Chapter 13: Oscillations

Exercises — Exercises

Q1 Which of the following examples represent periodic motion?

(a) A swimmer completing one (return) trip from one bank of a river to the other and back.

(b) A freely suspended bar magnet displaced from its N-S direction and released.

(c) A hydrogen molecule rotating about its centre of mass.

(d) An arrow released from a bow.

SOLUTION

A periodic motion is one that repeats itself identically after a fixed interval of time (the period).

(a) A swimmer's return trip: the swimmer goes from one bank to the other and comes back. However, the motion depends on external factors (current, fatigue, stroke variation) and does not repeat identically. This is NOT periodic motion.

(b) A freely suspended bar magnet displaced from its N-S direction: when displaced and released, the magnet experiences a restoring torque (due to Earth's magnetic field) and oscillates about the N-S equilibrium direction. This motion repeats identically — it IS periodic (and approximately SHM for small displacements).

(c) A hydrogen molecule rotating about its centre of mass: the rotation repeats with a fixed angular period. This IS periodic motion.

(d) An arrow released from a bow: the arrow flies through the air and does not return or repeat its motion. This is NOT periodic.

Answer: (b) and (c) represent periodic motion.

Q2 Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?
(a) the rotation of earth about its axis.
(b) motion of an oscillating mercury column in a U-tube.
(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.
(d) general vibrations of a polyatomic molecule about its equilibrium position.

SOLUTION

SHM requires the restoring force (or torque) to be directly proportional to the displacement from the equilibrium position.

(a) Rotation of Earth about its axis: this is uniform circular motion (or nearly so), not oscillatory motion about an equilibrium position. It is periodic but NOT simple harmonic.

(b) Oscillating mercury column in a U-tube: when one column is depressed by a small amount x, the restoring force is proportional to x (weight of extra mercury column = 2ρAgx). The equation of motion is $\ddot{x} = -(2g/L)x$, which is exactly of the SHM form. This IS (nearly) simple harmonic motion.

(c) Ball bearing in a smooth curved bowl, released from slightly above the lowest point: for small displacements, the component of gravity along the bowl's surface provides a restoring force proportional to displacement (analogous to a pendulum). This is (nearly) simple harmonic motion.

(d) General vibrations of a polyatomic molecule: such vibrations involve several normal modes at different frequencies and the net restoring force is not simply proportional to a single displacement coordinate. This is periodic but NOT simple harmonic.

Answer: (b) and (c) are nearly SHM. (a) and (d) are periodic but not SHM.

Q3 Fig. 13.18 depicts four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?

SOLUTION

Reading the four x-t graphs from Fig. 13.18:

(a) The displacement x increases continuously with time (monotonically). The

motion never repeats. This is NOT periodic.

(b) The graph shows a sinusoidal oscillation. The time axis is marked at t = –3, –1, 0, 1, 3 s. The pattern repeats every 2 s. This IS periodic with period T = 2 s.

(c) The graph shows irregular, non-repeating variations. Even though the motion appears to go up and down, the pattern does not identically repeat. This is NOT periodic.

(d) The graph shows a regular sinusoidal oscillation. The time axis is marked at t = –3, –2, –1, 0, 1, 2, 3 s and the waveform clearly repeats every 2 s. This IS periodic with period T = 2 s.

Answer: Plots (b) and (d) represent periodic motion, each with period T = 2 s. Plots (a) and (c) are non-periodic.

Q4 Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):
(a) $\sin \omega t - \cos \omega t$
(b) $\sin^3 \omega t$
(c) $3 \cos(\pi/4 - 2\omega t)$
(d) $\cos \omega t + \cos 3\omega t + \cos 5\omega t$
(e) $\exp(-\omega^2 t^2)$
(f) $1 + \omega t + \omega^2 t^2$

SOLUTION

SHM requires displacement of the form $A \sin(\omega_o t + \varphi)$ or $A \cos(\omega_o t + \varphi)$ — a single sinusoid.

(a) $\sin \omega t - \cos \omega t$
$= \sqrt{2} [\sin \omega t \times (1/\sqrt{2}) - \cos \omega t \times (1/\sqrt{2})]$
$= \sqrt{2} [\sin \omega t \cos(\pi/4) - \cos \omega t \sin(\pi/4)]$
$= \sqrt{2} \sin(\omega t - \pi/4)$
This is a single sinusoid of amplitude $\sqrt{2}$ and angular frequency ω.
=> SIMPLE HARMONIC MOTION, period $T = 2\pi/\omega$.

(b) $\sin^3 \omega t$
Using the identity: $\sin^3\theta = (3 \sin\theta - \sin 3\theta)/4$
$= (3/4) \sin \omega t - (1/4) \sin 3\omega t$
This is a superposition of two SHMs with different frequencies (ω and 3ω). The

resultant is periodic (period = LCM of 2π/ω and 2π/3ω = 2π/ω) but NOT a single sinusoid.
=> PERIODIC BUT NOT SIMPLE HARMONIC, period T = 2π/ω.

(c) 3 cos(π/4 – 2ωt)
= 3 cos(2ωt – π/4) [since cos(–θ) = cos θ]
This is a single sinusoid with amplitude 3 and angular frequency 2ω.
=> SIMPLE HARMONIC MOTION, period T = 2π/2ω = π/ω.

(d) cos ωt + cos 3ωt + cos 5ωt
This is a sum of three SHMs with angular frequencies ω, 3ω, 5ω. The resultant is periodic (period = LCM of 2π/ω, 2π/3ω, 2π/5ω = 2π/ω) but NOT a single sinusoid.
=> PERIODIC BUT NOT SIMPLE HARMONIC, period T = 2π/ω.

(e) exp(–ω²t²)
At t = 0, this equals 1. As t increases (or decreases), the function decreases monotonically toward 0 (a Gaussian curve). It never repeats its value.
=> NON-PERIODIC.

(f) 1 + ωt + ω²t²
This is a polynomial in t that increases indefinitely with time. It never repeats.
=> NON-PERIODIC.

Answer: SHM: (a) T = 2π/ω; (c) T = π/ω. Periodic but not SHM: (b) T = 2π/ω; (d) T = 2π/ω. Non-periodic: (e) and (f).

Q5 A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is
(a) at the end A,
(b) at the end B,
(c) at the mid-point of AB going towards A,
(d) at 2 cm away from B going towards A,
(e) at 3 cm away from A going towards B, and
(f) at 4 cm away from B going towards A.

SOLUTION

Setup: A and B are the extreme ends of SHM, 10 cm apart.
Take A→B as positive direction (+ve x).

The mid-point (mean position) O is at the centre: OA = OB = 5 cm.
Taking O as origin: A is at x = –5 cm, B is at x = +5 cm.

Key rules:
• At extreme positions (A or B), velocity v = 0; the particle momentarily stops and reverses.
• Acceleration $a = -\omega^2 x$, always directed towards O (mean position).
• Force $F = ma = -m\omega^2 x$, same direction as acceleration.
• Velocity is in the direction of motion.

(a) At end A (x = –5 cm):
Velocity = 0 (extreme position, particle momentarily at rest).
Acceleration = $-\omega^2(-5)$ = +ve (directed towards O, i.e., towards B).
Force = positive (towards B).

(b) At end B (x = +5 cm):
Velocity = 0 (extreme position, momentarily at rest).
Acceleration = $-\omega^2(+5)$ = –ve (directed towards O, i.e., towards A).
Force = negative (towards A).

(c) At mid-point of AB (x = 0) going towards A (i.e., in –ve direction):
Velocity = –ve (moving towards A).
Acceleration = $-\omega^2(0) = 0$ (at mean position).
Force = 0.

(d) At 2 cm from B going towards A:
x = +5 – 2 = +3 cm (still positive, between O and B).
Velocity = –ve (moving towards A).
Acceleration = $-\omega^2(+3)$ = –ve (directed towards O, i.e., towards A — same direction as motion here).
Force = negative.

(e) At 3 cm from A going towards B:
x = –5 + 3 = –2 cm (negative, between O and A).
Velocity = +ve (moving towards B).
Acceleration = $-\omega^2(-2)$ = +ve (directed towards O, i.e., towards B — same direction as motion).
Force = positive.

(f) At 4 cm from B going towards A:
x = +5 – 4 = +1 cm (positive, between O and B).

Velocity = –ve (moving towards A).
Acceleration = $-\omega^2(+1)$ = –ve (directed towards O, i.e., towards A).
Force = negative.

Answer: (a) v = 0, a = +ve, F = +ve. (b) v = 0, a = –ve, F = –ve. (c) v = –ve, a = 0, F = 0. (d) v = –ve, a = –ve, F = –ve. (e) v = +ve, a = +ve, F = +ve. (f) v = –ve, a = –ve, F = –ve.

Q6 Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?
(a) a = 0.7x
(b) $a = -200x^2$
(c) a = –10x
(d) $a = 100x^3$

SOLUTION

For SHM, the acceleration must satisfy: $a = -\omega^2 x$
Requirements: (i) a must be proportional to x (linear in x), and (ii) the proportionality constant must be negative (restoring character).

(a) a = 0.7x:
Linear in x, but the constant (+0.7) is positive.
This means force is directed away from the origin (repulsive/unstable), NOT restoring.
=> NOT SHM.

(b) $a = -200x^2$:
Proportional to x^2 — not linear in x.
=> NOT SHM.

(c) a = –10x:
Linear in x with a negative proportionality constant.
This is of the form $a = -\omega^2 x$ with $\omega^2 = 10$, i.e., $\omega = \sqrt{10}$ rad/s.
=> REPRESENTS SHM.

(d) $a = 100x^3$:
Proportional to x^3 — not linear in x. Also positive coefficient (non-restoring).
=> NOT SHM.

Answer: Only (c) a = –10x represents SHM (with $\omega^2 = 10$, i.e., $\omega = \sqrt{10}$ rad/s).

Q7 The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A \cos(\omega t + \varphi)$. If the initial ($t = 0$) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is $\pi\ s^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM: $x = B \sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

SOLUTION

Given: $\omega = \pi\ s^{-1}$, $x(0) = 1$ cm, $v(0) = \omega$ cm/s $= \pi$ cm/s.

Using $x(t) = A \cos(\omega t + \varphi)$:
At $t = 0$: $x(0) = A \cos \varphi = 1$ ... (1)
Velocity: $v(t) = -A\omega \sin(\omega t + \varphi)$
At $t = 0$: $v(0) = -A\omega \sin \varphi = \pi$
$\Rightarrow -A \sin \varphi = \pi/\omega = 1 \Rightarrow A \sin \varphi = -1$ ... (2)

From (1) and (2):
$A^2 = (A \cos \varphi)^2 + (A \sin \varphi)^2 = 1^2 + (-1)^2 = 2$
$A = \sqrt{2}$ cm

$\tan \varphi = (A \sin \varphi)/(A \cos \varphi) = -1/1 = -1$
φ is in the fourth quadrant ($\cos \varphi > 0$, $\sin \varphi < 0$): $\varphi = -\pi/4$

Verification: $x(0) = \sqrt{2} \cos(-\pi/4) = \sqrt{2} \times (1/\sqrt{2}) = 1$ cm ✓
$v(0) = -\sqrt{2} \times \pi \times \sin(-\pi/4) = -\sqrt{2}\pi \times (-1/\sqrt{2}) = \pi$ cm/s ✓

Using $x(t) = B \sin(\omega t + \alpha)$:
At $t = 0$: $x(0) = B \sin \alpha = 1$ ... (3)
Velocity: $v(t) = B\omega \cos(\omega t + \alpha)$
At $t = 0$: $v(0) = B\omega \cos \alpha = \pi$
$\Rightarrow B \cos \alpha = \pi/\omega = 1$ ... (4)

From (3) and (4):
$B^2 = 1^2 + 1^2 = 2 \Rightarrow B = \sqrt{2}$ cm
$\tan \alpha = (B \sin \alpha)/(B \cos \alpha) = 1/1 = 1$
α is in the first quadrant: $\alpha = \pi/4$

Verification: $x(0) = \sqrt{2} \sin(\pi/4) = 1$ cm ✓
$v(0) = \sqrt{2} \times \pi \times \cos(\pi/4) = \pi$ cm/s ✓

Answer: Cosine form: amplitude $A = \sqrt{2}$ cm, initial phase $\varphi = -\pi/4$. Sine form: amplitude $B = \sqrt{2}$ cm, initial phase $\alpha = \pi/4$.

Q8 A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

SOLUTION

Given: Scale range 0–50 kg, length of scale = 20 cm = 0.20 m, period T = 0.6 s.

Step 1: Find the spring constant k.
Maximum load = 50 kg, which produces maximum extension = 0.20 m.
Using Hooke's law: F = kx
$k = F/x = (50 \times 9.8)\ N / 0.20\ m = 490/0.20 = 2450\ N\ m^{-1}$

Step 2: Find the mass m of the oscillating body.
Period: $T = 2\pi \sqrt{(m/k)}$
$\Rightarrow m = k T^2 / (4\pi^2)$
$= 2450 \times (0.6)^2 / (4\pi^2)$
$= 2450 \times 0.36 / 39.478$
$= 882.0 / 39.478$
$= 22.34\ kg$

Step 3: Find weight.
$W = mg = 22.34 \times 9.8 = 218.9\ N \approx 219\ N$

Answer: Weight of the body ≈ 219 N (mass ≈ 22.3 kg).

Q9 A spring having with a spring constant 1200 N m^{-1} is mounted on a horizontal table as shown in Fig. 13.19. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released. Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.

SOLUTION

Given: $k = 1200\ N\ m^{-1}$, m = 3 kg, amplitude A = 2.0 cm = 0.020 m.

Angular frequency:
$\omega = \sqrt{(k/m)} = \sqrt{(1200/3)} = \sqrt{400} = 20.0\ rad\ s^{-1}$

(i) Frequency of oscillations:
$f = \omega/(2\pi) = 20.0/(2 \times 3.14159) = 20.0/6.2832 = 3.18\ Hz$

(ii) Maximum acceleration:
$a_{max} = \omega^2 A = (20.0)^2 \times 0.020 = 400 \times 0.020 = 8.0\ m\ s^{-2}$

(iii) Maximum speed:
$v_{max} = \omega A = 20.0 \times 0.020 = 0.40 \text{ m s}^{-1}$

Answer: (i) $f \approx 3.18$ Hz; (ii) $a_{max} = 8.0 \text{ m s}^{-2}$; (iii) $v_{max} = 0.40 \text{ m s}^{-1}$.

Q10 In Exercise 13.9, let us take the position of mass when the spring is unstretched as x = 0, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is
(a) at the mean position,
(b) at the maximum stretched position, and
(c) at the maximum compressed position.
In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

SOLUTION

From Exercise 13.9: $\omega = 20.0 \text{ rad s}^{-1}$, amplitude $A = 0.020 \text{ m} = 2.0 \text{ cm}$.

General form: $x(t) = A \cos(\omega t + \varphi_o)$

(a) At t = 0, mass is at the mean position (x = 0):
$x(0) = A \cos(\varphi_o) = 0 \Rightarrow \varphi_o = \pm\pi/2$
The mass was pulled to the right (+A) and released, so at the mean position it moves in the negative direction (towards left). Taking $v(0) < 0$: $v(0) = -A\omega \sin(\varphi_o) < 0 \Rightarrow \sin(\varphi_o) > 0 \Rightarrow \varphi_o = +\pi/2$.
$x(t) = 0.02 \cos(20t + \pi/2) = -0.02 \sin(20t)$ m
Equivalently, if the mass moves in the positive x direction at the mean position: $x(t) = 0.02 \sin(20t)$ m.

(b) At t = 0, mass is at the maximum stretched position (x = +A):
$x(0) = A \cos(\varphi_o) = A \Rightarrow \varphi_o = 0$.
$x(t) = 0.02 \cos(20t)$ m

(c) At t = 0, mass is at the maximum compressed position (x = –A):
$x(0) = A \cos(\varphi_o) = -A \Rightarrow \varphi_o = \pi$.
$x(t) = 0.02 \cos(20t + \pi) = -0.02 \cos(20t)$ m

Comparison:
All three functions have the same frequency ($\omega = 20 \text{ rad s}^{-1}$, $f = 3.18$ Hz) and the same amplitude (0.02 m). They differ only in the initial phase angle: (a) $\varphi_o = \pm\pi/2$,

(b) $\varphi_o = 0$, (c) $\varphi_o = \pi$.

Answer: (a) $x(t) = \pm 0.02 \sin(20t)$ m; (b) $x(t) = 0.02 \cos(20t)$ m; (c) $x(t) = -0.02 \cos(20t)$ m. All have the same frequency and amplitude; they differ only in initial phase.

Q11 Figures 13.20 correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure.
Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

SOLUTION

The x-projection of a particle P moving on a circle of radius r with angular speed ω is given by:
$x(t) = r \cos(\text{angle at time } t)$
For anticlockwise rotation: angle = $\omega t + \varphi_o$; for clockwise rotation: angle = $-\omega t + \varphi_o$,
where φ_o is the initial angle of OP with the +x axis.

Figure 13.20(a): radius = 3 cm, T = 2 s, P(t=0) at the bottom of the circle (i.e., on the –y axis), anticlockwise rotation.

$\omega = 2\pi/T = 2\pi/2 = \pi$ rad s^{-1}

Initial angle of OP from +x axis: $\varphi_o = -\pi/2$ (i.e., pointing down along –y)

For anticlockwise rotation: $x(t) = 3 \cos(\pi t - \pi/2) = 3 \sin(\pi t)$ cm

Figure 13.20(b): radius = 2 m, T = 4 s, P(t=0) on the –x axis (leftmost point), clockwise rotation.

$\omega = 2\pi/T = 2\pi/4 = \pi/2$ rad s^{-1}

Initial angle of OP from +x axis: $\varphi_o = \pi$ (P points along –x)

For clockwise rotation, angle decreases with time: angle = $\varphi_o - \omega t = \pi - (\pi/2)t$

$x(t) = 2 \cos(\pi - \pi t/2) = -2 \cos(\pi t/2)$ m [since $\cos(\pi - \theta) = -\cos\theta$]

Check: at t = 0, x = –2 m (P is at the leftmost point, x = –2 m). ✓

Answer: (a) $x(t) = 3 \sin(\pi t)$ cm; (b) $x(t) = -2 \cos(\pi t/2)$ m.

Q12 Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).
(a) $x = -2 \sin(3t + \pi/3)$
(b) $x = \cos(\pi/6 - t)$
(c) $x = 3 \sin(2\pi t + \pi/4)$
(d) $x = 2 \cos \pi t$

SOLUTION

For anticlockwise rotation, the SHM x-projection is $x(t) = r \cos(\omega t + \varphi_0)$.
Rewrite each expression in cosine form to read off r (radius), ω (angular speed), φ_0 (initial angle from +x axis).

(a) $x = -2 \sin(3t + \pi/3)$
Using $-\sin\theta = \cos(\theta + \pi/2)$:
$x = 2 \cos(3t + \pi/3 + \pi/2) = 2 \cos(3t + 5\pi/6)$
=> Radius r = 2 cm, $\omega = 3$ rad s^{-1}, initial angle $\varphi_0 = 5\pi/6$ (= 150° from +x axis, in the second quadrant).
At t = 0: P is at $(r \cos(5\pi/6), r \sin(5\pi/6)) = (-\sqrt{3}, 1)$ cm from centre.

(b) $x = \cos(\pi/6 - t)$
Since $\cos(-\theta) = \cos\theta$: $x = \cos(t - \pi/6)$.
In the anticlockwise form $x = r \cos(\omega t + \varphi_0)$: here $\omega = 1$ rad s^{-1}, $\varphi_0 = -\pi/6$.
=> Radius r = 1 cm, $\omega = 1$ rad s^{-1}, initial angle $\varphi_0 = -\pi/6$ (= –30°, or equivalently 330° from +x axis, in the fourth quadrant).

(c) $x = 3 \sin(2\pi t + \pi/4)$
Using $\sin\theta = \cos(\theta - \pi/2)$:
$x = 3 \cos(2\pi t + \pi/4 - \pi/2) = 3 \cos(2\pi t - \pi/4)$
=> Radius r = 3 cm, $\omega = 2\pi$ rad s^{-1}, initial angle $\varphi_0 = -\pi/4$ (= –45°, or 315° from +x axis, in the fourth quadrant).

(d) $x = 2 \cos \pi t$
Already in standard cosine form with $\varphi_0 = 0$.
=> Radius r = 2 cm, $\omega = \pi$ rad s^{-1}, initial angle $\varphi_0 = 0$ (P starts on the +x axis).

Answer: (a) r = 2 cm, ω = 3 rad/s, $\varphi_0 = 5\pi/6$ (150°). (b) r = 1 cm, ω = 1 rad/s, $\varphi_0 = -\pi/6$ (330°). (c) r = 3 cm, ω = 2π rad/s, $\varphi_0 = -\pi/4$ (315°). (d) r = 2 cm, ω = π rad/s, $\varphi_0 = 0°$.

Q13 Figure 13.21(a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure 13.21(b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. 13.21(b) is stretched by the same force F.
(a) What is the maximum extension of the spring in the two cases?
(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?

SOLUTION

(a) Maximum extension:

Case (a): Spring fixed at one end, force F applied at the other end.
By Hooke's law, extension x = F/k.

Case (b): Spring free at both ends, force F applied at each end (equal and opposite).
The spring is in equilibrium under equal forces F at each end.
The tension throughout the spring is F (same as Case a).
Total extension = F/k.
Both cases give the same extension F/k.

(b) Period of oscillation after release:

Case (a): Mass m on spring of constant k.
$T_a = 2\pi \sqrt{(m/k)}$

Case (b): Two equal masses m connected by spring k, no external supports.
The centre of mass remains stationary. By symmetry, each mass oscillates with amplitude equal to half the total spring extension.
To find the effective equation of motion: let x_1 and x_2 be displacements from equilibrium.
By symmetry: $x_1 = -x_2 = \xi$ (say).
Spring extension = $x_1 - x_2 = 2\xi$.
Restoring force on mass 1 = $-k(2\xi) = -2k\xi$.
Equation of motion: $m\ddot{\xi} = -2k\xi$
=> $\ddot{\xi} = -(2k/m)\,\xi$, which is SHM with $\omega^2 = 2k/m$.
$T_b = 2\pi \sqrt{(m/(2k))} = 2\pi/\sqrt{(2k/m)}$

Alternatively, using the concept of reduced mass $\mu = m \cdot m/(m+m) = m/2$ on spring k:

$T_b = 2\pi \sqrt{(\mu/k)} = 2\pi \sqrt{(m/2k)}$ — same result.

Answer: (a) Maximum extension = F/k in both cases. (b) Period in case (a): $T = 2\pi \sqrt{(m/k)}$; Period in case (b): $T = 2\pi \sqrt{(m/2k)}$.

Q14 The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min, what is its maximum speed?

SOLUTION

Given: stroke = 2A = 1.0 m => amplitude A = 0.5 m.
Angular frequency: $\omega = 200$ rad $min^{-1} = 200/60$ rad $s^{-1} = 10/3$ rad s^{-1}.

Maximum speed in SHM:
$v_{max} = \omega A = (10/3) \times 0.5 = 5/3$ m $s^{-1} \approx 1.67$ m s^{-1}

Answer: Maximum speed = $\omega A = (200/60) \times 0.5 \approx 1.67$ m s^{-1}.

Q15 The acceleration due to gravity on the surface of moon is 1.7 m s^{-2}. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (g on the surface of earth is 9.8 m s^{-2})

SOLUTION

Given: $g_{moon} = 1.7$ m s^{-2}, $g_{earth} = 9.8$ m s^{-2}, $T_{earth} = 3.5$ s.

Time period of a simple pendulum: $T = 2\pi \sqrt{(L/g)}$
For a given pendulum (same L):
$T_{moon} / T_{earth} = \sqrt{(g_{earth} / g_{moon})}$

$T_{moon} = T_{earth} \times \sqrt{(g_{earth} / g_{moon})}$
$= 3.5 \times \sqrt{(9.8 / 1.7)}$
$= 3.5 \times \sqrt{5.7647}$
$= 3.5 \times 2.401$
$= 8.40$ s

Answer: Time period on Moon ≈ 8.4 s.

Q16 A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

SOLUTION

The car moves on a circular track of radius R with constant speed v.
In the non-inertial frame of the car, the bob experiences two accelerations:

1. Gravitational acceleration g (vertically downward).
2. Centripetal acceleration v^2/R (directed radially inward towards the centre of the circular track), which appears as a centrifugal pseudo-force v^2/R directed radially outward.

These two accelerations are perpendicular to each other (g is vertical, v^2/R is horizontal).
The effective gravitational acceleration (determining the restoring force for pendulum oscillations) is:
$g_{eff} = \sqrt{(g^2 + (v^2/R)^2)}$

The time period of a simple pendulum under effective gravity g_{eff} is:
$T = 2\pi \sqrt{(l / g_{eff})}$
$= 2\pi \sqrt{(l / \sqrt{(g^2 + v^4/R^2)})}$

Answer: $T = 2\pi \sqrt{[l / (g^2 + v^4/R^2)^{(1/2)}]}$.

Q17 A cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_l. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period
$T = 2\pi \sqrt{(h\rho / \rho_l g)}$
where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

SOLUTION

Let the cork have base area A, height h, and density ρ.
The liquid has density $\rho_l > \rho$ (so the cork floats).

At equilibrium, the cork floats with depth x_o submerged.
Weight of cork = buoyant force
$\rho \times A \times h \times g = \rho_l \times A \times x_o \times g$
$\Rightarrow x_o = \rho h/\rho_l$ (equilibrium submerged depth)

Now suppose the cork is pushed down by a small displacement y from its

equilibrium position.
New submerged depth = $x_0 + y$.

Net upward force on cork:
F_{net} = Buoyancy – Weight
$= \rho_l \times A \times (x_0 + y) \times g - \rho \times A \times h \times g$
$= \rho_l \times A \times x_0 \times g + \rho_l \times A \times y \times g - \rho \times A \times h \times g$

The first and third terms cancel (equilibrium condition $\rho_l A x_0 g = \rho A h g$), leaving:
$F_{net} = \rho_l \times A \times g \times y$ (upward)

This force acts upward when $y > 0$ (pushed down) and downward when $y < 0$ (pulled up),
so it is always a restoring force:
$F_{net} = -\rho_l A g y$

Mass of cork: $m = \rho A h$
Equation of motion: $m \ddot{y} = -\rho_l A g y$
$\rho A h \ddot{y} = -\rho_l A g y$
$\ddot{y} = -(\rho_l g / \rho h) y$

This is of the form $\ddot{y} = -\omega^2 y$ with $\omega^2 = \rho_l g / (\rho h)$.
Hence the motion is simple harmonic.

Time period:
$T = 2\pi/\omega = 2\pi \sqrt{(\rho h / \rho_l g)} = 2\pi \sqrt{(h\rho / \rho_l g)}$ (proved)

Answer: The cork executes SHM with $T = 2\pi \sqrt{(h\rho / \rho_l g)}$. (Derived above by showing $F_{net} = -\rho_l A g y$, giving $\omega^2 = \rho_l g/\rho h$.)

Q18 One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

SOLUTION

Let the total length of the mercury column in the U-tube be L, the cross-sectional area of the tube be A, and the density of mercury be ρ.

When the pressure difference is removed, the mercury column is displaced from the equilibrium position. Let x be the displacement of the mercury in one arm

below the equilibrium level (and simultaneously x above equilibrium in the other arm).

The height difference between the two columns becomes 2x.

Restoring force (due to the extra weight of the mercury column of height 2x):
$F = -\rho \times A \times 2x \times g = -2\rho A g\, x$
(Negative sign: the force acts to restore equilibrium, opposing the displacement.)

Total mass of mercury: $m = \rho AL$

Equation of motion (Newton's second law):
$\rho AL \times \ddot{x} = -2\rho A g\, x$
$\ddot{x} = -(2g/L)\, x$

This is of the form $\ddot{x} = -\omega^2 x$ with $\omega^2 = 2g/L$, which is the defining equation of SHM.

Therefore, when the suction pump is removed, the mercury column executes simple harmonic motion.

Time period: $T = 2\pi/\omega = 2\pi \sqrt{(L/2g)} = \pi \sqrt{(2L/g)}$

Answer: The mercury executes SHM with $\ddot{x} = -(2g/L)x$, giving $T = \pi\sqrt{(2L/g)}$. (Proved by showing the restoring force is proportional to displacement.)