Chapter 12: Linear Programming

Exercise 12.1 — Exercise 12.1

Q1 Solve the following Linear Programming Problem graphically:
Maximise Z = 3x + 4y subject to the constraints: x + y ≤ 4, x ≥ 0, y ≥ 0.

SOLUTION

Step 1: Identify the feasible region.
The constraints are x + y ≤ 4, x ≥ 0, y ≥ 0. Graph the line x + y = 4, which meets the x-axis at (4, 0) and the y-axis at (0, 4). The feasible region is the triangular area in the first quadrant on and below this line.

Step 2: Find the corner points of the feasible region.
The corner points are O(0, 0), A(4, 0), and B(0, 4).

Step 3: Evaluate Z = 3x + 4y at each corner point.
At O(0, 0): Z = 3(0) + 4(0) = 0
At A(4, 0): Z = 3(4) + 4(0) = 12
At B(0, 4): Z = 3(0) + 4(4) = 16

Step 4: The feasible region is bounded, so the maximum value of Z is the largest among these values.
Maximum Z = 16 at B(0, 4).

Answer: Maximum Z = 16 at (0, 4).

Q2 Solve the following Linear Programming Problem graphically:
Minimise Z = –3x + 4y subject to x + 2y ≤ 8, 3x + 2y ≤ 12, x ≥ 0, y ≥ 0.

SOLUTION

Step 1: Graph the boundary lines and find the feasible region.
Line 1: x + 2y = 8 meets axes at (8, 0) and (0, 4).
Line 2: 3x + 2y = 12 meets axes at (4, 0) and (0, 6).
The feasible region is the quadrilateral in the first quadrant satisfying both ≤ constraints.

Step 2: Find the corner points.
O(0, 0): intersection of the axes.
A(4, 0): intersection of 3x + 2y = 12 with x-axis.
B: intersection of x + 2y = 8 and 3x + 2y = 12.

Subtracting the first from the second: 2x = 4 => x = 2, then 2 + 2y = 8 => y = 3.
So B = (2, 3).
C(0, 4): intersection of x + 2y = 8 with y-axis.

Step 3: Evaluate Z = –3x + 4y at each corner point.
At O(0, 0): Z = 0
At A(4, 0): Z = –3(4) + 4(0) = –12
At B(2, 3): Z = –3(2) + 4(3) = –6 + 12 = 6
At C(0, 4): Z = –3(0) + 4(4) = 16

Step 4: The feasible region is bounded. The minimum value is –12 at A(4, 0).

Answer: Minimum Z = –12 at (4, 0).

Q3 Solve the following Linear Programming Problem graphically:
Maximise Z = 5x + 3y subject to 3x + 5y ≤ 15, 5x + 2y ≤ 10, x ≥ 0, y ≥ 0.

SOLUTION

Step 1: Graph the boundary lines and find the feasible region.
Line 1: 3x + 5y = 15 meets axes at (5, 0) and (0, 3).
Line 2: 5x + 2y = 10 meets axes at (2, 0) and (0, 5).
The feasible region is the area in the first quadrant satisfying both ≤ constraints.

Step 2: Find the corner points.
O(0, 0): origin.
A(2, 0): intersection of 5x + 2y = 10 with x-axis.
B: intersection of 3x + 5y = 15 and 5x + 2y = 10.
From 3x + 5y = 15: x = (15 – 5y)/3.
Substitute into 5x + 2y = 10: 5(15 – 5y)/3 + 2y = 10
=> (75 – 25y)/3 + 2y = 10 => 75 – 25y + 6y = 30 => –19y = –45 => y = 45/19.
Then x = (15 – 5(45/19))/3 = (15 – 225/19)/3 = (285/19 – 225/19)/3 = (60/19)/3 = 20/19.
So B = (20/19, 45/19).
C(0, 3): intersection of 3x + 5y = 15 with y-axis.

Step 3: Evaluate Z = 5x + 3y at each corner point.
At O(0, 0): Z = 0
At A(2, 0): Z = 5(2) + 3(0) = 10
At B(20/19, 45/19): Z = 5(20/19) + 3(45/19) = 100/19 + 135/19 = 235/19 ≈ 12.37
At C(0, 3): Z = 5(0) + 3(3) = 9

Step 4: The feasible region is bounded. The maximum value is 235/19 at B(20/19, 45/19).

Answer: Maximum Z = 235/19 at (20/19, 45/19).

Q4 Solve the following Linear Programming Problem graphically:
Minimise Z = 3x + 5y such that x + 3y ≥ 3, x + y ≥ 2, x, y ≥ 0.

SOLUTION

Step 1: Graph the boundary lines and identify the feasible region.
Line 1: x + 3y = 3 meets axes at (3, 0) and (0, 1).
Line 2: x + y = 2 meets axes at (2, 0) and (0, 2).
Since both constraints are ≥, the feasible region lies above and to the right of both lines. The region is unbounded.

Step 2: Find the corner points.
A(3, 0): intersection of x + 3y = 3 with x-axis. Check x + y = 3 ≥ 2 ✓.
B: intersection of x + 3y = 3 and x + y = 2.
Subtracting: 2y = 1 => y = 1/2, then x = 2 – 1/2 = 3/2. So B = (3/2, 1/2).
C(0, 2): intersection of x + y = 2 with y-axis. Check x + 3y = 0 + 6 = 6 ≥ 3 ✓.

Step 3: Evaluate Z = 3x + 5y at each corner point.
At A(3, 0): Z = 3(3) + 5(0) = 9
At B(3/2, 1/2): Z = 3(3/2) + 5(1/2) = 9/2 + 5/2 = 14/2 = 7
At C(0, 2): Z = 3(0) + 5(2) = 10

Step 4: The smallest value of Z at the corner points is 7 at B(3/2, 1/2). Since the feasible region is unbounded, check whether the open half plane 3x + 5y < 7 has any point in common with the feasible region. The line 3x + 5y = 7 does not intersect the feasible region in a way that gives 3x + 5y < 7 inside it. Therefore, the minimum value of Z is 7.

Answer: Minimum Z = 7 at (3/2, 1/2).

Q5 Solve the following Linear Programming Problem graphically:
Maximise Z = 3x + 2y subject to x + 2y ≤ 10, 3x + y ≤ 15, x, y ≥ 0.

SOLUTION

Step 1: Graph the boundary lines and find the feasible region.
Line 1: x + 2y = 10 meets axes at (10, 0) and (0, 5).
Line 2: 3x + y = 15 meets axes at (5, 0) and (0, 15).
The feasible region is the quadrilateral in the first quadrant satisfying both ≤ constraints.

Step 2: Find the corner points.
O(0, 0): origin.
A(5, 0): intersection of 3x + y = 15 with x-axis.
B: intersection of x + 2y = 10 and 3x + y = 15.
From x + 2y = 10: x = 10 – 2y.
Substitute: 3(10 – 2y) + y = 15 => 30 – 6y + y = 15 => –5y = –15 => y = 3.
Then x = 10 – 2(3) = 4. So B = (4, 3).
C(0, 5): intersection of x + 2y = 10 with y-axis.

Step 3: Evaluate Z = 3x + 2y at each corner point.
At O(0, 0): Z = 0
At A(5, 0): Z = 3(5) + 2(0) = 15
At B(4, 3): Z = 3(4) + 2(3) = 12 + 6 = 18
At C(0, 5): Z = 3(0) + 2(5) = 10

Step 4: The feasible region is bounded. The maximum value is 18 at B(4, 3).

Answer: Maximum Z = 18 at (4, 3).

Q6 Solve the following Linear Programming Problem graphically:
Minimise Z = x + 2y subject to 2x + y ≥ 3, x + 2y ≥ 6, x, y ≥ 0.
Show that the minimum of Z occurs at more than two points.

SOLUTION

Step 1: Graph the boundary lines and identify the feasible region.
Line 1: 2x + y = 3 meets axes at (3/2, 0) and (0, 3).
Line 2: x + 2y = 6 meets axes at (6, 0) and (0, 3).
Since both constraints are ≥, the feasible region lies above and to the right of both lines. The region is unbounded.

Step 2: Find the corner points.

Find the intersection of 2x + y = 3 and x + 2y = 6.
From 2x + y = 3: y = 3 – 2x.
Substitute: x + 2(3 – 2x) = 6 => x + 6 – 4x = 6 => –3x = 0 => x = 0, y = 3.
So the two lines intersect at (0, 3).

The corner points of the feasible region are: A(0, 3) and B(6, 0).
A(0, 3): intersection of both boundary lines (they meet at the y-axis point).
B(6, 0): intersection of x + 2y = 6 with x-axis. Check 2(6) + 0 = 12 ≥ 3 ✓.

Step 3: Evaluate Z = x + 2y at each corner point.
At A(0, 3): Z = 0 + 2(3) = 6
At B(6, 0): Z = 6 + 2(0) = 6

Step 4: Both corner points give Z = 6. Since the feasible region is unbounded, verify that the open half plane x + 2y < 6 has no common point with the feasible region. The constraint x + 2y ≥ 6 ensures no feasible point has x + 2y < 6. Therefore, minimum Z = 6.

Since the minimum value Z = 6 occurs at both A(0, 3) and B(6, 0), every point on the line segment joining A and B also gives Z = 6. This is because on the segment, parametrically x = 6t, y = 3 – 3t for t ∈ [0, 1], so Z = 6t + 2(3 – 3t) = 6t + 6 – 6t = 6. Thus the minimum occurs at infinitely many points (the entire segment AB), confirming it occurs at more than two points.

Answer: Minimum Z = 6 at every point on the line segment joining (0, 3) and (6, 0).

Q7 Solve the following Linear Programming Problem graphically:
Minimise and Maximise Z = 5x + 10y subject to x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0.

SOLUTION

Step 1: Graph the boundary lines and find the feasible region.
Line 1: x + 2y = 120 (≤ constraint — feasible region below this line).
Line 2: x + y = 60 (≥ constraint — feasible region above this line).
Line 3: x – 2y = 0, i.e., x = 2y (≥ constraint — feasible region to the right/above this line, meaning x ≥ 2y).

Step 2: Find the corner points by solving pairs of boundary equations.
A: intersection of x + y = 60 and x – 2y = 0 (x = 2y).
2y + y = 60 => 3y = 60 => y = 20, x = 40. So A = (40, 20).
Check x + 2y = 40 + 40 = 80 ≤ 120 ✓.

B: intersection of x + y = 60 with x-axis (y = 0).
x = 60. So B = (60, 0).
Check x – 2y = 60 ≥ 0 ✓, x + 2y = 60 ≤ 120 ✓.

C: intersection of x + 2y = 120 with x-axis (y = 0).
x = 120. So C = (120, 0).
Check x + y = 120 ≥ 60 ✓, x – 2y = 120 ≥ 0 ✓.

D: intersection of x + 2y = 120 and x – 2y = 0 (x = 2y).
2y + 2y = 120 => 4y = 120 => y = 30, x = 60. So D = (60, 30).
Check x + y = 90 ≥ 60 ✓.

The corner points of the feasible region are A(40, 20), B(60, 0), C(120, 0), D(60, 30).

Step 3: Evaluate Z = 5x + 10y at each corner point.
At A(40, 20): Z = 5(40) + 10(20) = 200 + 200 = 400
At B(60, 0): Z = 5(60) + 10(0) = 300
At C(120, 0): Z = 5(120) + 10(0) = 600
At D(60, 30): Z = 5(60) + 10(30) = 300 + 300 = 600

Step 4: The feasible region is bounded.
Minimum Z = 300 at B(60, 0).
Maximum Z = 600 at both C(120, 0) and D(60, 30) — every point on the segment CD also gives Z = 600.

Answer: Minimum Z = 300 at (60, 0); Maximum Z = 600 at every point on the line segment joining (120, 0) and (60, 30).

Q8 Solve the following Linear Programming Problem graphically:
Minimise and Maximise Z = x + 2y subject to x + 2y ≥ 100, 2x – y ≤ 0, 2x + y ≤ 200; x, y ≥ 0.

SOLUTION

Step 1: Rewrite constraints and graph the feasible region.
Constraint 1: x + 2y ≥ 100.
Constraint 2: 2x – y ≤ 0, i.e., y ≥ 2x.
Constraint 3: 2x + y ≤ 200.
x, y ≥ 0.

Step 2: Find the corner points.

A: intersection of x + 2y = 100 and y-axis (x = 0).
x = 0, 2y = 100, y = 50. So A = (0, 50).
Check y ≥ 2x: 50 ≥ 0 ✓; 2x + y = 50 ≤ 200 ✓.

B: intersection of x + 2y = 100 and 2x – y = 0 (y = 2x).
x + 2(2x) = 100 => 5x = 100 => x = 20, y = 40. So B = (20, 40).
Check 2x + y = 40 + 40 = 80 ≤ 200 ✓.

C: intersection of 2x – y = 0 (y = 2x) and 2x + y = 200.
2x + 2x = 200 => 4x = 200 => x = 50, y = 100. So C = (50, 100).
Check x + 2y = 50 + 200 = 250 ≥ 100 ✓.

D: intersection of 2x + y = 200 and y-axis (x = 0).
y = 200. So D = (0, 200).
Check y ≥ 2x: 200 ≥ 0 ✓; x + 2y = 400 ≥ 100 ✓.

The corner points are A(0, 50), B(20, 40), C(50, 100), D(0, 200).

Step 3: Evaluate Z = x + 2y at each corner point.
At A(0, 50): Z = 0 + 2(50) = 100
At B(20, 40): Z = 20 + 2(40) = 20 + 80 = 100
At C(50, 100): Z = 50 + 2(100) = 250
At D(0, 200): Z = 0 + 2(200) = 400

Step 4: The feasible region is bounded.
Minimum Z = 100 at both A(0, 50) and B(20, 40) — every point on the segment AB also gives Z = 100.
Maximum Z = 400 at D(0, 200).

Answer: Minimum Z = 100 at every point on the segment joining (0, 50) and (20, 40); Maximum Z = 400 at (0, 200).

Q9 Solve the following Linear Programming Problem graphically:
Maximise Z = –x + 2y, subject to the constraints: x ≥ 3, x + y ≥ 5, x + 2y ≥ 6, y ≥ 0.

SOLUTION

Step 1: Identify the feasible region.
The constraints are: x ≥ 3 (right of the vertical line x = 3), x + y ≥ 5 (above the line x + y = 5), x + 2y ≥ 6 (above the line x + 2y = 6), y ≥ 0.
There is no upper bound on x or y, so the feasible region is unbounded.

Step 2: Find the corner points.

A: intersection of $x = 3$ and $x + y = 5$.
$y = 5 – 3 = 2$. Check $x + 2y = 3 + 4 = 7 \geq 6$ ✓. So A = (3, 2).

B: intersection of $x + y = 5$ and $x + 2y = 6$.
Subtracting: $y = 1$, then $x = 4$. Check $x \geq 3$ ✓. So B = (4, 1).

Check the intersection of $x = 3$ and $x + 2y = 6$: $y = 3/2$, giving (3, 3/2).
At this point, $x + y = 3 + 3/2 = 9/2 = 4.5 < 5$. NOT feasible.

The corner points of the feasible region are A(3, 2) and B(4, 1).

Step 3: Evaluate $Z = –x + 2y$ at each corner point.
At A(3, 2): $Z = –3 + 2(2) = –3 + 4 = 1$
At B(4, 1): $Z = –4 + 2(1) = –2$

Step 4: Since the feasible region is unbounded, check whether Z has a maximum.
The largest value at corner points is $Z = 1$ at A(3, 2). Check whether the open half plane $–x + 2y > 1$ has points in common with the feasible region.
At the feasible point (3, y) for large y: $Z = –3 + 2y \to \infty$ as $y \to \infty$.
For example, (3, 100) is feasible and gives $Z = –3 + 200 = 197 > 1$.
Therefore, $Z = –x + 2y$ has no maximum value subject to the given constraints.

Answer: $Z = –x + 2y$ has no maximum value (the feasible region is unbounded and Z can be made arbitrarily large).

Q10 Solve the following Linear Programming Problem graphically:
Maximise $Z = x + y$, subject to $x – y \leq –1$, $–x + y \leq 0$, $x, y \geq 0$.

SOLUTION

Step 1: Analyse the constraints.
Constraint 1: $x – y \leq –1$, which means $y \geq x + 1$ (the feasible region is above the line $y = x + 1$).
Constraint 2: $–x + y \leq 0$, which means $y \leq x$ (the feasible region is below or on the line $y = x$).
$x, y \geq 0$.

Step 2: Determine if a feasible region exists.
Constraint 1 requires $y \geq x + 1$, so $y > x$.
Constraint 2 requires $y \leq x$.
It is impossible to have simultaneously $y > x$ and $y \leq x$.
Therefore, there is no point (x, y) that satisfies all the constraints at the same

time.

Step 3: Conclusion.

The feasible region is empty — no feasible solution exists. Hence the linear programming problem has no solution, and Z = x + y has no maximum value.

Answer: The problem has no feasible solution (the constraints are inconsistent), so Z = x + y has no maximum value.