Chapter 10: Thermal Properties of Matter

Exercises — Exercises

Q1 The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.

SOLUTION

Given:
Triple point of neon: $T_{Ne} = 24.57$ K
Triple point of CO_2: $T_{CO_2} = 216.55$ K

Conversion formula:
Celsius: $t_c = T - 273.15$
Fahrenheit: $t_F = (9/5) \times t_c + 32$

For Neon:
$t_c = 24.57 - 273.15 = -248.58$ °C
$t_F = (9/5) \times (-248.58) + 32 = -447.44 + 32 = -415.44$ °F

For Carbon Dioxide:
$t_c = 216.55 - 273.15 = -56.60$ °C
$t_F = (9/5) \times (-56.60) + 32 = -101.88 + 32 = -69.88$ °F

Answer: Neon: −248.58 °C and −415.44 °F; CO_2: −56.60 °C and −69.88 °F

Q2 Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between TA and TB?

SOLUTION

Both scales are absolute scales, so they have the same physical zero (absolute zero).
The triple point of water is a unique fixed physical state. Its temperature in Kelvin is 273.16 K.

On scale A: 200 A = 273.16 K => 1 A = 273.16/200 K
On scale B: 350 B = 273.16 K => 1 B = 273.16/350 K

Any temperature T corresponds to TA units on scale A and TB units on scale B:
T (in K) = TA × (273.16/200) = TB × (273.16/350)

Dividing both sides (273.16 cancels):
TA / 200 = TB / 350

=> TA / TB = 200 / 350 = 4 / 7

=> TA = (4/7) × TB

Answer: TA = (4/7) TB, or equivalently 7 TA = 4 TB

Q3 The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law: $R = R_o [1 + \alpha (T - T_o)]$. The resistance is 101.6 Ω at the triple-point of water 273.16 K, and 165.5 Ω at the normal melting point of lead (600.5 K). What is the temperature when the resistance is 123.4 Ω?

SOLUTION

Given:

R_o = 101.6 Ω at T_o = 273.16 K

R_{lead} = 165.5 Ω at T_{lead} = 600.5 K

Find T when R = 123.4 Ω

Step 1: Find α from the lead data.

$R_{lead} = R_o [1 + \alpha (T_{lead} - T_o)]$

$165.5 = 101.6 \times [1 + \alpha \times (600.5 - 273.16)]$

$165.5 / 101.6 = 1 + \alpha \times 327.34$

$1.6289... = 1 + 327.34\ \alpha$

$\alpha = 0.6289 / 327.34 = 1.921 \times 10^{-3}\ K^{-1}$

Step 2: Find T when R = 123.4 Ω.

$123.4 = 101.6 \times [1 + 1.921 \times 10^{-3} \times (T - 273.16)]$

$123.4 / 101.6 = 1 + 1.921 \times 10^{-3} \times (T - 273.16)$

$1.2146 = 1 + 1.921 \times 10^{-3} \times (T - 273.16)$

$T - 273.16 = 0.2146 / (1.921 \times 10^{-3}) = 111.7$ K

T = 273.16 + 111.7 = 384.8 K

Answer: T ≈ 384.8 K

Q4 Answer the following:
(a) The triple-point of water is a standard fixed point in modern thermometry. Why? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale)?
(b) There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0 °C and 100 °C respectively. On the absolute scale, one of the fixed points is the triple-point of water, which on the Kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (Kelvin) scale?
(c) The absolute temperature (Kelvin scale) T is related to the temperature tc on the Celsius scale by tc = T − 273.15. Why do we have 273.15 in this relation, and not 273.16?
(d) What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale?

SOLUTION

(a) The triple-point of water is a unique condition (a specific temperature and pressure: 273.16 K and 611.73 Pa) at which all three phases — solid, liquid, and vapour — coexist in equilibrium. This state is perfectly reproducible anywhere in the world, independent of external conditions.
The melting point of ice and the boiling point of water, by contrast, depend on pressure. The melting point shifts slightly with applied pressure, and the boiling point changes significantly with altitude (atmospheric pressure). These dependences make them less reliable as universal fixed points.

(b) On the Kelvin scale, only one fixed point is needed: the triple-point of water (273.16 K). The scale is then defined by the size of the unit (1 K = 1/273.16 of the triple-point temperature). There is no second fixed point assigned a specific numerical value; absolute zero (0 K) is the theoretical lower limit determined by extrapolation, not a separately assigned fixed point.

(c) The triple-point of water is assigned exactly 273.16 K on the Kelvin scale. The melting point of ice at standard atmospheric pressure (the ice point) occurs at a slightly lower temperature than the triple-point — specifically 273.15 K (the triple-point is at 273.16 K and 611.73 Pa, while the ice point is at 273.15 K and 101.325 kPa; the higher pressure very slightly lowers the melting point of ice). Since the Celsius scale defines 0 °C as the ice point (melting of ice at 1 atm), we have: 0 °C = 273.15 K, giving tc = T − 273.15.

(d) The Fahrenheit scale has 180 degree-intervals between the ice point (32 °F) and steam point (212 °F), whereas the Kelvin/Celsius scale has 100 intervals over the same range. So 1 Fahrenheit degree = 100/180 = 5/9 Celsius degree.
On the desired absolute scale (F-sized units, starting from absolute zero):
T_abs = T_kelvin × (9/5) = 273.16 × (9/5) = 491.69
The triple-point of water on this scale = 491.69 (in Fahrenheit-sized absolute units)

Answer: (a) Triple-point is uniquely reproducible (fixed T and P); ice/boiling points depend on pressure. (b) No second fixed point is assigned; only the triple-point (273.16 K) is defined. (c) The ice point (0 °C) equals 273.15 K, not 273.16 K, due to the small pressure difference between the triple-point and the ice point. (d) 491.69 (in Fahrenheit-sized absolute units)

Q5 Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made:

Temperature	Pressure thermometer A	Pressure thermometer B
Triple-point of water	1.250×10^5 Pa	0.200×10^5 Pa
Normal melting point of sulphur	1.797×10^5 Pa	0.287×10^5 Pa

(a) What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B?
(b) What do you think is the reason behind the slight difference in answers of thermometers A and B? (The thermometers are not faulty). What further procedure is needed in the experiment to reduce the discrepancy between the two readings?

SOLUTION

Given:
Triple-point of water: T_tp = 273.16 K
PA (at triple-point) = 1.250 × 10⁵ Pa
PB (at triple-point) = 0.200 × 10⁵ Pa
PA (sulphur) = 1.797 × 10⁵ Pa
PB (sulphur) = 0.287 × 10⁵ Pa

(a) For an ideal gas at constant volume, P ∝ T:
T = T_tp × (P_sulphur / P_tp)

For thermometer A (oxygen):

$T_A = 273.16 \times (1.797 \times 10^5) / (1.250 \times 10^5)$
$T_A = 273.16 \times 1.4376 = 392.69$ K

For thermometer B (hydrogen):
$T_B = 273.16 \times (0.287 \times 10^5) / (0.200 \times 10^5)$
$T_B = 273.16 \times 1.435 = 391.98$ K

(b) The slight difference arises because neither oxygen nor hydrogen is a perfectly ideal gas at the pressures used in the experiment. Real gases deviate from ideal behaviour — the deviations differ between oxygen and hydrogen because they have different intermolecular forces and molecular sizes.

To reduce the discrepancy: repeat the measurements at progressively lower gas pressures (both gases) and extrapolate the readings to zero pressure (zero-pressure limit). At very low pressures all gases approach ideal behaviour, and both thermometers would give the same temperature.

Answer: (a) $T_A = 392.69$ K; $T_B = 391.98$ K. (b) Real gas deviations differ between O_2 and H_2; extrapolate to zero-pressure limit to get the ideal-gas value.

Q6 A steel tape 1 m long is correctly calibrated for a temperature of 27.0 °C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0 °C. What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is 27.0 °C? Coefficient of linear expansion of steel = 1.20×10^{-5} K^{-1}.

SOLUTION

Given:
Tape calibrated correctly at $T_0 = 27.0$ °C
Tape reading on hot day = 63.0 cm at $T = 45.0$ °C
$\alpha_{steel} = 1.20 \times 10^{-5}$ K^{-1}

The tape itself has expanded at 45 °C. Each centimetre mark on the tape is actually larger than 1 cm at this temperature.

Scale factor of tape at 45 °C:
1 division = $(1 + \alpha \times \Delta T)$ cm = $(1 + 1.20 \times 10^{-5} \times 18)$ cm = 1.000216 cm

Actual length of the rod at 45 °C:
$L(45\ °C) = 63.0 \times 1.000216 = 63.0136$ cm ≈ 63.01 cm

Both the tape and the rod are made of the same material (steel). So if the tape

reads 63.0 cm for the rod at 45 °C, and both have expanded by the same factor since 27 °C, the tape would also read 63.0 cm for the rod at 27 °C (the expansion cancels).

Length of the rod at 27.0 °C:
$L(27\ °C) = L(45\ °C) / (1 + \alpha \times \Delta T) = 63.0136 / 1.000216 = 63.00$ cm

Alternatively: at 27 °C the tape is correctly calibrated, so the tape reading directly gives the true length. If the rod and tape are the same material, the tape always reads 63.0 cm for this rod regardless of temperature — so L(27 °C) = 63.0 cm.

Answer: Actual length at 45 °C ≈ 63.01 cm; length at 27 °C = 63.00 cm

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Q7 A large steel wheel is to be fitted on to a shaft of the same material. At 27 °C, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of steel to be constant over the required temperature range: $\alpha_{steel} = 1.20 \times 10^{-5}\ K^{-1}$.

SOLUTION

Given:
$T_1 = 27\ °C$
$d_{shaft} = 8.70$ cm at 27 °C
$d_{hole} = 8.69$ cm (wheel hole, unchanged since only the shaft is cooled)
$\alpha_{steel} = 1.20 \times 10^{-5}\ K^{-1}$

The shaft must be cooled until its diameter equals the wheel hole diameter (8.69 cm).

$d_{shaft} \times (1 + \alpha \times (T - T_1)) = d_{hole}$
$8.70 \times (1 + 1.20 \times 10^{-5} \times (T - 27)) = 8.69$

$1 + 1.20 \times 10^{-5} \times (T - 27) = 8.69 / 8.70 = 0.998851$
$1.20 \times 10^{-5} \times (T - 27) = -0.001149$
$T - 27 = -0.001149 / (1.20 \times 10^{-5}) = -95.77\ K$
$T = 27 - 95.77 = -68.77\ °C \approx -69\ °C$

Answer: The shaft must be cooled to approximately −69 °C for the wheel to slip on.

Q8 A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0 °C. What is the change in the diameter of the hole when the sheet is heated to 227 °C? Coefficient of linear expansion of copper = $1.70 \times 10^{-5}\ K^{-1}$.

SOLUTION

Given:

$d_0 = 4.24$ cm at $T_1 = 27.0$ °C

$T_2 = 227$ °C, $\Delta T = 227 - 27 = 200$ K

$\alpha_{Cu} = 1.70 \times 10^{-5}\ K^{-1}$

A hole in a material expands (or contracts) exactly as if it were filled with the same material — the diameter changes by the same fractional amount $\alpha_l \times \Delta T$.

$\Delta d = \alpha_{Cu} \times d_0 \times \Delta T$

$\Delta d = 1.70 \times 10^{-5} \times 4.24 \times 200$

$\Delta d = 1.70 \times 10^{-5} \times 848$

$\Delta d = 0.01442$ cm $\approx 1.44 \times 10^{-2}$ cm $= 0.144$ mm

Answer: Change in diameter $\approx 1.44 \times 10^{-2}$ cm (0.144 mm); the hole expands.

Q9 A brass wire 1.8 m long at 27 °C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of −39 °C, what is the tension developed in the wire, if its diameter is 2.0 mm? Co-efficient of linear expansion of brass = $2.0 \times 10^{-5}\ K^{-1}$; Young's modulus of brass = 0.91×10^{11} Pa.

SOLUTION

Given:

$L = 1.8$ m, $T_1 = 27$ °C, $T_2 = -39$ °C

$\Delta T = T_2 - T_1 = -66$ K (cooling by 66 K)

Diameter $d = 2.0$ mm $= 2.0 \times 10^{-3}$ m $\Rightarrow$ $r = 1.0 \times 10^{-3}$ m

$\alpha_{brass} = 2.0 \times 10^{-5}\ K^{-1}$

$Y_{brass} = 0.91 \times 10^{11}$ Pa

Step 1: Cross-sectional area of wire.

$A = \pi r^2 = \pi \times (1.0 \times 10^{-3})^2 = 3.14159 \times 10^{-6}\ m^2$

Step 2: Thermal strain.

When the wire cools, it would naturally contract by $\Delta L = \alpha \times L \times$	ΔT	, but the rigid supports prevent this. The prevented contraction sets up a tensile strain:
strain = $\alpha \times$	ΔT	$= 2.0 \times 10^{-5} \times 66 = 1.32 \times 10^{-3}$

Step 3: Thermal stress.
stress = $Y \times$ strain = $0.91 \times 10^{11} \times 1.32 \times 10^{-3} = 1.2012 \times 10^{8}$ Pa

Step 4: Tension (force).
F = stress $\times A = 1.2012 \times 10^{8} \times 3.14159 \times 10^{-6} = 377.4$ N $\approx 3.77 \times 10^{2}$ N

Answer: Tension developed $\approx 3.77 \times 10^{2}$ N (≈ 377 N)

Q10 A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250 °C, if the original lengths are at 40.0 °C? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand (Co-efficient of linear expansion of brass = 2.0×10^{-5} K^{-1}, steel = 1.2×10^{-5} K^{-1}).

SOLUTION

Given:
$L_{brass} = L_{steel} = 50$ cm $= 0.50$ m
$T_1 = 40.0$ °C, $T_2 = 250$ °C, $\Delta T = 210$ K
$\alpha_{brass} = 2.0 \times 10^{-5}$ K^{-1}, $\alpha_{steel} = 1.2 \times 10^{-5}$ K^{-1}
Ends are free to expand.

Change in length of brass rod:
$\Delta L_{brass} = \alpha_{brass} \times L \times \Delta T = 2.0 \times 10^{-5} \times 0.50 \times 210 = 2.10 \times 10^{-3}$ m $= 0.21$ cm

Change in length of steel rod:
$\Delta L_{steel} = \alpha_{steel} \times L \times \Delta T = 1.2 \times 10^{-5} \times 0.50 \times 210 = 1.26 \times 10^{-3}$ m $= 0.126$ cm

Total change in length of combined rod:
$\Delta L_{total} = \Delta L_{brass} + \Delta L_{steel} = 0.21 + 0.126 = 0.336$ cm ≈ 3.36 mm

Thermal stress at junction:
Since the ends are free to expand, both rods expand freely. No external constraint prevents expansion. Therefore, no thermal stress is developed at the junction.
The brass rod expands more than the steel rod, but since both ends are free, the two rods simply expand independently — there is no compressive or tensile

stress at the junction.

Answer: Total change in length = 0.336 cm (3.36 mm); no thermal stress is developed at the junction.

Q11 The coefficient of volume expansion of glycerine is $49 \times 10^{-5}\ K^{-1}$. What is the fractional change in its density for a 30 °C rise in temperature?

SOLUTION

Given:

α_V (glycerine) = $49 \times 10^{-5}\ K^{-1} = 4.9 \times 10^{-4}\ K^{-1}$

$\Delta T = 30\ °C = 30\ K$

For a fixed mass m, density $\rho = m/V$.

When volume increases by ΔV: $\Delta V/V = \alpha_V \times \Delta T$

New density: $\rho' = m / (V + \Delta V) = \rho / (1 + \alpha_V \times \Delta T)$

Fractional change in density:

$\Delta\rho/\rho = (\rho' - \rho)/\rho = 1/(1 + \alpha_V \times \Delta T) - 1 = -\alpha_V \times \Delta T / (1 + \alpha_V \times \Delta T)$

Since $\alpha_V \times \Delta T = 4.9 \times 10^{-4} \times 30 = 1.47 \times 10^{-2} << 1$, we use the approximation:

$\Delta\rho/\rho \approx -\alpha_V \times \Delta T = -4.9 \times 10^{-4} \times 30 = -1.47 \times 10^{-2}$

Answer: Fractional change in density ≈ -1.47×10^{-2} (density decreases by about 1.47%)

Q12 A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminium = 0.91 J g^{-1} K^{-1}.

SOLUTION

Given:

P = 10 kW = 10,000 W

m = 8.0 kg

t = 2.5 min = 150 s

50% of power heats the aluminium block

$s_{Al} = 0.91\ J\ g^{-1}\ K^{-1} = 910\ J\ kg^{-1}\ K^{-1}$

Energy delivered to the block:

Q = 0.50 × P × t = 0.50 × 10,000 × 150 = 750,000 J

Temperature rise: Q = m × s × ΔT
ΔT = Q / (m × s) = 750,000 / (8.0 × 910)
ΔT = 750,000 / 7280 = 103.02 K ≈ 103 °C

Answer: Rise in temperature ≈ 103 °C

Q13 A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500 °C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of copper = 0.39 J g^{-1} K^{-1}; heat of fusion of water = 335 J g^{-1}).

SOLUTION

Given:
m_{Cu} = 2.5 kg
T_{Cu} = 500 °C; ice is at 0 °C (large block so temperature stays 0 °C)
s_{Cu} = 0.39 J g^{-1} K^{-1} = 390 J kg^{-1} K^{-1}
L_f = 335 J g^{-1} = 335,000 J kg^{-1}

Heat released by copper as it cools from 500 °C to 0 °C:
$Q = m_{Cu} \times s_{Cu} \times (T_{Cu} - T_{ice})$
$Q = 2.5 \times 390 \times (500 - 0) = 2.5 \times 390 \times 500$
Q = 487,500 J

Mass of ice melted:
$m_{ice} = Q / L_f = 487{,}500 / 335{,}000 = 1.455$ kg ≈ 1.46 kg

Answer: Maximum ice that can melt ≈ 1.46 kg

Q14 In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150 °C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm^3 of water at 27 °C. The final temperature is 40 °C. Compute the specific heat of the metal. If heat losses to the surroundings are not negligible, is your answer greater or smaller than the actual value for specific heat of the metal?

SOLUTION

Given:
m_{metal} = 0.20 kg, T_{metal} = 150 °C
Water equivalent of calorimeter = 0.025 kg

Volume of water = 150 cm^3 => m_{water} = 0.150 kg (density of water = 1 g/cm^3)
$T_{initial}$ (water + calorimeter) = 27 °C
T_{final} = 40 °C
s_{water} = 4186 J kg^{-1} K^{-1}

Heat gained by water and calorimeter:
$Q_{gained} = (m_{water} + water_{equivalent}) \times s_{water} \times (T_{final} - T_{initial})$
$Q_{gained} = (0.150 + 0.025) \times 4186 \times (40 - 27)$
$Q_{gained} = 0.175 \times 4186 \times 13 = 9523$ J

Heat lost by metal:
$Q_{lost} = m_{metal} \times s_{metal} \times (T_{metal} - T_{final})$
$Q_{lost} = 0.20 \times s_{metal} \times (150 - 40) = 0.20 \times s_{metal} \times 110$

Equating (assuming no heat loss to surroundings):
$0.20 \times s_{metal} \times 110 = 9523$
$s_{metal} = 9523 / (0.20 \times 110) = 9523 / 22 = 432.9$ J kg^{-1} K^{-1} ≈ 433 J kg^{-1} K^{-1}

If heat losses to surroundings are not negligible:
The actual heat gained by water + calorimeter is less than the heat lost by the metal (some heat escapes to surroundings). Our calculation assumes all metal heat goes to water/calorimeter. Therefore, the denominator (Q_{gained}) is less than the actual Q_{lost}, making our calculated s_{metal} smaller than the true value.
=> Our answer is smaller than the actual specific heat of the metal.

Answer: Specific heat of metal ≈ 433 J kg^{-1} K^{-1}; with heat losses, the computed value is smaller than the actual value.

Q15 Given below are observations on molar specific heats at room temperature of some common gases.

Gas	Molar specific heat (Cv) (cal mol^{-1} K^{-1})
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon monoxide	5.01
Chlorine	6.17

The measured molar specific heats of these gases are markedly different from those for monatomic gases. Typically, molar specific heat of a monatomic gas is 2.92 cal/mol K. Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine?

SOLUTION

For a monatomic ideal gas (e.g., He, Ar), Cv = (3/2)R = 2.98 cal mol^{-1} K^{-1} ≈ 2.92 cal mol^{-1} K^{-1} (only three translational degrees of freedom contribute).

The gases listed — H_2, N_2, O_2, NO, CO — are diatomic molecules. In addition to three translational degrees of freedom, diatomic molecules have two rotational degrees of freedom (rotation about the two axes perpendicular to the molecular axis). At room temperature, vibrational modes are not significantly excited.

Using the law of equipartition of energy: each active degree of freedom contributes (1/2)R to Cv.
For diatomic gases (5 active degrees at room temperature):
Cv = (5/2)R = 5/2 × 1.987 = 4.97 cal mol^{-1} K^{-1}

This matches well with the experimental values (4.87–5.02 cal mol^{-1} K^{-1}) for H_2, N_2, O_2, NO, and CO, confirming five active degrees of freedom.

For chlorine (Cl_2), Cv = 6.17 cal mol^{-1} K^{-1}, which is significantly higher than the diatomic value of ≈ 5.0.
Inference: The value close to (7/2)R = 6.95 cal mol^{-1} K^{-1} (all seven degrees — 3 trans + 2 rot + 2 vib) suggests that vibrational modes are partially excited in Cl_2 at room temperature. This is because Cl_2 has a relatively weak bond (lower vibrational frequency), so the vibrational degrees of freedom begin to become active even at room temperature. For H_2, N_2, O_2, CO, the bond is stiffer and vibrational modes are not excited at room temperature.

Answer: Diatomic molecules have 5 active degrees of freedom (3 translational + 2 rotational) at room temperature, giving Cv ≈ (5/2)R ≈ 4.97 cal mol^{-1} K^{-1}, matching the data. Chlorine's higher Cv (6.17) indicates vibrational modes are partially active at room temperature due to its weaker, lower-frequency bond.

Q16 A child running a temperature of 101 °F is given an antipyrin (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98 °F in 20 minutes, what is the average rate of extra evaporation caused by the drug. Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about 580 cal g^{-1}.

SOLUTION

Given:
T_1 = 101 °F, T_2 = 98 °F, time = 20 min
m = 30 kg = 30,000 g
s = 1 cal g^{-1} $°C^{-1}$ (same as water)
Lv = 580 cal g^{-1}

Temperature drop in Celsius:
$\Delta T = (101 - 98) \times (5/9) = 3 \times 5/9 = 5/3$ °C

Heat that must be removed from the child's body:
$Q = m \times s \times \Delta T = 30{,}000 \times 1 \times (5/3) = 50{,}000$ cal

Total mass of sweat evaporated:
$m_{evap} = Q / Lv = 50{,}000 / 580 = 86.21$ g

Average rate of extra evaporation:
Rate = m_{evap} / time = 86.21 g / 20 min = 4.31 g/min

Answer: Average rate of extra evaporation ≈ 4.3 g/min

Q17 A 'thermacole' icebox is a cheap and an efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is 45 °C, and co-efficient of thermal conductivity of thermacole is 0.01 J s^{-1} m^{-1} K^{-1}. [Heat of fusion of water = 335 × 10^3 J kg^{-1}]

SOLUTION

Given:
Cubical box: side = 30 cm = 0.30 m
Thickness L = 5.0 cm = 0.05 m
m_{ice} = 4.0 kg

$T_{outside} = 45\ °C$, $T_{inside} = 0\ °C$ (ice melts at 0 °C, temperature stays constant)
$K = 0.01\ J\ s^{-1}\ m^{-1}\ K^{-1}$
$L_f = 335 \times 10^3\ J\ kg^{-1}$
$t = 6\ h = 6 \times 3600 = 21{,}600\ s$

Total surface area of cube (6 faces):
$A = 6 \times (0.30)^2 = 6 \times 0.09 = 0.54\ m^2$

Rate of heat flow into the box:
$H = K \times A \times (T_{outside} - T_{inside}) / L$
$H = 0.01 \times 0.54 \times 45 / 0.05$
$H = 0.01 \times 0.54 \times 900 = 4.86\ J/s = 4.86\ W$

Total heat entering in 6 hours:
$Q = H \times t = 4.86 \times 21{,}600 = 104{,}976\ J$

Mass of ice melted:
$m_{melted} = Q / L_f = 104{,}976 / 335{,}000 = 0.3134\ kg \approx 0.313\ kg$

Ice remaining:
$m_{remaining} = 4.0 - 0.313 = 3.687\ kg \approx 3.69\ kg$

Answer: Ice remaining after 6 h ≈ 3.69 kg

Q18 A brass boiler has a base area of 0.15 m² and thickness 1.0 cm. It boils water at the rate of 6.0 kg/min when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = 109 J s^{-1} m^{-1} K^{-1}; Heat of vaporisation of water = 2256 × 10^3 J kg^{-1}.

SOLUTION

Given:
$A = 0.15\ m^2$, $L = 1.0\ cm = 0.01\ m$
Boiling rate = 6.0 kg/min = 6.0/60 kg/s = 0.10 kg/s
$K_{brass} = 109\ J\ s^{-1}\ m^{-1}\ K^{-1}$
$L_v = 2256 \times 10^3\ J\ kg^{-1}$
T_{water} (boiling) = 100 °C

Rate of heat required to vaporise water:
$H = boil_rate \times L_v = 0.10 \times 2256 \times 10^3 = 225{,}600\ W$

In steady state, this heat is conducted through the brass base:
$H = K \times A \times (T_{flame} - T_{water}) / L$

225,600 = 109 × 0.15 × (T_flame − 100) / 0.01
225,600 = 1635 × (T_flame − 100)
T_flame − 100 = 225,600 / 1635 = 137.98 °C
T_flame = 100 + 138 = 238 °C

Answer: Temperature of the flame in contact with the boiler ≈ 238 °C

Q19 Explain why:
(a) a body with large reflectivity is a poor emitter
(b) a brass tumbler feels much colder than a wooden tray on a chilly day
(c) an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace
(d) the earth without its atmosphere would be inhospitably cold
(e) heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water

SOLUTION

(a) By Kirchhoff's law of thermal radiation, at any given temperature and wavelength, the emissivity of a body equals its absorptivity. A body with large reflectivity absorbs very little radiation (low absorptivity). Therefore, by Kirchhoff's law, it also emits very little radiation (low emissivity) — it is a poor emitter.

(b) The brass tumbler and the wooden tray are both at the same room temperature (below body temperature on a chilly day). However, brass is a good conductor of heat (high thermal conductivity ~100 W m^{-1} K^{-1}), while wood is a poor conductor (~0.12 W m^{-1} K^{-1}). When you touch brass, it rapidly conducts heat away from your hand, making your hand cool faster and the brass feels colder. Wood conducts heat away very slowly, so the sensation is warmer.

(c) An optical pyrometer works by matching the brightness (intensity at a specific wavelength) of the hot object to a calibrated standard using Stefan-Boltzmann's law for a perfect blackbody.
For a red hot iron piece in the open (emissivity e < 1): the actual radiation emitted = e × σT^4. The pyrometer interprets this lower intensity as if it were coming from a blackbody at a lower temperature T', giving a reading lower than the true temperature T.
When the same iron piece is inside a furnace (a closed enclosure): the furnace

acts like a blackbody cavity regardless of the emissivity of its walls. The radiation emerging from a small hole in the furnace is true blackbody radiation at the furnace temperature. So the pyrometer gives the correct temperature.

(d) The atmosphere acts like a thermal blanket around the Earth (the 'greenhouse effect'). It absorbs and re-radiates the infrared radiation emitted by the Earth's surface, reducing the net heat loss to space. Without the atmosphere, the Earth would lose heat rapidly by radiation into space at night and would not be warmed adequately, making it extremely cold (estimated mean surface temperature would be about −18 °C instead of the current +15 °C).

(e) Steam at 100 °C carries a large amount of latent heat (Lv ≈ 2256 kJ kg^{-1}) in addition to sensible heat. When steam condenses in the radiator, it releases this latent heat as well as the sensible heat as the water cools further. Hot water at ~100 °C can only release sensible heat as it cools. For the same mass flow rate, steam delivers far more heat per kilogram than hot water. Therefore, steam-based heating systems are more efficient.

Answer: (a) By Kirchhoff's law: high reflectivity → low absorptivity → low emissivity (poor emitter). (b) Brass has high thermal conductivity, rapidly removes heat from the hand, feeling colder than wood (low conductivity). (c) Open iron (e < 1) emits less than a blackbody; pyrometer under-reads. In a furnace cavity, radiation is true blackbody emission; reading is correct. (d) Without the atmosphere's greenhouse effect, Earth radiates heat to space unchecked and would be freezing cold. (e) Steam releases additional latent heat (~2256 kJ/kg) on condensation, delivering far more heat than hot water of the same mass.

Q20 A body cools from 80 °C to 50 °C in 5 minutes. Calculate the time it takes to cool from 60 °C to 30 °C. The temperature of the surroundings is 20 °C.

SOLUTION

Given:

$T_{surr} = 20$ °C

Stage 1: cools from 80 °C to 50 °C in $t_1 = 5$ min

Stage 2: cools from 60 °C to 30 °C in $t_2 = ?$

Using Newton's law of cooling in the average-temperature form:

$\Delta T/\Delta t = K (T_{avg} - T_{surr})$

Stage 1:

$T_{avg1} = (80 + 50) / 2 = 65\ °C$

Excess over surroundings = $65 - 20 = 45\ °C$

Rate = $(80 - 50) / 5 = 6\ °C/min$

$6 = K \times 45 \Rightarrow K = 6/45 = 2/15\ min^{-1}$

Stage 2:

$T_{avg2} = (60 + 30) / 2 = 45\ °C$

Excess over surroundings = $45 - 20 = 25\ °C$

Rate = $(60 - 30) / t_2 = 30/t_2\ °C/min$

$30/t_2 = K \times 25 = (2/15) \times 25 = 50/15 = 10/3$

$t_2 = 30 \times 3 / 10 = 9\ min$

Answer: Time to cool from 60 °C to 30 °C = 9 minutes