Chapter 11: Three Dimensional Geometry

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Exercise 11.1 — Exercise 11.1

Q1 If a line makes angles 90°, 135°, 45° with the x, y and z-axes respectively, find its direction cosines.

SOLUTION

Let the direction cosines be l, m, n corresponding to angles α = 90°, β = 135°, γ = 45°.

l = cos 90° = 0

m = cos 135° = -1/√2

n = cos 45° = 1/√2

Verification: $l^2 + m^2 + n^2 = 0 + 1/2 + 1/2 = 1$. ✓

Answer: l = 0, m = -1/√2, n = 1/√2

Q2 Find the direction cosines of a line which makes equal angles with the coordinate axes.

SOLUTION

Let the line make equal angles α with all three coordinate axes, so l = m = n = cos α.

Using the identity $l^2 + m^2 + n^2 = 1$:

$3\cos^2 \alpha = 1 \Rightarrow \cos \alpha = \pm 1/\sqrt{3}$

Therefore the direction cosines are l = m = n = 1/√3 or l = m = n = -1/√3.

There are also mixed-sign cases such as (1/√3, 1/√3, -1/√3), etc. In general, l = m = n = ±1/√3 with any combination of signs giving the same magnitude.

Answer: l = m = n = ±1/√3

Q3 If a line has the direction ratios -18, 12, -4, then what are its direction cosines?

SOLUTION

Given direction ratios: a = -18, b = 12, c = -4.
Magnitude: $\sqrt{(a^2 + b^2 + c^2)} = \sqrt{(324 + 144 + 16)} = \sqrt{484} = 22$.
Direction cosines:
l = -18/22 = -9/11
m = 12/22 = 6/11
n = -4/22 = -2/11
Verification: $(9/11)^2 + (6/11)^2 + (2/11)^2 = (81+36+4)/121 = 121/121 = 1$. ✓

Answer: l = -9/11, m = 6/11, n = -2/11

Q4 Show that the points (2, 3, 4), (– 1, – 2, 1), (5, 8, 7) are collinear.

SOLUTION

Let A = (2, 3, 4), B = (-1, -2, 1), C = (5, 8, 7).
Direction ratios of AB: (-1-2, -2-3, 1-4) = (-3, -5, -3).
Direction ratios of BC: (5-(-1), 8-(-2), 7-1) = (6, 10, 6).
Check proportionality: -3/6 = -5/10 = -3/6 = -1/2.
The direction ratios of AB and BC are proportional (ratio -1:2), so AB is parallel to BC.
Since B is a point common to both segments, A, B, C are collinear.

Answer: Points (2,3,4), (-1,-2,1), (5,8,7) are collinear since direction ratios of AB and BC are proportional.

Q5 Find the direction cosines of the sides of the triangle whose vertices are (3, 5, – 4), (– 1, 1, 2) and (– 5, – 5, – 2).

SOLUTION

Let A = (3, 5, -4), B = (-1, 1, 2), C = (-5, -5, -2).

Side AB: direction ratios = (-1-3, 1-5, 2-(-4)) = (-4, -4, 6).
|AB| = $\sqrt{(16 + 16 + 36)} = \sqrt{68} = 2\sqrt{17}$.
Direction cosines of AB: $(-4)/(2\sqrt{17}), (-4)/(2\sqrt{17}), 6/(2\sqrt{17}) = -2/\sqrt{17}, -2/\sqrt{17}, 3/\sqrt{17}$.

Side BC: direction ratios = (-5-(-1), -5-1, -2-2) = (-4, -6, -4).
|BC| = $\sqrt{(16 + 36 + 16)} = \sqrt{68} = 2\sqrt{17}$.
Direction cosines of BC: $(-4)/(2\sqrt{17}), (-6)/(2\sqrt{17}), (-4)/(2\sqrt{17}) = -2/\sqrt{17}, -3/\sqrt{17},$

-2/√17.

Side CA: direction ratios = (3-(-5), 5-(-5), -4-(-2)) = (8, 10, -2).
|CA| = √(64 + 100 + 4) = √168 = 2√42.
Direction cosines of CA: 8/(2√42), 10/(2√42), (-2)/(2√42) = 4/√42, 5/√42, -1/√42.

Answer: AB: -2/√17, -2/√17, 3/√17 | BC: -2/√17, -3/√17, -2/√17 | CA: 4/√42, 5/√42, -1/√42

Exercise 11.2 — Exercise 11.2

Q1 Show that the three lines with direction cosines 12/13, -3/13, -4/13; 4/13, 12/13, 3/13; 3/13, -4/13, 12/13 are mutually perpendicular.

SOLUTION

Let L1, L2, L3 have the given direction cosines. Two lines are perpendicular if the sum of products of their direction cosines is zero.

L1 · L2 = (12/13)(4/13) + (-3/13)(12/13) + (-4/13)(3/13)
= (48 - 36 - 12)/169 = 0/169 = 0.

L2 · L3 = (4/13)(3/13) + (12/13)(-4/13) + (3/13)(12/13)
= (12 - 48 + 36)/169 = 0/169 = 0.

L1 · L3 = (12/13)(3/13) + (-3/13)(-4/13) + (-4/13)(12/13)
= (36 + 12 - 48)/169 = 0/169 = 0.

Since each pair of dot products equals zero, the three lines are mutually perpendicular.

Answer: All three dot products equal zero, so the lines are mutually perpendicular.

Q2 Show that the line through the points (1, – 1, 2), (3, 4, – 2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

SOLUTION

Direction ratios of first line (through (1,-1,2) and (3,4,-2)):
a1 = 3-1 = 2, b1 = 4-(-1) = 5, c1 = -2-2 = -4.

Direction ratios of second line (through (0,3,2) and (3,5,6)):
a2 = 3-0 = 3, b2 = 5-3 = 2, c2 = 6-2 = 4.

Dot product: a1*a2 + b1*b2 + c1*c2 = 2(3) + 5(2) + (-4)(4) = 6 + 10 - 16 = 0.

Since the dot product is zero, the two lines are perpendicular.

Answer: a1·a2 + b1·b2 + c1·c2 = 6 + 10 - 16 = 0, so the lines are perpendicular.

Q3 Show that the line through the points (4, 7, 8), (2, 3, 4) is parallel to the line through the points (– 1, – 2, 1), (1, 2, 5).

SOLUTION

Direction ratios of first line (through (4,7,8) and (2,3,4)):
a1 = 2-4 = -2, b1 = 3-7 = -4, c1 = 4-8 = -4.

Direction ratios of second line (through (-1,-2,1) and (1,2,5)):
a2 = 1-(-1) = 2, b2 = 2-(-2) = 4, c2 = 5-1 = 4.

Check proportionality: a1/a2 = -2/2 = -1, b1/b2 = -4/4 = -1, c1/c2 = -4/4 = -1.
All ratios are equal (= -1), so the direction ratios are proportional.

Therefore the two lines are parallel.

Answer: Direction ratios (-2,-4,-4) and (2,4,4) are proportional (ratio -1:1), so the lines are parallel.

Q4 Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector 3 î + 2 ĵ – 2 k̂.

SOLUTION

The position vector of the point (1, 2, 3) is a = î + 2ĵ + 3k̂.
The direction vector is b = 3î + 2ĵ - 2k̂.

Vector equation of the line:
r = (î + 2ĵ + 3k̂) + λ(3î + 2ĵ - 2k̂)

Cartesian form: (x-1)/3 = (y-2)/2 = (z-3)/(-2)

Answer: r = (î + 2ĵ + 3k̂) + λ(3î + 2ĵ - 2k̂); Cartesian: (x-1)/3 = (y-2)/2 = (z-3)/(-2)

Q5 Find the equation of the line in vector and in cartesian form that passes through the point with position vector 2 î – ĵ + 4 k̂ and is in the direction î + 2ĵ – k̂.

SOLUTION

The point has position vector a = 2î - ĵ + 4k̂ (i.e., point (2, -1, 4)).
The direction vector is b = î + 2ĵ - k̂.

Vector equation:
r = (2î - ĵ + 4k̂) + λ(î + 2ĵ - k̂)

Cartesian form: (x-2)/1 = (y+1)/2 = (z-4)/(-1)

Answer: r = (2î - ĵ + 4k̂) + λ(î + 2ĵ - k̂); Cartesian: (x-2)/1 = (y+1)/2 = (z-4)/(-1)

Q6 Find the cartesian equation of the line which passes through the point (– 2, 4, – 5) and parallel to the line given by (x + 3)/3 = (y – 4)/5 = (z + 8)/6.

SOLUTION

The given line has direction ratios 3, 5, 6. A parallel line through (-2, 4, -5) has the same direction ratios.

Cartesian equation of the required line:
(x - (-2))/3 = (y - 4)/5 = (z - (-5))/6

=> (x + 2)/3 = (y - 4)/5 = (z + 5)/6

Answer: (x + 2)/3 = (y - 4)/5 = (z + 5)/6

Q7 The cartesian equation of a line is (x – 5)/3 = (y + 4)/7 = (z – 6)/2. Write its vector form.

SOLUTION

The Cartesian equation (x-5)/3 = (y+4)/7 = (z-6)/2 shows:
The line passes through the point (5, -4, 6), with direction ratios (3, 7, 2).

Position vector of the point: a = 5î - 4ĵ + 6k̂.
Direction vector: b = 3î + 7ĵ + 2k̂.

Vector equation: r = (5î - 4ĵ + 6k̂) + λ(3î + 7ĵ + 2k̂)

Answer: r = (5î - 4ĵ + 6k̂) + λ(3î + 7ĵ + 2k̂)

Q8 Find the angle between the following pairs of lines:
(i) r = 2î – 5ĵ + k̂ + λ(3î + 2ĵ + 6k̂) and r = 7î – 6k̂ + μ(î + 2ĵ + 2k̂)
(ii) r = 3î + ĵ – 2k̂ + λ(î – ĵ – 2k̂) and r = 2î – ĵ – 56k̂ + μ(3î – 5ĵ – 4k̂)

SOLUTION

(i) Direction vectors: b1 = 3î + 2ĵ + 6k̂, b2 = î + 2ĵ + 2k̂.
b1 · b2 = 3(1) + 2(2) + 6(2) = 3 + 4 + 12 = 19.

	b1	= √(9 + 4 + 36) = √49 = 7.				
	b2	= √(1 + 4 + 4) = √9 = 3.				

	b1	= √(9 + 4 + 36) = √49 = 7.				
cos θ =	b1 · b2	/ (	b1		b2	) = 19 / (7 × 3) = 19/21.

θ = cos^(-1)(19/21).

(ii) Direction vectors: b1 = î – ĵ – 2k̂, b2 = 3î – 5ĵ – 4k̂.
b1 · b2 = 1(3) + (-1)(-5) + (-2)(-4) = 3 + 5 + 8 = 16.

	b1	= √(1 + 1 + 4) = √6.
	b2	= √(9 + 25 + 16) = √50 = 5√2.

cos θ = 16 / (√6 × 5√2) = 16 / (5√12) = 16 / (10√3) = 8/(5√3) = 8√3/15.
θ = cos^(-1)(8√3/15).

Answer: (i) θ = cos^(-1)(19/21) (ii) θ = cos^(-1)(8√3/15)

Q9 Find the angle between the following pair of lines:
(i) (x – 2)/2 = (y – 1)/5 = (z + 3)/(–3) and (x + 2)/(–1) = (y – 4)/8 = (z – 5)/4
(ii) x/2 = y/2 = z/1 and (x – 5)/4 = (y – 2)/1 = (z – 3)/8

SOLUTION

(i) Direction ratios: a1 = (2, 5, -3), a2 = (-1, 8, 4).
a1 · a2 = 2(-1) + 5(8) + (-3)(4) = -2 + 40 - 12 = 26.

	a1	= √(4 + 25 + 9) = √38.
	a2	= √(1 + 64 + 16) = √81 = 9.
cos θ =	26	/ (√38 × 9) = 26/(9√38).

θ = cos^(-1)(26 / (9√38)).

(ii) Direction ratios: a1 = (2, 2, 1), a2 = (4, 1, 8).
a1 · a2 = 2(4) + 2(1) + 1(8) = 8 + 2 + 8 = 18.

	a1	= √(4 + 4 + 1) = √9 = 3.
	a2	= √(16 + 1 + 64) = √81 = 9.

cos θ = 18 / (3 × 9) = 18/27 = 2/3.
θ = cos^(-1)(2/3).

Answer: (i) θ = cos^(-1)(26/(9√38)) (ii) θ = cos^(-1)(2/3)

Q10 Find the values of p so that the lines (1 – x)/3 = (7y – 14)/(2p) = (z – 3)/2 and (7 – 7x)/(3p) = (y – 5)/1 = (6 – z)/5 are at right angles.

SOLUTION

Rewrite the lines in standard form.

First line: (1-x)/3 = (7y-14)/(2p) = (z-3)/2
=> -(x-1)/3 = 7(y-2)/(2p) = (z-3)/2
=> (x-1)/(-3) = (y-2)/(2p/7) = (z-3)/2.
Direction ratios: a1 = -3, b1 = 2p/7, c1 = 2.

Second line: (7-7x)/(3p) = (y-5)/1 = (6-z)/5
=> -7(x-1)/(3p) = (y-5)/1 = -(z-6)/5
=> (x-1)/(-3p/7) = (y-5)/1 = (z-6)/(-5).
Direction ratios: a2 = -3p/7, b2 = 1, c2 = -5.

For perpendicular lines: a1*a2 + b1*b2 + c1*c2 = 0.
(-3)(-3p/7) + (2p/7)(1) + (2)(-5) = 0
9p/7 + 2p/7 - 10 = 0
11p/7 = 10
p = 70/11.

Answer: p = 70/11

Q11 Show that the lines (x – 5)/7 = (y + 2)/(–5) = z/1 and x/1 = y/2 = z/3 are perpendicular to each other.

SOLUTION

Direction ratios of first line: a1 = 7, b1 = -5, c1 = 1.
Direction ratios of second line: a2 = 1, b2 = 2, c2 = 3.

a1*a2 + b1*b2 + c1*c2 = 7(1) + (-5)(2) + 1(3) = 7 - 10 + 3 = 0.

Since the dot product of direction ratios is zero, the lines are perpendicular.

Answer: a1·a2 + b1·b2 + c1·c2 = 7 - 10 + 3 = 0, so the lines are perpendicular.

Q12 Find the shortest distance between the lines

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

SOLUTION

Identify the vectors:

$\vec{a_1} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b_1} = \hat{i} - \hat{j} + \hat{k}$

$\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$

$\vec{a_2} - \vec{a_1} = (2-1)\hat{i} + (-1-2)\hat{j} + (-1-1)\hat{k} = \hat{i} - 3\hat{j} - 2\hat{k}$.

$\vec{b_1} \times \vec{b_2}$ =	$\hat{i}$ $\hat{j}$ $\hat{k}$	
	1 -1 1	
	2 1 2	

$= \hat{i}[(-1)(2) - (1)(1)] - \hat{j}[(1)(2) - (1)(2)] + \hat{k}[(1)(1) - (-1)(2)]$

$= \hat{i}(-2 - 1) - \hat{j}(2 - 2) + \hat{k}(1 + 2)$

$= -3\hat{i} + 0\hat{j} + 3\hat{k}$.

$|\vec{b_1} \times \vec{b_2}| = \sqrt{(9 + 0 + 9)} = \sqrt{18} = 3\sqrt{2}$.

$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (-3)(1) + (0)(-3) + (3)(-2) = -3 + 0 - 6 = -9$.

Shortest distance d =	$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} -$ $\vec{a_1})$	/	$\vec{b_1} \times$ $\vec{b_2}$	
=	-9	/ $(3\sqrt{2}) = 9/(3\sqrt{2}) = 3/\sqrt{2} =$ $3\sqrt{2}/2$.		

Answer: Shortest distance = $3\sqrt{2}/2$

Q13 Find the shortest distance between the lines $(x + 1)/7 = (y + 1)/(-6) = (z + 1)/1$ and $(x - 3)/1 = (y - 5)/(-2) = (z - 7)/1$.

SOLUTION

Line 1 passes through (-1, -1, -1) with direction ratios (7, -6, 1): $\vec{a_1} = (-1,-1,-1)$, $\vec{b_1} = (7,-6,1)$.

Line 2 passes through (3, 5, 7) with direction ratios (1, -2, 1): $\vec{a_2} = (3,5,7)$, $\vec{b_2} = (1,-2,1)$.

$\vec{a_2} - \vec{a_1} = (3-(-1), 5-(-1), 7-(-1)) = (4, 6, 8)$.

b1 × b2 =	î ĵ k̂	
	7 -6 1	
	1 -2 1	

= î[(-6)(1) – (1)(-2)] – ĵ[(7)(1) – (1)(1)] + k̂[(7)(-2) – (-6)(1)]
= î(-6 + 2) – ĵ(7 – 1) + k̂(-14 + 6)
= -4î – 6ĵ – 8k̂.

|b1 × b2| = √(16 + 36 + 64) = √116 = 2√29.

(b1 × b2) · (a2 – a1) = (-4)(4) + (-6)(6) + (-8)(8) = -16 – 36 – 64 = -116.

Shortest distance d = |-116| / (2√29) = 116/(2√29) = 58/√29 = 2√29.

Answer: Shortest distance = 2√29

Q14 Find the shortest distance between the lines whose vector equations are r = (î + 2ĵ + 3k̂) + λ(î – 3ĵ + 2k̂) and r = 4î + 5ĵ + 6k̂ + μ(2î + 3ĵ + k̂).

SOLUTION

a1 = î + 2ĵ + 3k̂, b1 = î – 3ĵ + 2k̂
a2 = 4î + 5ĵ + 6k̂, b2 = 2î + 3ĵ + k̂

a2 – a1 = (4-1)î + (5-2)ĵ + (6-3)k̂ = 3î + 3ĵ + 3k̂.

b1 × b2 =	î ĵ k̂	
	1 -3 2	
	2 3 1	

= î[(-3)(1) – (2)(3)] – ĵ[(1)(1) – (2)(2)] + k̂[(1)(3) – (-3)(2)]
= î(-3 – 6) – ĵ(1 – 4) + k̂(3 + 6)
= -9î + 3ĵ + 9k̂.

|b1 × b2| = √(81 + 9 + 81) = √171 = 3√19.

(b1 × b2) · (a2 – a1) = (-9)(3) + (3)(3) + (9)(3) = -27 + 9 + 27 = 9.

Shortest distance d = |9| / (3√19) = 3/√19 = 3√19/19.

Answer: Shortest distance = 3√19/19

Q15 Find the shortest distance between the lines whose vector equations are $\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$ and $\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$.

SOLUTION

Rewrite the first line by grouping terms in t:
$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t(-\hat{i} + \hat{j} - 2\hat{k})$.
So $\vec{a}_1 = (1, -2, 3)$ and $\vec{b}_1 = (-1, 1, -2)$.

Rewrite the second line by grouping terms in s:
$\vec{r} = \hat{i} - \hat{j} - \hat{k} + s(\hat{i} + 2\hat{j} - 2\hat{k})$.
So $\vec{a}_2 = (1, -1, -1)$ and $\vec{b}_2 = (1, 2, -2)$.

$\vec{a}_2 - \vec{a}_1 = (1-1, -1-(-2), -1-3) = (0, 1, -4)$.

$\vec{b}_1 \times \vec{b}_2 =$	$\hat{i}$ $\hat{j}$ $\hat{k}$	
	-1 1 -2	
	1 2 -2	

$= \hat{i}[(1)(-2) - (-2)(2)] - \hat{j}[(-1)(-2) - (-2)(1)] + \hat{k}[(-1)(2) - (1)(1)]$
$= \hat{i}(-2 + 4) - \hat{j}(2 + 2) + \hat{k}(-2 - 1)$
$= 2\hat{i} - 4\hat{j} - 3\hat{k}$.

$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(4 + 16 + 9)} = \sqrt{29}$.

$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (2)(0) + (-4)(1) + (-3)(-4) = 0 - 4 + 12 = 8$.

Shortest distance $d = |8| / \sqrt{29} = 8/\sqrt{29} = 8\sqrt{29}/29$.

Answer: Shortest distance = $8/\sqrt{29} = 8\sqrt{29}/29$

Misc — Miscellaneous Exercise

Q1 Find the angle between the lines whose direction ratios are a, b, c and b – c, c – a, a – b.

SOLUTION

Let d1 = (a, b, c) and d2 = (b-c, c-a, a-b).

Compute the dot product:
d1 · d2 = a(b-c) + b(c-a) + c(a-b)
= ab – ac + bc – ab + ca – cb
= 0.

Since d1 · d2 = 0, cos θ = 0, so θ = 90°.
The lines are perpendicular to each other.

Answer: The angle between the lines is 90°.

Q2 Find the equation of a line parallel to x-axis and passing through the origin.

SOLUTION

The x-axis has direction ratios (1, 0, 0).
A line parallel to the x-axis and passing through the origin (0, 0, 0) has:
Position vector of point: a = 0.
Direction vector: b = î.

Vector equation: r = λ î.

Cartesian form: x/1 = y/0 = z/0, i.e., the line is y = 0 and z = 0 (the x-axis itself).
Any line parallel to x-axis through the origin is the x-axis itself.

Answer: r = λ î (i.e., the x-axis: y = 0, z = 0)

Q3 If the lines (x – 1)/(–3) = (y – 2)/(2k) = (z – 3)/2 and (x – 1)/(3k) = (y – 1)/1 = (z – 6)/(–5) are perpendicular, find the value of k.

SOLUTION

Direction ratios of first line: a1 = -3, b1 = 2k, c1 = 2.
Direction ratios of second line: a2 = 3k, b2 = 1, c2 = -5.

For perpendicular lines: a1*a2 + b1*b2 + c1*c2 = 0.
(-3)(3k) + (2k)(1) + (2)(-5) = 0

-9k + 2k – 10 = 0

-7k = 10

k = -10/7.

Answer: k = -10/7

Q4 Find the shortest distance between lines r = 6î + 2ĵ + 2k̂ + λ(î – 2ĵ + 2k̂) and r = –4î – k̂ + μ(3î – 2ĵ – 2k̂).

SOLUTION

a1 = 6î + 2ĵ + 2k̂, b1 = î – 2ĵ + 2k̂

a2 = –4î + 0ĵ – k̂, b2 = 3î – 2ĵ – 2k̂

a2 – a1 = (-4-6)î + (0-2)ĵ + (-1-2)k̂ = -10î – 2ĵ – 3k̂.

b1 × b2 =	î ĵ k̂	
	1 -2 2	
	3 -2 -2	

= î[(-2)(-2) – (2)(-2)] – ĵ[(1)(-2) – (2)(3)] + k̂[(1)(-2) – (-2)(3)]

= î(4 + 4) – ĵ(-2 – 6) + k̂(-2 + 6)

= 8î + 8ĵ + 4k̂.

|b1 × b2| = √(64 + 64 + 16) = √144 = 12.

(b1 × b2) · (a2 – a1) = (8)(-10) + (8)(-2) + (4)(-3) = -80 – 16 – 12 = -108.

Shortest distance d = |-108| / 12 = 108/12 = 9.

Answer: Shortest distance = 9

Q5 Find the vector equation of the line passing through the point (1, 2, – 4) and perpendicular to the two lines: (x – 8)/3 = (y + 19)/(–16) = (z – 10)/7 and (x – 15)/3 = (y – 29)/8 = (z – 5)/(–5).

SOLUTION

The required line passes through (1, 2, -4) and is perpendicular to both given lines.

Position vector of point: a = î + 2ĵ – 4k̂.

Direction vectors of the two given lines:

$\vec{b_1} = 3\hat{i} - 16\hat{j} + 7\hat{k}$ (from first line)

$\vec{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$ (from second line)

The direction of the required line must be perpendicular to both $\vec{b_1}$ and $\vec{b_2}$, so it is $\vec{b} = \vec{b_1} \times \vec{b_2}$.

$\vec{b_1} \times \vec{b_2} =$	$\hat{i}$ $\hat{j}$ $\hat{k}$	
	3 -16 7	
	3 8 -5	

$= \hat{i}[(-16)(-5) - (7)(8)] - \hat{j}[(3)(-5) - (7)(3)] + \hat{k}[(3)(8) - (-16)(3)]$

$= \hat{i}(80 - 56) - \hat{j}(-15 - 21) + \hat{k}(24 + 48)$

$= 24\hat{i} + 36\hat{j} + 72\hat{k}$

$= 12(2\hat{i} + 3\hat{j} + 6\hat{k})$.

We can take direction vector as $2\hat{i} + 3\hat{j} + 6\hat{k}$.

Vector equation of the required line:

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$.

Answer: $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$