Chapter 9: Mechanical Properties of Fluids

Exercises — Exercises

Q1 Explain why

(a) The blood pressure in humans is greater at the feet than at the brain

(b) Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km

(c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.

SOLUTION

(a) Blood pressure at the feet versus the brain:

Pressure in a fluid at rest increases with depth: $P = P_o + \rho gh$, where h is measured downward from a reference level.

The feet are at a much lower level than the brain — roughly 1.3–1.5 m below the heart and even further below the brain.

The column of blood between the brain and the feet creates an additional hydrostatic pressure ρgh at the feet.

Therefore the blood pressure at the feet is greater than at the brain by approximately ρgh, where h is the vertical height difference.

(b) Atmospheric pressure halving at ~6 km despite the atmosphere extending beyond 100 km:

The atmosphere is not a uniform fluid of constant density. Air is compressible and its density decreases with altitude — air is denser near sea level and very rarified at great heights.

The bulk of the atmosphere's mass (and hence weight per unit area) is concentrated in the lower few kilometres.

Roughly half the total mass of the atmosphere lies below about 5–6 km altitude, so the weight of the air column above 6 km is approximately half of the total weight at sea level.

This is why pressure (= weight of air column per unit area) halves at about 6 km, even though the atmosphere extends far beyond that height.

(c) Hydrostatic pressure as a scalar despite being force/area:

Pressure is defined as the magnitude of the normal force per unit area. The 'force' in the numerator is the component of force normal to the surface — this is a scalar magnitude, not a vector.

At any point in a fluid at rest, the pressure is the same in all directions (Pascal's law). A scalar has no directional preference — the same value P acts on a surface regardless of how that surface is oriented.

If pressure were a vector, it would have different values in different directions, contradicting Pascal's law.

Therefore hydrostatic pressure is a scalar quantity.

Answer: (a) Height difference between feet and brain creates additional hydrostatic pressure ρgh at the feet. (b) About half the atmospheric mass lies below 6 km, so the overlying column weight (= pressure) is halved there. (c) Pressure is the magnitude of the normal force per unit area and is the same in all directions at a point — hence scalar.

Q2 Explain why

(a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.

(b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops.

(c) Surface tension of a liquid is independent of the area of the surface.

(d) Water with detergent dissolved in it should have small angles of contact.

(e) A drop of liquid under no external forces is always spherical in shape.

SOLUTION

(a) Contact angle of mercury (obtuse) versus water (acute) on glass:

The angle of contact θ satisfies: $S_{sa} = S_{sl} + S_{la} \cos\theta$, i.e., $\cos\theta = (S_{sa} - S_{sl})/S_{la}$.

For mercury on glass: the cohesive forces among mercury atoms are very strong (mercury is a liquid metal), while the adhesive force between mercury and glass is weak. So $S_{sl} > S_{sa}$, making $\cos\theta$ negative, giving $\theta > 90°$ (obtuse).

For water on glass: the adhesive force between water molecules and glass (due to polar interactions) is strong, while water's cohesive forces are weaker. So $S_{sl} < S_{sa}$, making $\cos\theta$ positive, giving $\theta < 90°$ (acute).

(b) Water spreads on glass; mercury forms drops:

When the angle of contact is acute (water-glass), the liquid-solid surface energy is low — it is energetically favourable to increase the contact area, so the liquid spreads out.

When the angle of contact is obtuse (mercury-glass), creating more liquid-solid

interface costs energy — the liquid minimises contact area by forming spherical (or near-spherical) drops.

(c) Surface tension is independent of surface area:
Surface tension S is defined as surface energy per unit area (or force per unit length). It is an intrinsic property of the liquid-air interface determined by intermolecular forces, not by the total extent of the surface.
Whether a soap film is large or small, the energy per unit area at the interface remains the same because the molecular environment at the surface is the same regardless of total area.

(d) Detergent makes contact angle small:
Detergent molecules are surfactants. They adsorb at the water-solid and water-air interfaces, significantly reducing the liquid-air surface tension S_{la} and the solid-liquid surface energy S_{sl}.
From $\cos\theta = (S_{sa} - S_{sl})/S_{la}$: the reduction in S_{la} and S_{sl} increases $\cos\theta$ (makes it more positive), reducing θ toward 0°.
A small contact angle means water wets the solid better, enabling cleaning agents to penetrate fabrics and surfaces effectively.

(e) A free liquid drop is spherical:
For a given volume, the shape with the minimum surface area is a sphere.
Since a liquid surface has surface energy proportional to its area, a free drop (with no external forces like gravity or air resistance) minimises its total surface energy by minimising surface area.
Therefore, under no external forces, a liquid drop always takes the spherical shape.

Answer: (a) Mercury: strong cohesion, weak adhesion to glass → $\cos\theta < 0$ → obtuse angle. Water: strong adhesion to glass → $\cos\theta > 0$ → acute angle. (b) Small contact angle (water) → liquid spreads; obtuse contact angle (mercury) → liquid forms drops. (c) S is energy per unit area — an intrinsic interfacial property, independent of total area. (d) Detergent reduces S_{la} and S_{sl} → $\cos\theta$ increases → θ decreases. (e) Sphere has minimum surface area for given volume; free drop minimises surface energy → spherical shape.

Q3 Fill in the blanks using the word(s) from the list appended with each statement:
(a) Surface tension of liquids generally . . . with temperatures (increases / decreases)
(b) Viscosity of gases . . . with temperature, whereas viscosity of liquids . . . with temperature (increases / decreases)
(c) For solids with elastic modulus of rigidity, the shearing force is proportional to . . . , while for fluids it is proportional to . . . (shear strain / rate of shear strain)
(d) For a fluid in a steady flow, the increase in flow speed at a constriction follows (conservation of mass / Bernoulli's principle)
(e) For the model of a plane in a wind tunnel, turbulence occurs at a . . . speed for turbulence for an actual plane (greater / smaller)

SOLUTION

(a) Surface tension of liquids generally decreases with temperature.
Reason: As temperature rises, molecular kinetic energy increases, weakening intermolecular attractions at the surface, so surface energy per unit area (surface tension) decreases.

(b) Viscosity of gases increases with temperature, whereas viscosity of liquids decreases with temperature.
Reason: In gases, viscosity arises from momentum transfer between molecules — faster molecules at higher temperature → greater momentum exchange → higher viscosity. In liquids, viscosity arises from intermolecular cohesive forces — higher temperature weakens these forces → lower viscosity.

(c) For solids with elastic modulus of rigidity, shearing force is proportional to shear strain; for fluids it is proportional to rate of shear strain.
Reason: A solid resists deformation (Hooke's law: stress $\propto$ strain). A fluid continues to deform as long as a shear stress is applied; the stress is proportional to the rate of strain (Newton's law of viscosity).

(d) The increase in flow speed at a constriction follows conservation of mass.
Reason: The equation of continuity (Av = constant) is a statement of conservation of mass for incompressible steady flow — at a narrower cross-section, speed must increase to maintain constant volume flux. Bernoulli's principle then relates the resulting pressure change to the speed change, but the speed increase itself is due to mass conservation.

(e) For the model plane in a wind tunnel, turbulence occurs at a greater speed than for the actual plane.

Reason: The Reynolds number Re = ρvl/η determines turbulence onset. The model has a smaller characteristic length l than the actual plane. For the same Re (same flow regime), the model must be flown at a proportionally higher speed to match the actual plane's conditions. Thus turbulence (occurring at the critical Re) is reached at a greater speed for the model.

Answer: (a) decreases (b) increases; decreases (c) shear strain; rate of shear strain (d) conservation of mass (e) greater

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Q4 Explain why
(a) To keep a piece of paper horizontal, you should blow over, not under, it
(b) When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers
(c) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection
(d) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel
(e) A spinning cricket ball in air does not follow a parabolic trajectory

SOLUTION

(a) Blowing over a piece of paper to keep it horizontal:
By Bernoulli's principle, the region of higher fluid velocity has lower pressure. Blowing air over the top of the paper increases the air speed above it, reducing the pressure on the top surface.
The atmospheric pressure below the paper (which is undisturbed) now exceeds the pressure above. The resulting net upward force keeps the paper horizontal.
Blowing under the paper would increase pressure below, pushing it further down and making it harder to keep horizontal.

(b) Fast jets of water through finger gaps when closing a tap:
When fingers partially close the tap, the water must pass through the narrower openings between the fingers.
By the equation of continuity (Av = constant for incompressible flow), the smaller cross-sectional area of these openings forces the water to flow at much higher speed.
This is consistent with Bernoulli's principle as well: the constricted area results in high-velocity jets.

(c) Needle size controls injection flow rate better than thumb pressure:

By Poiseuille's law, the volume flow rate through a tube is $Q = \pi R^4 \Delta P/(8\eta L)$. Flow rate varies as the fourth power of the radius R of the needle.
A small change in needle radius causes a very large change in flow rate (R^4 dependence), while a change in thumb pressure (ΔP) only gives a linear change in flow rate.
Therefore, the needle size (radius) is a far more sensitive and effective control of flow rate than thumb pressure.

(d) Backward thrust from fluid flowing out of a hole:
By conservation of momentum (Newton's third law): the fluid gains momentum in the direction of flow as it exits the hole. By Newton's third law, the vessel experiences an equal and opposite reaction force — a backward thrust in the direction opposite to the outflow.
This is the rocket principle: the ejected fluid carries forward momentum, and the vessel recoils backward.

(e) Spinning cricket ball deviates from parabolic trajectory (Magnus effect):
A spinning ball drags air along with it due to friction. On one side of the ball the dragged air motion adds to the ambient airflow (faster air), while on the other side the two flows oppose each other (slower air).
By Bernoulli's principle, the faster air has lower pressure and the slower air has higher pressure. This creates a net transverse force (Magnus force) perpendicular to the velocity of the ball.
This sideways force causes the ball to curve away from its parabolic path, following a curved trajectory — the basis of swing in cricket.

Answer: (a) Blowing over top → higher speed, lower pressure on top → net upward force holds paper up. (b) Constricted gaps → equation of continuity → higher speed jets. (c) $Q \propto R^4$ (Poiseuille) — needle radius has far stronger effect than pressure. (d) Newton's third law: ejected fluid carries forward momentum → vessel recoils backward. (e) Magnus effect: spin creates unequal air speeds on opposite sides → pressure difference → transverse force → curved path.

Q5 A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?

SOLUTION

Given: mass m = 50 kg, heel diameter d = 1.0 cm = 1.0×10^{-2} m, g = 9.8 m s^{-2}

Area of circular heel:

$A = \pi (d/2)^2 = \pi \times (0.5 \times 10^{-2})^2 = \pi \times 0.25 \times 10^{-4}$ m^2

$A = 7.854 \times 10^{-5}$ m^2

Force exerted by heel = weight of girl:

$F = mg = 50 \times 9.8 = 490$ N

Pressure:

$P = F/A = 490 / (7.854 \times 10^{-5})$

$P = 6.24 \times 10^{6}$ Pa

Answer: $P \approx 6.24 \times 10^{6}$ Pa

Q6 Torricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m^{-3}. Determine the height of the wine column for normal atmospheric pressure.

SOLUTION

Given: density of wine $\rho = 984$ kg m^{-3}, atmospheric pressure $P_a = 1.013 \times 10^{5}$ Pa, g = 9.8 m s^{-2}

For a barometer: $P_a = \rho g h$

$h = P_a / (\rho g)$

$h = (1.013 \times 10^{5}) / (984 \times 9.8)$

$h = (1.013 \times 10^{5}) / 9643.2$

h = 10.50 m

Answer: $h \approx 10.5$ m

Q7 A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.

SOLUTION

Given: maximum stress = 10^9 Pa, depth of ocean h = 3 km = 3000 m
Density of sea water $\rho \approx 1.03 \times 10^3$ kg m^{-3}, g = 9.8 m s^{-2}
Atmospheric pressure Pa = 1.013×10^5 Pa

Total pressure (absolute) at the ocean floor:
$P = Pa + \rho gh$
$P = 1.013 \times 10^5 + (1.03 \times 10^3 \times 9.8 \times 3000)$
$P = 1.013 \times 10^5 + 3.0282 \times 10^7$
$P \approx 3.04 \times 10^7$ Pa

Since 3.04×10^7 Pa << 10^9 Pa (maximum stress the structure can withstand), the structure is suitable.

Answer: Pressure at 3 km depth ≈ 3.04×10^7 Pa, which is well below the maximum stress of 10^9 Pa. Yes, the structure is suitable.

Q8 A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear?

SOLUTION

Given: mass of car m = 3000 kg, area of large piston A = 425 cm^2 = 425×10^{-4} m^2, g = 9.8 m s^{-2}

By Pascal's law, the pressure transmitted through the hydraulic fluid is the same everywhere.
The pressure required to support the car:

$P = F/A = mg/A$
$P = (3000 \times 9.8) / (425 \times 10^{-4})$
$P = 29400 / 0.0425$
$P = 6.92 \times 10^5$ Pa

This is the pressure the smaller piston must bear (same pressure throughout the fluid by Pascal's law).

Answer: $P \approx 6.92 \times 10^5$ Pa

Q9 A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit?

SOLUTION

Given: height of water column $h_w = 10.0$ cm, height of spirit column $h_s = 12.5$ cm

Mercury is at the same level in both arms.

Since mercury is at the same level, the pressure at the mercury surface must be equal from both sides.

Pressure from water arm = pressure from spirit arm (at the mercury level):

$\rho_w \times g \times h_w = \rho_s \times g \times h_s$

$\rho_s = \rho_w \times (h_w / h_s)$

$\rho_s = 1000 \times (10.0 / 12.5)$

$\rho_s = 1000 \times 0.8 = 800 \text{ kg m}^{-3}$

Specific gravity of spirit = $\rho_s / \rho_{water} = 800/1000 = 0.8$

Answer: Specific gravity of spirit = 0.8

Q10 In the previous problem, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (Specific gravity of mercury = 13.6)

SOLUTION

Given: specific gravity of spirit = 0.8 (from Q9.9), specific gravity of mercury = 13.6

After adding: water arm has 10 + 15 = 25 cm of water; spirit arm has 12.5 + 15 = 27.5 cm of spirit.

Let mercury drop by Δ cm in the water arm and rise by Δ cm in the spirit arm. (Both arms assumed equal cross-section, so mercury level difference = 2Δ.)

Pressure balance at the level of mercury in the lower arm (water side, reference level):

Pressure from water side = pressure from spirit side at the same reference level

$\rho_w \times g \times (25 + \Delta) = \rho_s \times g \times (27.5 - \Delta) + \rho_{Hg} \times g \times (2\Delta)$

Dividing through by $\rho_w \times g$ (using relative densities):

$1.0 \times (25 + \Delta) = 0.8 \times (27.5 - \Delta) + 13.6 \times (2\Delta)$
$25 + \Delta = 22 - 0.8\Delta + 27.2\Delta$
$25 + \Delta = 22 + 26.4\Delta$
$3 = 25.4\Delta$
$\Delta = 3/25.4 \approx 0.118$ cm

Difference in mercury levels = $2\Delta \approx 2 \times 0.118 = 0.236$ cm ≈ 0.24 cm

Answer: Difference in mercury levels ≈ 0.24 cm

Q11 Can Bernoulli's equation be used to describe the flow of water through a rapid in a river? Explain.

SOLUTION

No, Bernoulli's equation cannot be reliably used to describe the flow of water through a river rapid.

Bernoulli's equation applies under the following conditions:
1. The flow must be steady (streamline flow) — velocity at each point does not change with time.
2. The fluid must be non-viscous (inviscid).
3. The fluid must be incompressible.
4. The equation applies along a single streamline.

In a river rapid, the flow is turbulent — the velocity at any point fluctuates rapidly and randomly in magnitude and direction. Streamlines no longer exist in the usual sense, and the conditions for Bernoulli's equation are violated.

Additionally, viscous dissipation is significant in turbulent flow — kinetic energy is converted to heat by friction between turbulent eddies, so energy is not conserved in the form assumed by Bernoulli's equation.

Therefore, Bernoulli's equation does not apply to water flowing through a river rapid.

Answer: No. A river rapid has turbulent, non-steady flow with significant viscous dissipation — all three conditions required for Bernoulli's equation (steady, non-viscous, incompressible flow along a streamline) are violated.

Q12 Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain.

SOLUTION

No, it does not matter whether gauge or absolute pressures are used in Bernoulli's equation, provided the same type of pressure is used consistently throughout.

Bernoulli's equation is:
$P + (1/2)\rho v^2 + \rho gh = \text{constant}$

Gauge pressure $P_g = P_{absolute} - P_{atmospheric}$
If we substitute $P_{absolute} = P_g + P_a$ in Bernoulli's equation:
$(P_{g1} + Pa) + (1/2)\rho v1^2 + \rho gh1 = (P_{g2} + Pa) + (1/2)\rho v2^2 + \rho gh2$

The atmospheric pressure Pa cancels from both sides, giving:
$P_{g1} + (1/2)\rho v1^2 + \rho gh1 = P_{g2} + (1/2)\rho v2^2 + \rho gh2$

This is Bernoulli's equation in terms of gauge pressures, and it is identical in form and content.
So the choice of absolute or gauge pressure does not affect the results, as long as one type is used consistently (not mixed) throughout the calculation.

Answer: No, it does not matter. Atmospheric pressure appears on both sides of the equation and cancels out. Gauge and absolute pressures give identical results as long as one type is used consistently.

Q13 Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0×10^{-3} kg s^{-1}, what is the pressure difference between the two ends of the tube? (Density of glycerine = 1.3×10^3 kg m^{-3} and viscosity of glycerine = 0.83 Pa s). [You may also like to check if the assumption of laminar flow in the tube is correct].

SOLUTION

Given: L = 1.5 m, R = 1.0 cm = 0.01 m
Mass flow rate = 4.0×10^{-3} kg s^{-1}
$\rho = 1.3 \times 10^3$ kg m^{-3}, $\eta = 0.83$ Pa s

Volume flow rate:
$Q = (\text{mass flow rate})/\rho = (4.0 \times 10^{-3})/(1.3 \times 10^3) = 3.077 \times 10^{-6}$ m^3 s^{-1}

Using Poiseuille's formula for viscous flow through a cylindrical tube:

$Q = \pi R^4 \Delta P / (8\eta L)$

Solving for pressure difference ΔP:
$\Delta P = 8\eta LQ / (\pi R^4)$
$\Delta P = (8 \times 0.83 \times 1.5 \times 3.077 \times 10^{-6}) / (\pi \times (0.01)^4)$
$\Delta P = (3.065 \times 10^{-5}) / (3.1416 \times 10^{-8})$
$\Delta P \approx 975$ Pa

Checking for laminar flow (Reynolds number):
Average velocity $v = Q/(\pi R^2) = 3.077 \times 10^{-6} / (\pi \times 10^{-4}) = 9.79 \times 10^{-3}$ m s^{-1}
$Re = \rho \times v \times 2R / \eta = (1.3 \times 10^3 \times 9.79 \times 10^{-3} \times 0.02) / 0.83$
$Re \approx 0.31$

Since Re << 2000, the flow is indeed laminar. The use of Poiseuille's formula is justified.

Answer: $\Delta P \approx 975$ Pa. The flow is strongly laminar (Re ≈ 0.31 << 2000).

Q14 In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m s^{-1} and 63 m s^{-1} respectively. What is the lift on the wing if its area is 2.5 m^2? Take the density of air to be 1.3 kg m^{-3}.

SOLUTION

Given: $v_{upper} = 70$ m s^{-1}, $v_{lower} = 63$ m s^{-1}, wing area $A = 2.5$ m^2, $\rho_{air} = 1.3$ kg m^{-3}

Applying Bernoulli's equation between the upper and lower wing surfaces (same height):
$P_{lower} + (1/2)\rho\, v_{lower}^2 = P_{upper} + (1/2)\rho\, v_{upper}^2$

Pressure difference (lower − upper):
$\Delta P = P_{lower} - P_{upper} = (1/2)\rho(v_{upper}^2 - v_{lower}^2)$
$\Delta P = (1/2) \times 1.3 \times (70^2 - 63^2)$
$\Delta P = 0.65 \times (4900 - 3969)$
$\Delta P = 0.65 \times 931$
$\Delta P = 605.15$ Pa

Lift force on wing:
$F = \Delta P \times A = 605.15 \times 2.5 = 1512.9$ N
$F \approx 1.51 \times 10^3$ N

Answer: Lift force ≈ 1.51×10^3 N

Q15 Figures 9.20(a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect? Why?

SOLUTION

Figure 9.20(a) is incorrect.

Each figure shows a horizontal tube with a central constriction, fitted with two vertical manometer tubes: the left tube taps the wide (upstream) section, the right tube taps the narrow constriction. The height of liquid in each vertical tube indicates the pressure at that point in the flow.

Physics of the flow: By the equation of continuity (Av = constant), the fluid speed is higher at the constriction (smaller area). By Bernoulli's principle, higher speed means lower pressure. So the pressure — and hence the liquid column in the manometer — must be LOWER at the constriction than in the wide section.

In Fig. 9.20(a): the manometer tube at the constriction shows a HIGHER liquid column than the one in the wide section. This says the pressure is greater at the constriction, which contradicts Bernoulli's principle (higher speed → lower pressure). Hence (a) is incorrect.

In Fig. 9.20(b): the manometer tube at the constriction shows a LOWER liquid column than the one in the wide section — correctly indicating lower pressure where the flow speed is higher. Hence (b) is the correct figure.

Therefore, Fig. 9.20(a) is incorrect.

Answer: Figure 9.20(a) is incorrect. It shows a higher liquid column (higher pressure) in the manometer at the constriction, but at a constriction the flow speed increases, so by Bernoulli's principle the pressure must be lower there — the column should be lower at the constriction, as correctly shown in Fig. 9.20(b).

Q16 The cylindrical tube of a spray pump has a cross-section of 8.0 cm², one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min⁻¹, what is the speed of ejection of the liquid through the holes?

SOLUTION

Given: cross-section of tube $A_{tube} = 8.0\ cm^2 = 8.0 \times 10^{-4}\ m^2$
Number of holes n = 40, diameter of each hole d = 1.0 mm = 1.0×10^{-3} m
Flow speed inside tube $v_{tube} = 1.5\ m\ min^{-1} = 1.5/60\ m\ s^{-1} = 0.025\ m\ s^{-1}$

Area of each hole:
$A_{hole} = \pi(d/2)^2 = \pi \times (0.5 \times 10^{-3})^2 = 7.854 \times 10^{-7}\ m^2$

Total area of all 40 holes:
$A_{holes} = 40 \times 7.854 \times 10^{-7} = 3.14 \times 10^{-5}\ m^2$

By equation of continuity (conservation of mass):
$A_{tube} \times v_{tube} = A_{holes} \times v_{holes}$

$v_{holes} = (A_{tube} \times v_{tube}) / A_{holes}$
$v_{holes} = (8.0 \times 10^{-4} \times 0.025) / (3.14 \times 10^{-5})$
$v_{holes} = (2.0 \times 10^{-5}) / (3.14 \times 10^{-5})$
$v_{holes} \approx 0.64\ m\ s^{-1}$

Answer: Speed of ejection ≈ 0.64 m s⁻¹

Q17 A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and the light slider supports a weight of 1.5×10^{-2} N (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film?

SOLUTION

Given: weight supported W = 1.5×10^{-2} N, length of slider l = 30 cm = 0.30 m

A soap film has two surfaces (front and back). The total length of the film in contact with the slider is 2l.

The surface tension force supports the weight:
$S \times 2l = W$

$S = W / (2l) = (1.5 \times 10^{-2}) / (2 \times 0.30)$
$S = (1.5 \times 10^{-2}) / 0.60$
$S = 2.5 \times 10^{-2}\ N\ m^{-1}$

Answer: $S = 2.5 \times 10^{-2}$ N m^{-1}

Q18 Figure 9.21(a) shows a thin liquid film supporting a small weight = 4.5×10^{-2} N. What is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c)? Explain your answer physically.

SOLUTION

From Fig. 9.21: All three configurations (a), (b), (c) have films of the same width = 40 cm = 0.40 m.

In Fig. (a): A rectangular film of width 40 cm supports W = 4.5×10^{-2} N.
Surface tension: $S = W/(2l) = (4.5 \times 10^{-2})/(2 \times 0.40) = (4.5 \times 10^{-2})/0.80 = 5.625 \times 10^{-2}$ N m^{-1}

In Fig. (b): The film has a different shape but the same width 40 cm at the slider. The surface tension force depends only on the length of the slider (40 cm) and the surface tension S of the liquid (same liquid, same temperature → same S).
Force = $S \times 2l = 5.625 \times 10^{-2} \times 0.80 = 4.5 \times 10^{-2}$ N

In Fig. (c): Similarly, the slider length is 40 cm; S is the same.
Force = $S \times 2l = 4.5 \times 10^{-2}$ N

Physical explanation:
Surface tension is a property of the liquid-air interface and acts along the contact line (the slider edge). The force exerted on the slider is $F = S \times 2l$, where l is the slider length. It depends only on l and S — not on the shape or area of the film.
Since all three figures have the same slider length (40 cm) and the same liquid at the same temperature (same S), all three films support the same weight = 4.5×10^{-2} N.

Answer: 4.5×10^{-2} N in both Fig. (b) and Fig. (c). The surface tension force $F = S \times 2l$ depends only on the slider length l (same = 40 cm) and the surface tension S (same liquid, same temperature), not on the shape or area of the film.

Q19 What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature? Surface tension of mercury at that temperature (20 °C) is 4.65×10^{-1} N m^{-1}. The atmospheric pressure is 1.01×10^5 Pa. Also give the excess pressure inside the drop.

SOLUTION

Given: radius of drop r = 3.00 mm = 3.00×10^{-3} m
Surface tension S = 4.65×10^{-1} N m^{-1}
Atmospheric pressure Pa = 1.01×10^5 Pa

A mercury drop (liquid in air) has a single liquid-air interface.

Excess pressure inside a spherical drop:
$\Delta P = P_{inside} - P_{outside} = 2S/r$
$\Delta P = 2 \times 4.65 \times 10^{-1} / (3.00 \times 10^{-3})$
$\Delta P = 0.930 / (3.00 \times 10^{-3})$
$\Delta P = 310$ Pa

Total pressure inside the drop:
$P_{inside} = Pa + \Delta P = 1.01 \times 10^5 + 310$
$P_{inside} = 1.0131 \times 10^5$ Pa
$P_{inside} \approx 1.01 \times 10^5$ Pa (to 3 sig figs)

Answer: Excess pressure = 310 Pa; total pressure inside ≈ 1.0131×10^5 Pa

Q20 What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20 °C) is 2.50×10^{-2} N m^{-1}? If an air bubble of the same dimension were formed at depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atmospheric pressure is 1.01×10^5 Pa).

SOLUTION

Given: r = 5.00 mm = 5.00×10^{-3} m, S = 2.50×10^{-2} N m^{-1}
Relative density of soap solution = 1.20, so ρ = 1200 kg m^{-3}
Depth of air bubble = 40.0 cm = 0.40 m, Pa = 1.01×10^5 Pa, g = 9.8 m s^{-2}

Part 1: Excess pressure inside a soap bubble (two surfaces):
A soap bubble has two liquid-air surfaces (inner and outer).
Excess pressure = $4S/r = 4 \times 2.50 \times 10^{-2} / (5.00 \times 10^{-3})$
Excess pressure = $0.10 / (5.00 \times 10^{-3}) = 20$ Pa

Part 2: Pressure inside an air bubble at depth 40.0 cm in soap solution:
An air bubble in a liquid has only ONE liquid-air interface.
Excess pressure for air bubble = $2S/r = 2 \times 2.50 \times 10^{-2} / (5.00 \times 10^{-3}) = 10$ Pa

Pressure outside the air bubble at depth 0.40 m:
$P_{out} = Pa + \rho gh = 1.01 \times 10^5 + (1200 \times 9.8 \times 0.40)$
$P_{out} = 1.01 \times 10^5 + 4704$
$P_{out} = 1.0570 \times 10^5$ Pa

Total pressure inside the air bubble:
$P_{inside} = P_{out} + 2S/r = 1.0570 \times 10^5 + 10$
$P_{inside} = 1.0571 \times 10^5$ Pa
$P_{inside} \approx 1.06 \times 10^5$ Pa

Answer: Excess pressure in soap bubble = 20 Pa; pressure inside air bubble at 40 cm depth ≈ 1.06×10^5 Pa