Chapter 3: Production and Costs

Exercises — Exercises

Q1 Explain the concept of a production function.

A production function shows the maximum quantity of output a firm can produce from every possible combination of inputs, given the technology it uses. It is written as q = f(x1, x2), where x1 and x2 are quantities of two inputs such as labour and capital, and q is the maximum output. It is defined for a given state of technology: a better technology would give a different function. It is a purely technical (physical) relationship between inputs and output and does not involve prices. For a given input combination, the function tells us only the largest output that is technically possible, not any lower output.

Q2 What is the total product of an input?

The total product (TP) of an input is the quantity of output produced when that input is varied while all other inputs are held fixed. For example, if capital is held constant and labour is increased one unit at a time, the output obtained at each level of labour is the total product of labour. It is therefore the relationship between the quantity of one variable input and the resulting output, and it generally rises as more of the input is employed.

Q3 What is the average product of an input?

The average product (AP) of an input is the total product per unit of that input, found by dividing total product by the quantity of the input used. For labour, AP(L) = TP(L) / L. It tells us the output produced, on average, by each unit of the variable input while the other inputs stay fixed.

Q4 What is the marginal product of an input?

The marginal product (MP) of an input is the change in total product caused by employing one more unit of that input, with all other inputs held constant. For labour, MP(L) = TP(L) - TP(L-1), that is, the extra output from the last unit of labour. It measures the contribution of the additional unit alone.

Q5 Explain the relationship between the marginal products and the total product of an input.

Total product is the sum of the marginal products of all units of the input employed up to that level: TP = MP1 + MP2 + ... + MPL. So while marginal product is positive, total product rises. When marginal product is positive but falling, total product still rises, but at a decreasing rate. When marginal product becomes zero, total product is at its maximum. If marginal product turns negative, total product starts to fall. Total product therefore increases at an increasing rate while MP rises, and at a decreasing rate once MP begins to fall.

Q6 Explain the concepts of the short run and the long run.

The short run is a period of time in which at least one input cannot be changed, so it stays fixed while the others can be varied; the firm can alter output only by changing the variable inputs, such as labour. The long run is a period long enough for all inputs to be varied, so the firm can change any input, including capital and the scale of operation. There is no fixed number of months or years for either; it depends on the firm and the industry. In the short run a firm has both fixed and variable inputs, while in the long run every input is variable.

Q7 What is the law of diminishing marginal product?

The law states that, when the quantity of one input is increased by equal units while the other inputs are held fixed, the marginal product of the variable input eventually starts to fall. Beyond a certain point, each additional unit of the input adds less to total output than the one before. It arises because the fixed inputs become increasingly scarce relative to the variable input, so each extra unit has less of the fixed input to work with.

Q8 What is the law of variable proportions?

The law of variable proportions describes how the marginal product of a variable input behaves as its quantity is increased while other inputs are fixed. Initially, marginal product increases as the input is added, because the fixed factor is being used more fully. After a point, marginal product starts to decline (diminishing marginal product), and total product rises at a slower rate. Therefore the total product first increases at an increasing rate, then at a decreasing rate. This law combines the initial increasing phase with the later decreasing phase of marginal product.

Q9 When does a production function satisfy constant returns to scale?

A production function satisfies constant returns to scale when a proportional increase in all inputs leads to an increase in output by the same proportion. For example, if all inputs are doubled, output exactly doubles; if all inputs are tripled, output triples. In other words, the output rises in the same proportion as the scale of inputs.

Q10 When does a production function satisfy increasing returns to scale?

A production function satisfies increasing returns to scale when a proportional increase in all inputs leads to a more than proportional increase in output. For instance, if all inputs are doubled, output becomes more than double. Output therefore grows faster than the scale of inputs.

Q11 When does a production function satisfy decreasing returns to scale?

A production function satisfies decreasing returns to scale when a proportional increase in all inputs leads to a less than proportional increase in output. For instance, if all inputs are doubled, output becomes less than double. Output therefore grows more slowly than the scale of inputs.

Q12 Briefly explain the concept of the cost function.

A cost function shows the minimum cost of producing each level of output, given the prices of inputs and the technology. A firm chooses the least-cost combination of inputs for each output level, using its production function and input prices. The cost then depends on the level of output: C = f(q). In the short run, the cost function includes both fixed and variable costs; in the long run, all costs are variable.

Q13 What are the total fixed cost, total variable cost and total cost of a firm? How are they related?

Total fixed cost (TFC) is the cost of the fixed inputs; it does not change with output and has to be paid even at zero output. Total variable cost (TVC) is the cost of the variable inputs; it rises as output rises and is zero at zero output. Total cost (TC) is the overall cost of producing a given output. They are related by TC = TFC + TVC. Since TFC is constant, TC differs from TVC by a fixed amount at every output level, and at zero output TC equals TFC.

Q14 What are the average fixed cost, average variable cost and average cost of a firm? How are they related?

Average fixed cost (AFC) is total fixed cost per unit of output: AFC = TFC / q. Average variable cost (AVC) is total variable cost per unit of output: AVC = TVC / q. Average cost (AC), also called short run average cost (SAC), is total cost per unit of output: AC = TC / q. Because TC = TFC + TVC, dividing by q gives AC = AFC + AVC. So average cost is the sum of average fixed cost and average variable cost.

Q15 Can there be some fixed cost in the long run? If not, why?

No. In the long run all inputs can be varied, so no input is fixed. A firm can change the scale of its plant, hire any quantity of capital and labour, or even shut down and avoid every cost. Hence all costs in the long run are variable costs, and there is no fixed cost.

Q16 What does the average fixed cost curve look like? Why does it look so?

The average fixed cost curve slopes downward throughout; it is a rectangular-hyperbola-like curve that comes closer to the horizontal axis as output rises but never touches it. This happens because TFC is constant, so AFC = TFC / q falls steadily as output q increases: the same fixed cost is spread over more and more units. AFC becomes very large at very small outputs and approaches zero (but never reaches it) at very large outputs.

Q17 What do the short run marginal cost, average variable cost and short run average cost curves look like?

All three curves are U-shaped. As output rises, each first falls, reaches a minimum point, and then rises. The SMC curve cuts both the AVC curve and the SAC curve from below at their respective minimum points. The U-shape reflects the law of variable proportions: marginal product first increases and then diminishes, so marginal cost first falls and then rises. Since AVC and SAC are averages of marginal cost, they follow the same pattern.

Q18 Why does the SMC curve cut the AVC curve at the minimum point of the AVC curve?

AVC is an average and SMC is the marginal. When marginal cost is below average variable cost, each additional unit pulls the average down, so AVC falls. When marginal cost is above AVC, each additional unit pulls the average up, so AVC rises. AVC therefore stops falling and starts to rise exactly where SMC equals AVC, which is the minimum point of AVC. Thus SMC cuts the AVC curve from below at its minimum point.

Q19 At which point does the SMC curve cut the SAC curve? Give reason in support of your answer.

The SMC curve cuts the SAC curve from below at the minimum point of the SAC curve. Reason: SAC is the average of total cost per unit. When SMC is less than SAC, each extra unit costs less than the average, so SAC falls. When SMC exceeds SAC, each extra unit costs more than the average, so SAC rises. Hence SAC is at its minimum where SMC = SAC.

Q20 Why is the short run marginal cost curve 'U'-shaped?

The shape of the SMC curve follows from the law of variable proportions. Initially, when the variable input is added to fixed inputs, its marginal product rises, so each extra unit of output needs less of the variable input and marginal cost falls. After a point, the marginal product of the variable input begins to fall (diminishing returns), so each extra unit of output requires more of the input and marginal cost rises. Marginal cost therefore falls first and then rises, giving a U-shaped curve.

Q21 What do the long run marginal cost and the average cost curves look like?

Both the long run average cost (LRAC) and long run marginal cost (LRMC) curves are U-shaped. The U-shape arises from returns to scale: initially, increasing returns to scale make the curves fall; later, decreasing returns to scale make them rise. The LRMC curve cuts the LRAC curve from below at the minimum point of LRAC.

Q22 The following table gives the total product schedule of labour. Find the corresponding average product and marginal product schedules of labour.

SOLUTION

AP = TP / L and MP = TP(L) - TP(L-1).

L	TP	AP	MP
0	0	-	-
1	15	15/1 = 15	15 - 0 = 15
2	35	35/2 = 17.5	35 - 15 = 20
3	50	50/3 = 16.67	50 - 35 = 15
4	40	40/4 = 10	40 - 50 = -10
5	48	48/5 = 9.6	48 - 40 = 8

Result: AP = 15, 17.5, 16.67, 10, 9.6 and MP = 15, 20, 15, -10, 8 for L = 1 to 5 (AP at L = 0 is undefined).

Q23 The following table gives the average product schedule of labour. Find the total product and marginal product schedules. It is given that the total product is zero at zero level of labour employment.

SOLUTION

TP = AP x L and MP = TP(L) - TP(L-1); TP(0) = 0.

L	AP	TP	MP
1	2	2 x 1 = 2	2 - 0 = 2
2	3	3 x 2 = 6	6 - 2 = 4
3	4	4 x 3 = 12	12 - 6 = 6
4	4.25	4.25 x 4 = 17	17 - 12 = 5
5	4	4 x 5 = 20	20 - 17 = 3
6	3.5	3.5 x 6 = 21	21 - 20 = 1

Result: TP = 2, 6, 12, 17, 20, 21 and MP = 2, 4, 6, 5, 3, 1 for L = 1 to 6.

Q24 The following table gives the marginal product schedule of labour. It is also given that total product of labour is zero at zero level of employment. Calculate the total and average product schedules of labour.

SOLUTION

TP(L) = TP(L-1) + MP(L), with TP(0) = 0; AP = TP / L.

L	MP	TP	AP
1	3	0 + 3 = 3	3/1 = 3
2	5	3 + 5 = 8	8/2 = 4
3	7	8 + 7 = 15	15/3 = 5
4	5	15 + 5 = 20	20/4 = 5
5	3	20 + 3 = 23	23/5 = 4.6
6	1	23 + 1 = 24	24/6 = 4

Result: TP = 3, 8, 15, 20, 23, 24 and AP = 3, 4, 5, 5, 4.6, 4 for L = 1 to 6.

Q25 The following table shows the total cost schedule of a firm. What is the total fixed cost schedule of this firm? Calculate the TVC, AFC, AVC, SAC and SMC schedules of the firm.

SOLUTION

TFC equals TC at zero output = 10, so TFC = 10 at every output.

TVC = TC - 10; AFC = 10/Q; AVC = TVC/Q; SAC = TC/Q; SMC = TC(Q) - TC(Q-1).

Q	TC	TFC	TVC	AFC	AVC	SAC	SMC
0	10	10	0	-	-	-	-
1	30	10	20	10	20	30	30 - 10 = 20
2	45	10	35	5	17.5	22.5	45 - 30 = 15
3	55	10	45	3.33	15	18.33	55 - 45 = 10
4	70	10	60	2.5	15	17.5	70 - 55 = 15
5	90	10	80	2	16	18	90 - 70 = 20
6	120	10	110	1.67	18.33	20	120 - 90 = 30

Check: SAC = AFC + AVC at each output (e.g. Q = 3: 3.33 + 15 = 18.33).

Result: TFC = 10 throughout; the other schedules are as tabulated above.

Q26 The following table gives the total cost schedule of a firm. It is also given that the average fixed cost at 4 units of output is Rs 5. Find the TVC, TFC, AVC, AFC, SAC and SMC schedules of the firm for the corresponding values of output.

SOLUTION

AFC at Q = 4 is 5, so TFC = AFC x Q = 5 x 4 = 20, constant at all outputs.
TVC = TC - 20; AFC = 20/Q; AVC = TVC/Q; SAC = TC/Q; SMC = change in TC (SMC at Q = 1 is TVC(1) - 0).

Q	TC	TFC	TVC	AFC	AVC	SAC	SMC
1	50	20	30	20	30	50	30
2	65	20	45	10	22.5	32.5	65 - 50 = 15
3	75	20	55	6.67	18.33	25	75 - 65 = 10
4	95	20	75	5	18.75	23.75	95 - 75 = 20
5	130	20	110	4	22	26	130 - 95 = 35
6	185	20	165	3.33	27.5	30.83	185 - 130 = 55

Check: SAC = AFC + AVC (e.g. Q = 4: 5 + 18.75 = 23.75).
Result: TFC = Rs 20 throughout; the other schedules are as tabulated above.

Q27 A firm's SMC schedule is shown in the following table. The total fixed cost of the firm is Rs 100. Find the TVC, TC, AVC and SAC schedules of the firm.

SOLUTION

TVC at Q is the sum of SMC up to Q; TC = TVC + 100; AVC = TVC/Q; SAC = TC/Q. The SMC values given are 500, 300, 200, 300, 500, 800 for Q = 1 to 6.

Q	SMC	TVC	TC	AVC	SAC
0	-	0	100	-	-
1	500	500	600	500	600
2	300	500 + 300 = 800	900	400	450
3	200	800 + 200 = 1000	1100	333.33	366.67
4	300	1000 + 300 = 1300	1400	325	350
5	500	1300 + 500 = 1800	1900	360	380
6	800	1800 + 800 = 2600	2700	433.33	450

Result: TVC = 500, 800, 1000, 1300, 1800, 2600; TC = 600, 900, 1100, 1400, 1900, 2700 for Q = 1 to 6.

Q28 Let the production function of a firm be Q = 5 L^(1/2) K^(1/2). Find out the maximum possible output that the firm can produce with 100 units of L and 100 units of K.

SOLUTION

Q = 5 x (100)^(1/2) x (100)^(1/2)
(100)^(1/2) = 10, so Q = 5 x 10 x 10
Q = 500 units. Maximum possible output = 500 units.

Q29 Let the production function of a firm be Q = 2 L^2 K^2. Find out the maximum possible output that the firm can produce with 5 units of L and 2 units of K. What is the maximum possible output that the firm can produce with zero unit of L and 10 units of K?

SOLUTION

With L = 5, K = 2: Q = 2 x (5)^2 x (2)^2 = 2 x 25 x 4 = 200.
With L = 0, K = 10: Q = 2 x (0)^2 x (10)^2 = 2 x 0 x 100 = 0.
Maximum output is 200 units with 5 units of L and 2 units of K, and 0 units with zero L and 10 units of K.

Q30 Find out the maximum possible output for a firm with zero unit of L and 10 units of K when its production function is Q = 5L + 2K.

SOLUTION

Q = 5(0) + 2(10) = 0 + 20 = 20.
Maximum possible output = 20 units.