a) False — $Y_{rubber} << Y_{steel}$. (b) True — stretching a coil involves shear (torsion) of the wire, governed by the shear modulus G.

Q5 Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in Fig. 8.11. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.
[Fig. 8.11: Steel wire (1.5 m) hangs vertically with a 4.0 kg mass at the junction between the two wires; brass wire (1.0 m) hangs below the 4.0 kg mass with a 6.0 kg mass at its lower end.]

SOLUTION

Given:
Diameter of each wire d = 0.25 cm = 2.5×10^{-3} m => radius = 1.25×10^{-3} m

Cross-sectional area $A = \pi r^2 = \pi \times (1.25 \times 10^{-3})^2 = 4.91 \times 10^{-6}\ m^2$
$L_{steel} = 1.5\ m$, $Y_{steel} = 2.0 \times 10^{11}\ N\ m^{-2}$
$L_{brass} = 1.0\ m$, $Y_{brass} = 0.91 \times 10^{11}\ N\ m^{-2}$
Mass at junction = 4.0 kg; mass at bottom = 6.0 kg

The steel wire (top) supports both the 4.0 kg mass at the junction AND the 6.0 kg mass hanging below, so:
$F_{steel} = (4.0 + 6.0) \times 9.8 = 98.0\ N$

The brass wire (bottom) supports only the 6.0 kg mass:
$F_{brass} = 6.0 \times 9.8 = 58.8\ N$

Elongation formula: $\Delta L = F \times L / (Y \times A)$

$\Delta L_{steel} = (98.0 \times 1.5) / (2.0 \times 10^{11} \times 4.91 \times 10^{-6})$
$= 147.0 / (9.82 \times 10^{5})$
$= 1.50 \times 10^{-4}\ m$

$\Delta L_{brass} = (58.8 \times 1.0) / (0.91 \times 10^{11} \times 4.91 \times 10^{-6})$
$= 58.8 / (4.47 \times 10^{5})$
$= 1.32 \times 10^{-4}\ m$

Answer: $\Delta L_{steel} \approx 1.50 \times 10^{-4}$ m (0.150 mm); $\Delta L_{brass} \approx 1.32 \times 10^{-4}$ m (0.132 mm)

Q6 The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

SOLUTION

Given:
Edge length $L = 10\ cm = 0.10\ m$
Mass $m = 100\ kg$ => Shearing force $F = m \times g = 100 \times 9.8 = 980\ N$
Shear modulus $G = 25\ GPa = 25 \times 10^{9}\ N\ m^{-2}$

The face fixed to the wall has area $A = L \times L = (0.10)^2 = 0.01\ m^2$

Shear stress $= F / A = 980 / 0.01 = 9.8 \times 10^{4}\ N\ m^{-2}$

Shear strain θ = Shear stress / $G = (9.8 \times 10^{4}) / (25 \times 10^{9}) = 3.92 \times 10^{-6}$

Vertical deflection $\Delta x = \theta \times L = 3.92 \times 10^{-6} \times 0.10 = 3.92 \times 10^{-7}\ m$

Answer: Vertical deflection $\Delta x \approx 3.92 \times 10^{-7}$ m (≈ 0.39 μm)

Q7 Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.

SOLUTION

Given:
Total mass M = 50,000 kg; number of columns n = 4
Inner radius r_i = 30 cm = 0.30 m
Outer radius r_o = 60 cm = 0.60 m
Young's modulus of mild steel $Y = 2.0 \times 10^{11}$ N m^{-2}

Weight supported by each column:
$W = Mg / n = (50000 \times 9.8) / 4 = 122500$ N

Cross-sectional area of each hollow column:
$A = \pi(r_o^2 - r_i^2) = \pi(0.60^2 - 0.30^2) = \pi(0.36 - 0.09) = \pi \times 0.27$
$= 0.848$ m^2

Compressional stress = $W / A = 122500 / 0.848 = 1.44 \times 10^5$ N m^{-2}

Compressional strain = stress / $Y = (1.44 \times 10^5) / (2.0 \times 10^{11})$
$= 7.22 \times 10^{-7}$

Answer: Compressional strain of each column ≈ 7.22×10^{-7}

Q8 A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain.

SOLUTION

Given:
Cross-section: b = 15.2 mm = 15.2×10^{-3} m, h = 19.1 mm = 19.1×10^{-3} m
Tensile force F = 44,500 N
Young's modulus of copper $Y = 1.1 \times 10^{11}$ N m^{-2} (from NCERT Table 8.1)

Cross-sectional area $A = b \times h = (15.2 \times 10^{-3}) \times (19.1 \times 10^{-3})$
$= 2.903 \times 10^{-4}$ m^2

Tensile stress = $F / A = 44500 / (2.903 \times 10^{-4})$

$= 1.533 \times 10^{8}\ \text{N m}^{-2}$

Longitudinal strain = Stress / Y = $(1.533 \times 10^{8}) / (1.1 \times 10^{11})$
$= 1.39 \times 10^{-3}$

Answer: Longitudinal strain ≈ 1.39×10^{-3}

Q9 A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed $10^{8}\ \text{N m}^{-2}$, what is the maximum load the cable can support?

SOLUTION

Given:
Radius of cable r = 1.5 cm = 1.5×10^{-2} m
Maximum allowable stress $\sigma_{max} = 10^{8}\ \text{N m}^{-2}$

Cross-sectional area of cable:
$A = \pi r^{2} = \pi \times (1.5 \times 10^{-2})^{2}$
$= \pi \times 2.25 \times 10^{-4}$
$= 7.07 \times 10^{-4}\ \text{m}^{2}$

Maximum load = $\sigma_{max} \times A$
$= 10^{8} \times 7.07 \times 10^{-4}$
$= 7.07 \times 10^{4}$ N

Answer: Maximum load ≈ 7.07×10^{4} N (≈ 70.7 kN)

Q10 A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

SOLUTION

Given:
Three wires, each of length L = 2.0 m
End wires: copper, $Y_c = 1.1 \times 10^{11}\ \text{N m}^{-2}$
Middle wire: iron, $Y_i = 1.9 \times 10^{11}\ \text{N m}^{-2}$
Each wire must carry the same tension T.

Since the bar is rigid and symmetrically loaded, all three wires elongate by the same amount ΔL.
From $Y = (T \times L) / (A \times \Delta L)$, for same T, L, and ΔL:

$A \times Y = \text{constant}$
$\Rightarrow A_{copper} \times Y_{copper} = A_{iron} \times Y_{iron}$

Area $A = \pi d^2 / 4$, so:
$(d_c^2 / 4) \times Y_c = (d_i^2 / 4) \times Y_i$
$d_c^2 / d_i^2 = Y_i / Y_c$
$d_c / d_i = \sqrt{(Y_i / Y_c)}$
$= \sqrt{(1.9 \times 10^{11} / 1.1 \times 10^{11})}$
$= \sqrt{(1.9 / 1.1)}$
$= \sqrt{1.727}$
$= 1.31$

Answer: $d_{copper} / d_{iron} \approx 1.31$

Q11 A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path.

SOLUTION

Given:
Mass $m = 14.5$ kg
Length of wire $L = 1.0$ m
Angular velocity $n = 2$ rev/s $\Rightarrow \omega = 2\pi \times 2 = 4\pi$ rad/s
Cross-sectional area $A = 0.065\ cm^2 = 0.065 \times 10^{-4}\ m^2 = 6.5 \times 10^{-6}\ m^2$
$Y_{steel} = 2.0 \times 10^{11}\ N\ m^{-2}$
$g = 9.8\ m\ s^{-2}$

At the lowest point of the vertical circle, the tension T in the wire provides both the centripetal force and supports the weight of the mass:
$T - mg = m \omega^2 L$
$T = m(g + \omega^2 L)$

$\omega = 4\pi$ rad/s $\Rightarrow \omega^2 = 16\pi^2 = 157.91\ (rad/s)^2$

$T = 14.5 \times (9.8 + 157.91 \times 1.0)$
$= 14.5 \times 167.71$
$= 2431.8$ N

Elongation $\Delta L = T \times L / (Y \times A)$
$= (2431.8 \times 1.0) / (2.0 \times 10^{11} \times 6.5 \times 10^{-6})$

$= 2431.8 / (1.3 \times 10^{6})$
$= 1.87 \times 10^{-3}$ m

Answer: Elongation $\Delta L \approx 1.87 \times 10^{-3}$ m (≈ 1.87 mm)

Q12 Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013×10^{5} Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

SOLUTION

Given:
Initial volume $V_o = 100.0$ L
Final volume = 100.5 L (the 0.5 L represents the decrease in volume: the water is compressed from 100.0 L to 99.5 L, i.e., $\Delta V = -0.5$ L)
Pressure increase $\Delta p = 100.0 \times 1.013 \times 10^{5} = 1.013 \times 10^{7}$ Pa

Volume strain: $\Delta V/V = (-0.5) / 100.0 = -5.0 \times 10^{-3}$

Bulk modulus: $B = -\Delta p / (\Delta V/V)$
$= -(1.013 \times 10^{7}) / (-5.0 \times 10^{-3})$
$= 2.026 \times 10^{9}$ Pa
$\approx 2.03 \times 10^{9}$ Pa

Bulk modulus of air at constant temperature (isothermal process):
$B_{air} = p = 1.013 \times 10^{5}$ Pa (at standard atmospheric pressure)

Ratio: $B_{water} / B_{air} = (2.03 \times 10^{9}) / (1.013 \times 10^{5}) \approx 2.0 \times 10^{4}$

Water is about 20,000 times less compressible than air.
Reason: In water (a liquid), molecules are already closely packed and their intermolecular repulsive forces resist any further reduction in volume very strongly. In air (a gas), molecules are far apart with very little intermolecular force, so the volume can be reduced easily. This makes gases far more compressible than liquids.

Answer: $B_{water} \approx 2.03 \times 10^{9}$ Pa; $B_{water} / B_{air} \approx 2 \times 10^{4}$ (water is ~20,000 times less compressible than air)

Q13 What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03×10^3 kg m^{-3}?

SOLUTION

Given:

Pressure at depth $p = 80.0 \times 1.013 \times 10^5 = 8.104 \times 10^6$ Pa

Surface density $\rho_o = 1.03 \times 10^3$ kg m^{-3}

Bulk modulus of water $B = 2.2 \times 10^9$ N m^{-2} (from NCERT Table 8.3)

Bulk modulus: $B = -p / (\Delta V/V)$

$\Rightarrow \Delta V/V = -p/B = -(8.104 \times 10^6) / (2.2 \times 10^9) = -3.684 \times 10^{-3}$

The volume decreases (water is compressed), so $V_{new} = V_o (1 + \Delta V/V) = V_o (1 - p/B)$

Since mass is conserved: $\rho_{new} \times V_{new} = \rho_o \times V_o$

$\rho_{new} = \rho_o / (1 + \Delta V/V) = \rho_o / (1 - p/B)$

$\rho_{new} = (1.03 \times 10^3) / (1 - 3.684 \times 10^{-3})$

$= (1.03 \times 10^3) / 0.99632$

$= 1033.8$ kg m^{-3}

$\approx 1.034 \times 10^3$ kg m^{-3}

Answer: Density at 80 atm depth $\approx 1.034 \times 10^3$ kg m^{-3}

Q14 Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

SOLUTION

Given:

Hydraulic pressure $p = 10$ atm $= 10 \times 1.013 \times 10^5 = 1.013 \times 10^6$ Pa

Bulk modulus of glass $B = 37 \times 10^9$ N $m^{-2} = 37$ GPa (from NCERT Table 8.3)

From the definition of bulk modulus: $B = p /$	$\Delta V/V$	
$\Rightarrow$	$\Delta V/V$	$= p / B = (1.013 \times 10^6) / (37 \times 10^9)$

$= 2.74 \times 10^{-5}$

The volume decreases (compression), so $\Delta V/V = -2.74 \times 10^{-5}$

Answer: Fractional change in volume $|\Delta V/V| \approx 2.74 \times 10^{-5}$ (volume decreases by 2.74×10^{-5} of original)

Q15 Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa.

SOLUTION

Given:

Edge of cube a = 10 cm = 0.10 m

Original volume $V = a^3 = (0.10)^3 = 10^{-3}$ m³ = 1.0 L

Hydraulic pressure $p = 7.0 \times 10^6$ Pa

Bulk modulus of copper $B = 140 \times 10^9$ N m^{-2} = 140 GPa (from NCERT Table 8.3)

Volume strain: $|\Delta V/V| = p / B = (7.0 \times 10^6) / (140 \times 10^9) = 5.0 \times 10^{-5}$

Volume contraction: $|\Delta V| = |\Delta V/V| \times V$

$= 5.0 \times 10^{-5} \times 10^{-3}$

$= 5.0 \times 10^{-8}$ m³

Answer: Volume contraction $|\Delta V| = 5.0 \times 10^{-8}$ m³ (= 0.05 cm³)

Q16 How much should the pressure on a litre of water be changed to compress it by 0.10%?

SOLUTION

Given:

Volume of water = 1 litre

Fractional compression $|\Delta V/V| = 0.10\% = 0.10 / 100 = 1.0 \times 10^{-3}$

Bulk modulus of water $B = 2.2 \times 10^9$ N m^{-2} (from NCERT Table 8.3)

From bulk modulus definition: B = Δp /	ΔV/V	
=> Δp = B ×	ΔV/V	

$= 2.2 \times 10^9 \times 1.0 \times 10^{-3}$

$= 2.2 \times 10^6$ Pa

Converting to atm: $2.2 \times 10^6 / 1.013 \times 10^5 \approx 21.7$ atm

Answer: Pressure increase required $\Delta p \approx 2.2 \times 10^6$ Pa (≈ 21.7 atm)