Chapter 14: Waves

Exercises — Exercises

Q1 A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

SOLUTION

Given: mass m = 2.50 kg, tension T = 200 N, length L = 20.0 m

Linear mass density: $\mu = m/L = 2.50/20.0 = 0.125$ kg m^{-1}

Speed of transverse wave: $v = \sqrt{(T/\mu)} = \sqrt{(200/0.125)} = \sqrt{1600} = 40.0$ m s^{-1}

Time for disturbance to travel from one end to the other:
$t = L/v = 20.0/40.0 = 0.5$ s

Answer: 0.5 s

Q2 A stone dropped from the top of a tower of height 300 m splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 m s^{-1}? (g = 9.8 m s^{-2})

SOLUTION

Given: height h = 300 m, v_{sound} = 340 m s^{-1}, g = 9.8 m s^{-2}

Time for the stone to fall to the base (free fall from rest):
$h = \frac{1}{2} g t_1^2 \Rightarrow t_1 = \sqrt{(2h/g)} = \sqrt{(2 \times 300 / 9.8)} = \sqrt{(61.22)} = 7.82$ s

Time for the sound to travel from the base to the top:
$t_2 = h/v_{sound} = 300/340 = 0.88$ s

Total time after the stone is dropped:
$t = t_1 + t_2 = 7.82 + 0.88 = 8.70$ s

Answer: The splash is heard approximately 8.70 s after the stone is dropped.

Q3 A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 20 °C = 343 m s^{-1}.

SOLUTION

Given: L = 12.0 m, m = 2.10 kg, v = 343 m s^{-1}

Linear mass density: $\mu = m/L = 2.10/12.0 = 0.175$ kg m^{-1}

For a transverse wave: $v = \sqrt{(T/\mu)} \Rightarrow T = \mu v^2$

$T = 0.175 \times (343)^2 = 0.175 \times 117649 = 20589$ N

$T \approx 2.06 \times 10^4$ N

Answer: $T \approx 2.06 \times 10^4$ N

Q4 Use the formula $v = \sqrt{(\gamma P/\rho)}$ to explain why the speed of sound in air
(a) is independent of pressure,
(b) increases with temperature,
(c) increases with humidity.

SOLUTION

The speed of sound in an ideal gas is given by $v = \sqrt{(\gamma P/\rho)}$.

(a) Independent of pressure:
For an ideal gas at constant temperature, $PV = nRT \Rightarrow P = (n/V)RT = (\rho/M)RT$, so $P/\rho = RT/M$ = constant (at fixed T).
Therefore $v = \sqrt{(\gamma P/\rho)} = \sqrt{(\gamma RT/M)}$, which does not involve P.
When P increases at constant T, ρ increases proportionally, keeping P/ρ constant.
Hence v is independent of pressure.

(b) Increases with temperature:
$v = \sqrt{(\gamma RT/M)}$, where R, γ, M are constants for a given gas.
$v \propto \sqrt{T}$ (T is absolute temperature).
As temperature T increases, v increases.

(c) Increases with humidity:
Humid (moist) air is a mixture of dry air (M ≈ 29 g mol^{-1}) and water vapour (M_{water} = 18 g mol^{-1}).
Water vapour has a lower molar mass than dry air.
As humidity increases, the effective molar mass M of the air mixture decreases.

Since $v = \sqrt{(\gamma RT/M)}$, a smaller M gives a larger v.
Therefore, the speed of sound increases with humidity.

Answer: (a) v independent of P because $P/\rho = RT/M$ = constant at fixed T. (b) $v \propto \sqrt{T}$, so v increases with temperature. (c) Humid air has lower effective molar mass ($M_{water} = 18 < M_{air} = 29$), so $v = \sqrt{(\gamma RT/M)}$ is larger.

Q5 You have learnt that a travelling wave in one dimension is represented by a function $y = f(x, t)$ where x and t must appear in the combination $x - vt$ or $x + vt$, i.e. $y = f(x \pm vt)$. Is the converse true? Examine if the following functions for y can possibly represent a travelling wave:
(a) $(x - vt)^2$
(b) $\log[(x + vt)/x_0]$
(c) $1/(x + vt)$

SOLUTION

The converse is NOT necessarily true. A function of the form $f(x \pm vt)$ represents a travelling wave only if it is finite, single-valued, and periodic (or at least well-behaved) for all x and t. Functions of $x \pm vt$ that are not finite everywhere or not periodic do not represent physical travelling waves.

(a) $y = (x - vt)^2$:
This is of the form $f(x - vt)$, so it travels in the +x direction.
However, $y \to \infty$ as $|x - vt| \to \infty$. It is not bounded.
It does not represent a (periodic) travelling wave.

(b) $y = \log[(x + vt)/x_0]$:
This is of the form $f(x + vt)$, travelling in the –x direction.
However, log is undefined for $(x + vt) \leq 0$, so y is not defined for all x and t.
It does not represent a physical travelling wave.

(c) $y = 1/(x + vt)$:
This is of the form $f(x + vt)$, travelling in the –x direction.
However, $y \to \infty$ when $x + vt = 0$. It is not finite everywhere.
It does not represent a physical travelling wave.

Answer: No — none of (a), (b), (c) represents a valid travelling wave: (a) is unbounded, (b) is undefined for non-positive arguments, (c) diverges at $x + vt = 0$.

Q6 A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is 340 m s^{-1} and in water 1486 m s^{-1}.

SOLUTION

Given: frequency f = 1000 kHz = 1.0×10^6 Hz
v_{air} = 340 m s^{-1}, v_{water} = 1486 m s^{-1}

Frequency does not change when sound crosses a medium boundary.

(a) Reflected sound (travels back in air):
$\lambda_{reflected} = v_{air} / f = 340 / (1.0 \times 10^6) = 3.4 \times 10^{-4}$ m = 0.34 mm

(b) Transmitted sound (travels in water):
$\lambda_{transmitted} = v_{water} / f = 1486 / (1.0 \times 10^6) = 1.486 \times 10^{-3}$ m ≈ 1.49 mm

Answer: (a) $\lambda_{reflected} = 3.4 \times 10^{-4}$ m (0.34 mm) in air; (b) $\lambda_{transmitted} = 1.49 \times 10^{-3}$ m (1.49 mm) in water.

Q7 A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 km s^{-1}? The operating frequency of the scanner is 4.2 MHz.

SOLUTION

Given: v_{tissue} = 1.7 km s^{-1} = 1700 m s^{-1}, f = 4.2 MHz = 4.2×10^6 Hz

Wavelength: $\lambda = v/f = 1700 / (4.2 \times 10^6)$
$\lambda = 4.05 \times 10^{-4}$ m ≈ 0.4 mm

Answer: $\lambda \approx 4.05 \times 10^{-4}$ m (≈ 0.4 mm)

Q8 A transverse harmonic wave on a string is described by
$y(x, t) = 3.0 \sin (36\, t + 0.018\, x + \pi/4)$
where x and y are in cm and t in s. The positive direction of x is from left to right.
(a) Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation?
(b) What are its amplitude and frequency?
(c) What is the initial phase at the origin?
(d) What is the least distance between two successive crests in the wave?

SOLUTION

Comparing $y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4)$ with the standard form $y = a \sin(\omega t + kx + \varphi)$:
$a = 3.0$ cm, $\omega = 36$ rad s^{-1}, $k = 0.018$ rad cm^{-1}, $\varphi = \pi/4$

(a) The terms ωt and kx have the same sign (+), indicating a wave travelling in the NEGATIVE x-direction (from right to left).
Speed: $v = \omega/k = 36 / 0.018 = 2000$ cm $s^{-1} = 20$ m s^{-1}
Direction: negative x-direction (from right to left).

(b) Amplitude: $a = 3.0$ cm
Frequency: $f = \omega / (2\pi) = 36 / (2\pi) \approx 5.73$ Hz
Period: $T = 2\pi/\omega = 2\pi/36 \approx 0.175$ s

(c) Initial phase at the origin ($x = 0$, $t = 0$):
Phase $= \omega(0) + k(0) + \pi/4 = \pi/4$ rad

(d) The least distance between two successive crests equals the wavelength:
$\lambda = 2\pi/k = 2\pi / 0.018 \approx 349$ cm ≈ 3.49 m

Answer: (a) Travelling wave, speed = 20 m s^{-1} in the negative x-direction. (b) Amplitude = 3.0 cm, frequency ≈ 5.73 Hz. (c) Initial phase = $\pi/4$ rad. (d) $\lambda \approx 3.49$ m.

Q9 For the wave described in Exercise 14.8, plot the displacement (y) versus (t) graphs for x = 0, 2 and 4 cm. What are the shapes of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another: amplitude, frequency or phase?

SOLUTION

The wave is $y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4)$, with y in cm and t in s.

For $x = 0$: $y(0, t) = 3.0 \sin(36t + \pi/4)$

For x = 2 cm: y(2, t) = 3.0 sin(36t + 0.036 + π/4) = 3.0 sin(36t + π/4 + 0.036)
For x = 4 cm: y(4, t) = 3.0 sin(36t + 0.072 + π/4) = 3.0 sin(36t + π/4 + 0.072)

All three graphs are sinusoidal (sine curves).
They have the same amplitude (3.0 cm) and the same frequency (≈ 5.73 Hz).
They differ only in their initial phase.

Phase difference between x = 0 and x = 2 cm: k × Δx = 0.018 × 2 = 0.036 rad
Phase difference between x = 2 cm and x = 4 cm: 0.018 × 2 = 0.036 rad

Conclusion: In a travelling wave, all points oscillate with the same amplitude and the same frequency. The oscillatory motion differs from point to point only in PHASE.

Answer: All three y-t graphs are sinusoidal with the same amplitude (3.0 cm) and frequency (≈ 5.73 Hz). They differ only in phase. In a travelling wave, points differ in phase, not in amplitude or frequency.

Q10 For the travelling harmonic wave
y(x, t) = 2.0 cos 2π (10t – 0.0080 x + 0.35)
where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of
(a) 4 m,
(b) 0.5 m,
(c) λ/2,
(d) 3λ/4

SOLUTION

The wave is y(x, t) = 2.0 cos 2π(10t – 0.0080x + 0.35), with x in cm.
Angular wave number: k = 2π × 0.0080 rad cm^{-1}
Wavelength: λ = 1/0.0080 = 125 cm = 1.25 m

Phase difference for separation Δx: Δφ = k Δx = 2π × 0.0080 × Δx (Δx in cm)

(a) Δx = 4 m = 400 cm:
Δφ = 2π × 0.0080 × 400 = 2π × 3.2 = 6.4π rad ≈ 20.1 rad

(b) Δx = 0.5 m = 50 cm:
Δφ = 2π × 0.0080 × 50 = 2π × 0.4 = 0.8π rad ≈ 2.51 rad

(c) Δx = λ/2 = 62.5 cm:
Δφ = k × λ/2 = π rad

(d) $\Delta x = 3\lambda/4 = 93.75$ cm:
$\Delta\varphi = k \times 3\lambda/4 = 3\pi/2$ rad

Answer: (a) 6.4π rad (≈ 20.1 rad); (b) 0.8π rad (≈ 2.51 rad); (c) π rad; (d) $3\pi/2$ rad.

Q11 The transverse displacement of a string (clamped at its both ends) is given by
$y(x, t) = 0.06 \sin(2\pi x/3) \cos(120\pi t)$
where x and y are in m and t in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg.
Answer the following:
(a) Does the function represent a travelling wave or a stationary wave?
(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency, and speed of each wave?
(c) Determine the tension in the string.

SOLUTION

The function $y(x, t) = 0.06 \sin(2\pi x/3) \cos(120\pi t)$ is a product of a function of x only and a function of t only.

(a) This represents a STATIONARY (standing) wave. The spatial and temporal parts are separated; there is no propagating phase (kx – ωt).

(b) Using the identity: $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A–B)]$,
$y = 0.06 \sin(2\pi x/3) \cos(120\pi t)$
$= 0.03 [\sin(2\pi x/3 + 120\pi t) + \sin(2\pi x/3 – 120\pi t)]$
$= 0.03 \sin(2\pi x/3 + 120\pi t) + 0.03 \sin(2\pi x/3 – 120\pi t)$

Wave 1 (travelling in –x direction): $y_1 = 0.03 \sin(120\pi t + 2\pi x/3)$
Wave 2 (travelling in +x direction): $y_2 = 0.03 \sin(2\pi x/3 – 120\pi t)$

For each constituent wave:
$k = 2\pi/3$ rad m^{-1} => $\lambda = 2\pi/k = 3.0$ m
$\omega = 120\pi$ rad s^{-1} => $f = \omega/(2\pi) = 60$ Hz
Speed: $v = \omega/k = 120\pi / (2\pi/3) = 120\pi \times 3/(2\pi) = 180$ m s^{-1}

(c) Tension in the string:
Linear mass density: $\mu = m/L = (3.0 \times 10^{-2}) / 1.5 = 0.02$ kg m^{-1}
$v = \sqrt{(T/\mu)}$ => $T = \mu v^2 = 0.02 \times (180)^2 = 0.02 \times 32400 = 648$ N

Answer: (a) Stationary wave. (b) Each constituent: $\lambda = 3.0$ m, $f = 60$ Hz, speed = 180 m s^{-1}. (c) Tension T = 648 N.

Q12 (i) For the wave on a string described in Exercise 14.11, do all the points on the string oscillate with the same (a) frequency, (b) phase, (c) amplitude? Explain your answers. (ii) What is the amplitude of a point 0.375 m away from one end?

SOLUTION

The wave is $y(x, t) = 0.06 \sin(2\pi x/3) \cos(120\pi t)$.

(i)(a) Frequency: YES — all points oscillate at the same frequency, f = 60 Hz, determined by $\cos(120\pi t)$.

(i)(b) Phase: NOT all points have the same phase. The time dependence is the same [$\cos(120\pi t)$] for all points, but the spatial factor $\sin(2\pi x/3)$ changes sign at nodes. Points on opposite sides of a node are 180° (π rad) out of phase with each other. Points between the same pair of consecutive nodes are in phase.

(i)(c) Amplitude: NO — the amplitude at each point is $0.06|\sin(2\pi x/3)|$, which depends on x. It is zero at nodes and maximum (0.06 m) at antinodes.

(ii) Amplitude at x = 0.375 m:
$A(x) = 0.06\ |\sin(2\pi \times 0.375 / 3)| = 0.06\ |\sin(\pi/4)|$
$= 0.06 \times (1/\sqrt{2}) = 0.06 \times 0.7071 \approx 0.0424$ m

Answer: (i) (a) Same frequency: yes, 60 Hz. (b) Same phase: no — points across a node are π out of phase. (c) Same amplitude: no — amplitude varies as $0.06|\sin(2\pi x/3)|$. (ii) Amplitude at x = 0.375 m is ≈ 0.0424 m (= $3\sqrt{2}$ cm).

Q13 Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all:
(a) $y = 2 \cos(3x) \sin(10t)$
(b) $y = 2\sqrt{(x - vt)}$
(c) $y = 3 \sin(5x - 0.5t) + 4 \cos(5x - 0.5t)$
(d) $y = \cos x \sin t + \cos 2x \sin 2t$

SOLUTION

(a) $y = 2 \cos(3x) \sin(10t)$:
This is a product of a function of x alone and a function of t alone.
=> STATIONARY WAVE (ii)

(b) $y = 2\sqrt{(x - vt)}$:
Although this is of the form f(x – vt), the square root function is not defined for x –

$vt < 0$. The function is not periodic and diverges as $x - vt \to \infty$. It does not represent a physical wave.
=> NONE (iii)

(c) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$:
Both terms have the same argument $(5x - 0.5t)$, so they can be combined using the identity $A\sin\theta + B\cos\theta = \sqrt{(A^2+B^2)}\sin(\theta + \varphi)$:
$y = \sqrt{(3^2 + 4^2)}\sin(5x - 0.5t + \varphi) = 5\sin(5x - 0.5t + \varphi)$
This is a single harmonic travelling wave in the +x direction with amplitude 5.
=> TRAVELLING WAVE (i)

(d) $y = \cos x \sin t + \cos 2x \sin 2t$:
Each term separately is a standing wave (product of function of x and function of t), but they have different wavelengths (2π and π) and different frequencies (1 and 2 rad s^{-1}). Their sum is neither a single travelling wave nor a single standing wave.
=> NONE as a single wave type (iii) — it is a superposition of two standing waves with different modes.

Answer: (a) Stationary wave; (b) None — not a valid wave function; (c) Travelling wave (amplitude 5, same direction); (d) None — superposition of two standing waves of different wavelengths and frequencies.

Q14 A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear mass density is 4.0×10^{-2} kg m^{-1}. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?

SOLUTION

Given: $f_1 = 45$ Hz (fundamental), $m = 3.5 \times 10^{-2}$ kg, $\mu = 4.0 \times 10^{-2}$ kg m^{-1}

Length of wire: $L = m/\mu = (3.5 \times 10^{-2}) / (4.0 \times 10^{-2}) = 0.875$ m

(a) For fundamental mode of a string fixed at both ends: $f_1 = v/(2L)$
$v = 2 L f_1 = 2 \times 0.875 \times 45 = 78.75$ m $s^{-1} \approx 79$ m s^{-1}

(b) Tension: $T = \mu v^2 = (4.0 \times 10^{-2}) \times (78.75)^2 = 4.0 \times 10^{-2} \times 6201.6 = 248$ N

Answer: (a) Speed $v \approx 79$ m s^{-1}; (b) Tension $T \approx 248$ N.

Q15 A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.

SOLUTION

Given: tuning fork frequency f = 340 Hz
First resonance length: L_1 = 25.5 cm = 0.255 m
Second resonance length: L_2 = 79.3 cm = 0.793 m

The tube is closed at one end (piston) and open at the other.
Consecutive resonances of a closed pipe differ by λ/2:
$L_2 - L_1 = \lambda/2$
$\lambda = 2(L_2 - L_1) = 2 \times (0.793 - 0.255) = 2 \times 0.538 = 1.076$ m

Speed of sound:
$v = f \times \lambda = 340 \times 1.076 = 365.8$ m $s^{-1} \approx 366$ m s^{-1}

Answer: Speed of sound ≈ 366 m s^{-1}

Q16 A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod are given to be 2.53 kHz. What is the speed of sound in steel?

SOLUTION

Given: L = 100 cm = 1.0 m, f = 2.53 kHz = 2530 Hz

The rod is clamped at its middle. The clamped point is a node; both free ends are antinodes.
Distance from node to nearest antinode = λ/4.
$L/2 = \lambda/4 \Rightarrow \lambda = 2L = 2 \times 1.0 = 2.0$ m

Speed of sound in steel:
$v = f \times \lambda = 2530 \times 2.0 = 5060$ m s^{-1}

Answer: Speed of sound in steel = 5060 m s^{-1} ≈ 5.06 km s^{-1}

Q17 A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 m s^{-1}).

SOLUTION

Given: L = 20 cm = 0.20 m, f_{source} = 430 Hz, v = 340 m s^{-1}

For a pipe closed at one end, the resonant frequencies are:
$f_n = (2n - 1)\, v / (4L)$, for n = 1, 2, 3, ...

Fundamental (n = 1): $f_1 = 340 / (4 \times 0.20) = 340 / 0.80 = 425$ Hz
The source frequency 430 Hz is very close to the fundamental (425 Hz).
The first harmonic (fundamental) of the closed pipe resonates with the 430 Hz source.

For a pipe open at both ends, resonant frequencies are:
$f_n = n\, v / (2L) = n \times 340 / (2 \times 0.20) = n \times 850$ Hz
n = 1: 850 Hz, n = 2: 1700 Hz, ...
430 Hz does not match any harmonic of the open pipe.
The same source will NOT be in resonance with the open pipe.

Answer: Closed pipe: 430 Hz excites the first harmonic (fundamental ≈ 425 Hz). Open pipe: no resonance (fundamental = 850 Hz, no harmonic matches 430 Hz).

Q18 Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?

SOLUTION

Given: original beat frequency = 6 Hz, reduced beat frequency = 3 Hz after reducing tension in A.
Original frequency of A: f_A = 324 Hz

Reducing tension reduces the frequency of string A (f_A decreases).

Case 1: If $f_B = 324 + 6 = 330$ Hz ($f_B > f_A$ initially):
After reducing f_A, the difference $f_B - f_A$ increases => beats increase. This contradicts the observation.

Case 2: If $f_B = 324 - 6 = 318$ Hz ($f_B < f_A$ initially):
After reducing f_A, f_A approaches f_B, so $f_A - f_B$ decreases => beats

decrease.
This matches the observation (beats reduced from 6 to 3 Hz).

Therefore, f_B = 318 Hz.

Answer: Frequency of string B = 318 Hz

Q19 Explain why (or how):
(a) in a sound wave, a displacement node is a pressure antinode and vice versa,
(b) bats can ascertain distances, directions, nature, and sizes of the obstacles without any 'eyes',
(c) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,
(d) solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and
(e) the shape of a pulse gets distorted during propagation in a dispersive medium.

SOLUTION

(a) Displacement node is a pressure antinode (and vice versa):
In a sound standing wave, a displacement node is a point of zero displacement — neighbouring layers of air are being maximally pushed together or pulled apart. The rate of change of displacement with position ($\partial s/\partial x$) is maximum at a displacement node, which means the compressions and rarefactions (and hence pressure variation) are maximum there. Therefore, a displacement node is a pressure antinode.
Conversely, at a displacement antinode, displacement is maximum but its rate of change ($\partial s/\partial x$) is zero, so pressure variation is zero — a pressure node.

(b) How bats navigate (echolocation):
Bats emit ultrasonic pulses (very high frequency, ~40 kHz–100 kHz) and detect the echoes reflected from obstacles. From the time delay between emission and reception of the echo, a bat calculates the distance to the obstacle. The direction of arrival of the echo gives the direction. Differences in echo intensity and spectral content between the two ears give information about the shape, size, and surface texture of the obstacle. This biological sonar system lets bats navigate and hunt in complete darkness.

(c) Violin and sitar notes at same frequency — timbre (quality):
A string on a violin or sitar vibrates not only in its fundamental mode but also in

higher harmonics (overtones) simultaneously. Although both instruments may produce the same fundamental frequency, the relative intensities (amplitudes) of the higher harmonics are different for each instrument due to differences in shape, material, resonating body, and playing technique. This unique mixture of harmonics gives each instrument its characteristic quality or timbre, allowing us to distinguish them even at the same pitch.

(d) Why solids support transverse waves but gases do not:
Transverse waves require the medium to resist shear (tangential) deformation — this property is characterised by the shear modulus. Solids have a definite shape and a non-zero shear modulus, so they can sustain shear stress and support transverse waves.
Fluids (gases and liquids) have no shear modulus — they cannot resist tangential forces and simply flow. Therefore, transverse waves cannot propagate in gases. Longitudinal (compressional) waves, which involve bulk deformation and pressure variation, can propagate in any medium (solid, liquid, or gas) since all media resist compression.

(e) Pulse distortion in a dispersive medium:
In a dispersive medium, the wave speed depends on frequency. A pulse is a superposition of sinusoidal waves of many different frequencies (its Fourier components). In a dispersive medium, each frequency component travels at a different speed, so the components shift in phase relative to one another as they propagate. The pulse shape, which was formed by a specific superposition of these components, gets distorted because the relative phase relationships between them change continuously with distance.

Answer: (a) At a displacement node, $\partial s/\partial x$ is maximum => max pressure variation (pressure antinode); vice versa at antinodes. (b) Bats use echolocation: emit ultrasound, compute distance from echo delay, direction from echo angle, size/shape from echo spectrum. (c) Same fundamental but different overtone content (timbre) lets us tell violin from sitar. (d) Transverse waves need shear modulus (present in solids, absent in gases); longitudinal waves need bulk modulus (present in all media). (e) In a dispersive medium, different frequency components of the pulse travel at different speeds, changing their relative phases and distorting the pulse shape.