Chapter 13: Probability

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Exercise 13.1 — Exercise 13.1

Q1 Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, find P(E|F) and P(F|E).

SOLUTION

Using the definition of conditional probability:

P(E|F) = P(E ∩ F) / P(F) = 0.2 / 0.3 = 2/3

P(F|E) = P(E ∩ F) / P(E) = 0.2 / 0.6 = 1/3

Answer: P(E|F) = 2/3, P(F|E) = 1/3

Q2 Compute P(A|B), if P(B) = 0.5 and P(A ∩ B) = 0.32.

SOLUTION

P(A|B) = P(A ∩ B) / P(B) = 0.32 / 0.5

= 32/50 = 16/25 = 0.64

Answer: P(A|B) = 16/25 = 0.64

Q3 If P(A) = 0.8, P(B) = 0.5 and P(B|A) = 0.4, find (i) P(A ∩ B), (ii) P(A|B), (iii) P(A ∪ B).

SOLUTION

(i) P(A ∩ B) = P(A) · P(B|A) = 0.8 × 0.4 = 0.32

(ii) P(A|B) = P(A ∩ B) / P(B) = 0.32 / 0.5 = 16/25 = 0.64

(iii) P(A ∪ B) = P(A) + P(B) − P(A ∩ B) = 0.8 + 0.5 − 0.32 = 0.98

Answer: (i) P(A ∩ B) = 0.32, (ii) P(A|B) = 0.64, (iii) P(A ∪ B) = 0.98

Q4 Evaluate P(A ∪ B), if 2P(A) = P(B) = 5/13 and P(A|B) = 2/5.

SOLUTION

Given 2P(A) = P(B) = 5/13, so P(B) = 5/13 and P(A) = 5/26.

P(A ∩ B) = P(A|B) · P(B) = (2/5) × (5/13) = 2/13

P(A ∪ B) = P(A) + P(B) − P(A ∩ B)
= 5/26 + 5/13 − 2/13
= 5/26 + 10/26 − 4/26
= 11/26

Answer: P(A ∪ B) = 11/26

Q5 If P(A) = 6/11, P(B) = 5/11 and P(A ∪ B) = 7/11, find (i) P(A ∩ B), (ii) P(A|B), (iii) P(B|A).

SOLUTION

(i) P(A ∩ B) = P(A) + P(B) − P(A ∪ B) = 6/11 + 5/11 − 7/11 = 4/11

(ii) P(A|B) = P(A ∩ B) / P(B) = (4/11) / (5/11) = 4/5

(iii) P(B|A) = P(A ∩ B) / P(A) = (4/11) / (6/11) = 4/6 = 2/3

Answer: (i) P(A ∩ B) = 4/11, (ii) P(A|B) = 4/5, (iii) P(B|A) = 2/3

Q6 A coin is tossed three times, where
(i) E : head on third toss, F : heads on first two tosses
(ii) E : at least two heads, F : at most two heads
(iii) E : at most two tails, F : at least one tail
Determine P(E|F).

SOLUTION

The sample space is S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}, each with probability 1/8.

(i) E = {HHH, HTH, THH, TTH}, F = {HHH, HHT}
E ∩ F = {HHH}
P(E ∩ F) = 1/8, P(F) = 2/8 = 1/4
P(E|F) = (1/8) / (1/4) = 1/2

(ii) E = {HHH, HHT, HTH, THH} (at least 2 heads), F = {HHT, HTH, THH, HTT, THT, TTH, TTT} (at most 2 heads)
E ∩ F = {HHT, HTH, THH}
P(E ∩ F) = 3/8, P(F) = 7/8
P(E|F) = (3/8) / (7/8) = 3/7

(iii) E = {HHH, HHT, HTH, THH, HTT, THT, TTH} (at most 2 tails), F = {HHT, HTH, THH, HTT, THT, TTH, TTT} (at least 1 tail)
E ∩ F = {HHT, HTH, THH, HTT, THT, TTH}
P(E ∩ F) = 6/8 = 3/4, P(F) = 7/8
P(E|F) = (3/4) / (7/8) = (3/4) × (8/7) = 6/7

Answer: (i) P(E|F) = 1/2, (ii) P(E|F) = 3/7, (iii) P(E|F) = 6/7

Q7 Two coins are tossed once, where
(i) E : tail appears on one coin, F : one coin shows head
(ii) E : no tail appears, F : no head appears
Determine P(E|F).

SOLUTION

Sample space S = {HH, HT, TH, TT}, each with probability 1/4.

(i) E : exactly one tail = {HT, TH}, F : exactly one head = {HT, TH}
E ∩ F = {HT, TH}
P(E ∩ F) = 2/4 = 1/2, P(F) = 2/4 = 1/2
P(E|F) = (1/2) / (1/2) = 1

(ii) E : no tail = {HH}, F : no head = {TT}
E ∩ F = {} (empty set)
P(E ∩ F) = 0, P(F) = 1/4
P(E|F) = 0 / (1/4) = 0

Answer: (i) P(E|F) = 1, (ii) P(E|F) = 0

Q8 A die is thrown three times.
E : 4 appears on the third toss, F : 6 and 5 appears respectively on first two tosses.
Determine P(E|F).

SOLUTION

The sample space has $6^3 = 216$ equally likely outcomes.

F = {(6,5,1), (6,5,2), (6,5,3), (6,5,4), (6,5,5), (6,5,6)}, so P(F) = 6/216 = 1/36.

E ∩ F = {(6,5,4)}, so P(E ∩ F) = 1/216.

P(E|F) = P(E ∩ F) / P(F) = (1/216) / (6/216) = 1/6

Answer: P(E|F) = 1/6

Q9 Mother, father and son line up at random for a family picture.
E : son on one end, F : father in middle.
Determine P(E|F).

SOLUTION

The 3 people can be arranged in 3! = 6 equally likely ways.
All arrangements: (M,F,S), (M,S,F), (F,M,S), (F,S,M), (S,M,F), (S,F,M).

F (father in middle) = {(M,F,S), (S,F,M)}, so P(F) = 2/6 = 1/3.

E (son on one end) = {(M,F,S), (F,M,S), (S,M,F), (S,F,M)}.

E ∩ F = {(M,F,S), (S,F,M)}, so P(E ∩ F) = 2/6 = 1/3.

P(E|F) = P(E ∩ F) / P(F) = (1/3) / (1/3) = 1

Answer: P(E|F) = 1

Q10 A black and a red dice are rolled.
(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

SOLUTION

(a) Let A = event sum > 9, B = event black die = 5.
Given black die = 5, possible outcomes: (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) — total 6.
Sum > 9 with black = 5: (5,5) gives sum 10, (5,6) gives sum 11. So A ∩ B = {(5,5),(5,6)}.

P(A|B) = 2/6 = 1/3

(b) Let C = event sum = 8, D = event red die < 4 (red ∈ {1,2,3}).
Total outcomes with red < 4: 6 × 3 = 18.
Sum = 8 with red < 4: red=2 gives black=6 → (6,2); red=3 gives black=5 → (5,3). C ∩ D = {(6,2),(5,3)}.
P(C|D) = 2/18 = 1/9

Answer: (a) P(sum > 9 | black = 5) = 1/3, (b) P(sum = 8 | red < 4) = 1/9

Q11 A fair die is rolled. Consider events E = {1,3,5}, F = {2,3} and G = {2,3,4,5}. Find
(i) P(E|F) and P(F|E)
(ii) P(E|G) and P(G|E)
(iii) P((E ∪ F)|G) and P((E ∩ F)|G).

SOLUTION

Each outcome of a fair die has probability 1/6.
P(E) = 3/6 = 1/2, P(F) = 2/6 = 1/3, P(G) = 4/6 = 2/3.

(i) E ∩ F = {3}, P(E ∩ F) = 1/6.

P(E	F) = (1/6) / (1/3) = 1/2
P(F	E) = (1/6) / (1/2) = 1/3

(ii) E ∩ G = {3,5}, P(E ∩ G) = 2/6 = 1/3.

P(E	G) = (1/3) / (2/3) = 1/2
P(G	E) = (1/3) / (1/2) = 2/3

(iii) E ∪ F = {1,2,3,5}. (E ∪ F) ∩ G = {2,3,5}, so P((E ∪ F) ∩ G) = 3/6 = 1/2.
P((E ∪ F)|G) = (1/2) / (2/3) = 3/4

E ∩ F = {3}. (E ∩ F) ∩ G = {3}, so P((E ∩ F) ∩ G) = 1/6.
P((E ∩ F)|G) = (1/6) / (2/3) = 1/4

Answer: (i) P(E|F) = 1/2, P(F|E) = 1/3; (ii) P(E|G) = 1/2, P(G|E) = 2/3; (iii) P((E ∪ F)|G) = 3/4, P((E ∩ F)|G) = 1/4

Q12 Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl, (ii) at least one is a girl?

SOLUTION

Sample space S = {(B,B),(B,G),(G,B),(G,G)}, each with probability 1/4. (First child, second child.)

(i) Let A = both girls = {(G,G)}, B = youngest (second child) is a girl = {(B,G),(G,G)}.
A ∩ B = {(G,G)}, P(A ∩ B) = 1/4, P(B) = 2/4 = 1/2.
P(A|B) = (1/4) / (1/2) = 1/2

(ii) Let A = both girls = {(G,G)}, C = at least one girl = {(B,G),(G,B),(G,G)}.
A ∩ C = {(G,G)}, P(A ∩ C) = 1/4, P(C) = 3/4.
P(A|C) = (1/4) / (3/4) = 1/3

Answer: (i) 1/2, (ii) 1/3

Q13 An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

SOLUTION

Total questions = 300 + 200 + 500 + 400 = 1400.

Let A = easy question, B = multiple choice question.
P(B) = (500 + 400)/1400 = 900/1400 = 9/14.
P(A ∩ B) = 500/1400 = 5/14.

P(A|B) = P(A ∩ B) / P(B) = (5/14) / (9/14) = 5/9

Answer: P(easy | multiple choice) = 5/9

Q14 Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

SOLUTION

Let A = sum of numbers is 4, B = numbers on the two dice are different.

P(B) = 30/36 = 5/6 (all outcomes minus the 6 doubles).

Pairs summing to 4 with different numbers: (1,3) and (3,1). (The pair (2,2) is excluded since the numbers are the same.)
P(A ∩ B) = 2/36 = 1/18.

P(A|B) = P(A ∩ B) / P(B) = (1/18) / (5/6) = (1/18) × (6/5) = 1/15

Answer: P(sum = 4 | numbers are different) = 1/15

Q15 Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

SOLUTION

The multiples of 3 on a die are 3 and 6.

Sample space description:
- If first roll is 3 or 6 (multiple of 3): roll the die again. Outcomes (3,1),(3,2),...,(3,6),(6,1),...,(6,6), each with probability (1/6)×(1/6) = 1/36.
- If first roll is 1,2,4,5: toss a coin. Outcomes (1,H),(1,T),(2,H),(2,T),(4,H),(4,T),(5,H),(5,T), each with probability (1/6)×(1/2) = 1/12.

Let E = coin shows tail, F = at least one die shows 3.

E occurs only when first roll ∈ {1,2,4,5} (the coin-toss branch). In this branch, no die shows a multiple of 3, so no die can show 3.
F occurs when: first die = 3 (all of (3,1),...,(3,6)) OR (first die = 6 AND second die = 3), i.e., (6,3).

Since E and F involve mutually exclusive branches, E ∩ F = empty set.
P(E ∩ F) = 0.

P(F) = 6×(1/36) + 1×(1/36) = 7/36.

P(E|F) = P(E ∩ F) / P(F) = 0 / (7/36) = 0

Answer: P(coin shows tail | at least one die shows 3) = 0

Q16 If P(A) = 1/2, P(B) = 0, then P(A|B) is
(A) 0
(B) 1/2
(C) not defined
(D) 1

SOLUTION

P(A	B) = P(A ∩ B) / P(B), which requires P(B) ≠ 0.
Since P(B) = 0, P(A	B) is not defined.

Answer: (C) not defined

Q17 If A and B are events such that P(A|B) = P(B|A), then
(A) A ⊂ B but A ≠ B
(B) A = B
(C) A ∩ B = φ
(D) P(A) = P(B)

SOLUTION

P(A|B) = P(A ∩ B)/P(B) and P(B|A) = P(A ∩ B)/P(A).

Setting them equal: P(A ∩ B)/P(B) = P(A ∩ B)/P(A).

Assuming P(A ∩ B) ≠ 0 (otherwise both sides are 0 for any P(A), P(B)), we get P(A) = P(B).

Answer: (D) P(A) = P(B)

Exercise 13.2 — Exercise 13.2

Q1 If P(A) = 3/5 and P(B) = 1/5, find P(A ∩ B) if A and B are independent events.

SOLUTION

For independent events, P(A ∩ B) = P(A) · P(B).

P(A ∩ B) = (3/5) × (1/5) = 3/25

Answer: P(A ∩ B) = 3/25

Q2 Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

SOLUTION

There are 26 black cards in a pack of 52.

P(first card is black) = 26/52 = 1/2.

Given first is black, 25 black cards remain among 51 total.
P(second is black | first is black) = 25/51.

By multiplication rule:
P(both black) = (26/52) × (25/51) = (1/2) × (25/51) = 25/102

Answer: P(both black) = 25/102

Q3 A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

SOLUTION

The box is approved if all 3 selected oranges are good.

P(all 3 good) = C(12,3) / C(15,3)

C(12,3) = (12 × 11 × 10) / (3 × 2 × 1) = 220
C(15,3) = (15 × 14 × 13) / (3 × 2 × 1) = 455

P(approved) = 220/455 = 44/91

Answer: P(approved) = 44/91

Q4 A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not.

SOLUTION

P(A) = 1/2, P(B) = 1/6.

P(A ∩ B) = P(head and 3) = 1/12 (out of 12 equally likely outcomes).

P(A) · P(B) = (1/2) × (1/6) = 1/12.

Since P(A ∩ B) = P(A) · P(B), A and B are independent events.

Answer: A and B are independent events.

Q5 A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent?

SOLUTION

S = {1,2,3,4,5,6}.
A = {2,4,6} (even numbers), P(A) = 3/6 = 1/2.
B = {1,2,3} (red numbers), P(B) = 3/6 = 1/2.
A ∩ B = {2}, P(A ∩ B) = 1/6.

P(A) · P(B) = (1/2) × (1/2) = 1/4.

Since P(A ∩ B) = 1/6 ≠ 1/4 = P(A) · P(B), A and B are NOT independent.

Answer: A and B are not independent.

Q6 Let E and F be events with P(E) = 3/5, P(F) = 3/10 and P(E ∩ F) = 1/5. Are E and F independent?

SOLUTION

P(E) · P(F) = (3/5) × (3/10) = 9/50.

P(E ∩ F) = 1/5 = 10/50.

Since 9/50 ≠ 10/50, P(E ∩ F) ≠ P(E) · P(F), so E and F are NOT independent.

Answer: E and F are not independent.

Q7 Given that the events A and B are such that P(A) = 1/2, P(A ∪ B) = 3/5 and P(B) = p. Find p if they are (i) mutually exclusive, (ii) independent.

SOLUTION

(i) Mutually exclusive: P(A ∩ B) = 0.

P(A ∪ B) = P(A) + P(B) − P(A ∩ B)

3/5 = 1/2 + p − 0

p = 3/5 − 1/2 = 6/10 − 5/10 = 1/10

(ii) Independent: P(A ∩ B) = P(A) · P(B) = p/2.

P(A ∪ B) = P(A) + P(B) − P(A ∩ B)

3/5 = 1/2 + p − p/2 = 1/2 + p/2

p/2 = 3/5 − 1/2 = 1/10

p = 1/5

Answer: (i) p = 1/10, (ii) p = 1/5

Q8 Let A and B be independent events with P(A) = 0.3 and P(B) = 0.4. Find (i) P(A ∩ B), (ii) P(A ∪ B), (iii) P(A|B), (iv) P(B|A).

SOLUTION

(i) P(A ∩ B) = P(A) · P(B) = 0.3 × 0.4 = 0.12

(ii) P(A ∪ B) = P(A) + P(B) − P(A ∩ B) = 0.3 + 0.4 − 0.12 = 0.58

(iii) Since A and B are independent, P(A|B) = P(A) = 0.3

(iv) Since A and B are independent, P(B|A) = P(B) = 0.4

Answer: (i) 0.12, (ii) 0.58, (iii) 0.3, (iv) 0.4

Q9 If A and B are two events such that P(A) = 1/4, P(B) = 1/2 and P(A ∩ B) = 1/8, find P(not A and not B).

SOLUTION

P(not A and not B) = P(A' ∩ B') = P((A ∪ B)') = 1 − P(A ∪ B).

P(A ∪ B) = P(A) + P(B) − P(A ∩ B) = 1/4 + 1/2 − 1/8 = 2/8 + 4/8 − 1/8 = 5/8.

P(A' ∩ B') = 1 − 5/8 = 3/8

Answer: P(not A and not B) = 3/8

Q10 Events A and B are such that P(A) = 1/2, P(B) = 7/12 and P(not A or not B) = 1/4. State whether A and B are independent.

SOLUTION

P(not A or not B) = P(A' ∪ B') = 1/4.
By De Morgan's law: P(A' ∪ B') = 1 − P(A ∩ B).
So P(A ∩ B) = 1 − 1/4 = 3/4.

But P(A) · P(B) = (1/2) × (7/12) = 7/24.

Since 3/4 ≠ 7/24, P(A ∩ B) ≠ P(A) · P(B), so A and B are NOT independent.

Answer: A and B are not independent.

Q11 Given two independent events A and B such that P(A) = 0.3, P(B) = 0.6. Find (i) P(A and B), (ii) P(A and not B), (iii) P(A or B), (iv) P(neither A nor B).

SOLUTION

(i) P(A and B) = P(A ∩ B) = P(A) · P(B) = 0.3 × 0.6 = 0.18

(ii) P(A and not B) = P(A) · P(B') = 0.3 × (1 − 0.6) = 0.3 × 0.4 = 0.12

(iii) P(A or B) = P(A ∪ B) = P(A) + P(B) − P(A ∩ B) = 0.3 + 0.6 − 0.18 = 0.72

(iv) P(neither A nor B) = P(A' ∩ B') = P(A') · P(B') = 0.7 × 0.4 = 0.28

Answer: (i) 0.18, (ii) 0.12, (iii) 0.72, (iv) 0.28

Q12 A die is tossed thrice. Find the probability of getting an odd number at least once.

SOLUTION

P(odd on a single throw) = 3/6 = 1/2.
P(not odd on a single throw) = 1/2.

P(no odd number in three throws) = $(1/2)^3$ = 1/8.

P(odd at least once) = 1 − P(no odd) = 1 − 1/8 = 7/8

Answer: P(odd at least once) = 7/8

Q13 Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that
(i) both balls are red,
(ii) first ball is black and second is red,
(iii) one of them is black and other is red.

SOLUTION

Total balls = 18. P(red) = 8/18 = 4/9. P(black) = 10/18 = 5/9.
Draws are independent (with replacement).

(i) P(both red) = (4/9) × (4/9) = 16/81

(ii) P(first black, second red) = (5/9) × (4/9) = 20/81

(iii) P(one black, one red) = P(BR) + P(RB) = 20/81 + 20/81 = 40/81

Answer: (i) 16/81, (ii) 20/81, (iii) 40/81

Q14 Probability of solving specific problem independently by A and B are 1/2 and 1/3 respectively. If both try to solve the problem independently, find the probability that
(i) the problem is solved,
(ii) exactly one of them solves the problem.

SOLUTION

P(A solves) = 1/2, P(B solves) = 1/3.
P(A fails) = 1/2, P(B fails) = 2/3.

(i) P(problem solved) = 1 − P(neither solves) = 1 − P(A fails) · P(B fails)
= 1 − (1/2)(2/3) = 1 − 1/3 = 2/3

(ii) P(exactly one solves) = P(A solves, B fails) + P(A fails, B solves)
= (1/2)(2/3) + (1/2)(1/3)
= 1/3 + 1/6 = 2/6 + 1/6 = 1/2

Answer: (i) P(problem solved) = 2/3, (ii) P(exactly one solves) = 1/2

Q15 One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?
(i) E : 'the card drawn is a spade', F : 'the card drawn is an ace'
(ii) E : 'the card drawn is black', F : 'the card drawn is a king'
(iii) E : 'the card drawn is a king or queen', F : 'the card drawn is a queen or jack'.

SOLUTION

(i) P(E) = 13/52 = 1/4, P(F) = 4/52 = 1/13.
P(E ∩ F) = 1/52 (ace of spades).
P(E) · P(F) = (1/4)(1/13) = 1/52 = P(E ∩ F). => E and F are independent.

(ii) P(E) = 26/52 = 1/2, P(F) = 4/52 = 1/13.
P(E ∩ F) = 2/52 = 1/26 (black kings).
P(E) · P(F) = (1/2)(1/13) = 1/26 = P(E ∩ F). => E and F are independent.

(iii) P(E) = 8/52 = 2/13 (4 kings + 4 queens), P(F) = 8/52 = 2/13 (4 queens + 4 jacks).
P(E ∩ F) = 4/52 = 1/13 (only queens are in both).
P(E) · P(F) = (2/13)(2/13) = 4/169.
Since 1/13 = 13/169 ≠ 4/169, E and F are NOT independent.

Answer: (i) Independent, (ii) Independent, (iii) Not independent

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Q16 In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.
(a) Find the probability that she reads neither Hindi nor English newspapers.
(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.
(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

SOLUTION

Let H = Hindi, E = English. P(H) = 0.6, P(E) = 0.4, P(H ∩ E) = 0.2.

(a) P(H ∪ E) = P(H) + P(E) − P(H ∩ E) = 0.6 + 0.4 − 0.2 = 0.8.
P(neither) = 1 − 0.8 = 0.2

(b) P(E|H) = P(H ∩ E) / P(H) = 0.2 / 0.6 = 1/3

(c) P(H|E) = P(H ∩ E) / P(E) = 0.2 / 0.4 = 1/2

Answer: (a) 0.2, (b) 1/3, (c) 1/2

Q17 The probability of obtaining an even prime number on each die, when a pair of dice is rolled is
(A) 0
(B) 1/3
(C) 1/12
(D) 1/36

SOLUTION

The only even prime number is 2.
P(2 on one die) = 1/6.
P(2 on both dice) = (1/6) × (1/6) = 1/36.

Answer: (D) 1/36

Q18 Two events A and B will be independent, if
(A) A and B are mutually exclusive
(B) P(A'B') = [1 − P(A)][1 − P(B)]
(C) P(A) = P(B)
(D) P(A) + P(B) = 1

SOLUTION

Option (B): $P(A' \cap B') = P(A') \cdot P(B') = [1 - P(A)][1 - P(B)]$.
This holds if and only if A' and B' are independent, which is equivalent to A and B being independent.

(A) is incorrect: mutually exclusive events with nonzero probabilities are not independent.
(C) is incorrect: equal probabilities do not imply independence.
(D) is incorrect: probabilities summing to 1 do not imply independence.

Answer: (B) P(A'B') = [1 − P(A)][1 − P(B)]

Exercise 13.3 — Exercise 13.3

Q1 An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

SOLUTION

Case 1: First ball drawn is red.
P(first red) = 5/10 = 1/2.
After returning and adding 2 red balls: urn has 7 red and 5 black (12 total).
P(second red | first red) = 7/12.

Case 2: First ball drawn is black.
P(first black) = 5/10 = 1/2.
After returning and adding 2 black balls: urn has 5 red and 7 black (12 total).
P(second red | first black) = 5/12.

By total probability theorem:
P(second red) = (1/2)(7/12) + (1/2)(5/12) = 7/24 + 5/24 = 12/24 = 1/2

Answer: P(second ball is red) = 1/2

Q2 A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

SOLUTION

Let E1 = first bag selected, E2 = second bag selected, A = red ball drawn.
P(E1) = P(E2) = 1/2.
P(A|E1) = 4/8 = 1/2, P(A|E2) = 2/8 = 1/4.

By Bayes' theorem:
P(E1|A) = P(E1) P(A|E1) / [P(E1) P(A|E1) + P(E2) P(A|E2)]
= (1/2)(1/2) / [(1/2)(1/2) + (1/2)(1/4)]
= (1/4) / (1/4 + 1/8)
= (1/4) / (3/8)
= (1/4) × (8/3) = 2/3

Answer: P(first bag | red ball) = 2/3

Q3 Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?

SOLUTION

Let H = hostler, D = day scholar, A = student has A grade.
P(H) = 0.6, P(D) = 0.4.
P(A|H) = 0.3, P(A|D) = 0.2.

P(A) = P(H) P(A|H) + P(D) P(A|D) = 0.6 × 0.3 + 0.4 × 0.2 = 0.18 + 0.08 = 0.26.

By Bayes' theorem:
P(H|A) = P(H) P(A|H) / P(A) = 0.18 / 0.26 = 18/26 = 9/13

Answer: P(student is hostler | A grade) = 9/13

Q4 In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/4. What is the probability that the student knows the answer given that he answered it correctly?

SOLUTION

Let K = student knows the answer, G = student guesses, C = student answers correctly.
P(K) = 3/4, P(G) = 1/4.
P(C|K) = 1 (knows => answers correctly), P(C|G) = 1/4.

P(C) = P(K) P(C|K) + P(G) P(C|G) = (3/4)(1) + (1/4)(1/4) = 3/4 + 1/16 = 12/16 + 1/16 = 13/16.

By Bayes' theorem:
P(K|C) = P(K) P(C|K) / P(C) = (3/4) / (13/16) = (3/4) × (16/13) = 12/13

Answer: P(knows answer | correct) = 12/13

Q5 A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy persons tested. If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

SOLUTION

Let D = person has the disease, D' = person is healthy, P = test result is positive.
P(D) = 0.1/100 = 0.001, P(D') = 0.999.
P(P|D) = 0.99, P(P|D') = 0.005.

P(P) = P(D) P(P|D) + P(D') P(P|D') = 0.001 × 0.99 + 0.999 × 0.005
= 0.00099 + 0.004995 = 0.005985.

P(D|P) = P(D) P(P|D) / P(P) = 0.00099 / 0.005985

Using fractions: P(D) = 1/1000, P(P|D) = 99/100, P(P|D') = 5/1000.
Numerator = (1/1000)(99/100) = 99/100000.
Denominator = 99/100000 + (999/1000)(5/1000) = 99/100000 + 4995/1000000
= 990/1000000 + 4995/1000000 = 5985/1000000.

P(D|P) = (99/100000) / (5985/1000000) = (99 × 1000000) / (100000 × 5985) = 990/5985 = 22/133

Answer: P(disease | positive test) = 22/133 ≈ 0.165

Q6 There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

SOLUTION

Let E1 = two-headed coin chosen, E2 = biased coin chosen, E3 = unbiased coin chosen, H = head observed.
P(E1) = P(E2) = P(E3) = 1/3.
P(H|E1) = 1, P(H|E2) = 3/4, P(H|E3) = 1/2.

P(H) = P(E1) P(H|E1) + P(E2) P(H|E2) + P(E3) P(H|E3)
= (1/3)(1) + (1/3)(3/4) + (1/3)(1/2)
= 1/3 + 1/4 + 1/6
= 4/12 + 3/12 + 2/12 = 9/12 = 3/4.

By Bayes' theorem:
P(E1|H) = P(E1) P(H|E1) / P(H) = (1/3)(1) / (3/4) = (1/3) × (4/3) = 4/9

Answer: P(two-headed coin | head shows) = 4/9

Q7 An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

SOLUTION

Total insured = 2000 + 4000 + 6000 = 12000.
Let E1 = scooter driver, E2 = car driver, E3 = truck driver, A = accident.
P(E1) = 2000/12000 = 1/6, P(E2) = 4000/12000 = 1/3, P(E3) = 6000/12000 = 1/2.
P(A|E1) = 0.01, P(A|E2) = 0.03, P(A|E3) = 0.15.

P(A) = (1/6)(0.01) + (1/3)(0.03) + (1/2)(0.15)
= 0.01/6 + 0.03/3 + 0.15/2
= 1/600 + 1/100 + 3/40
= 1/600 + 6/600 + 45/600 = 52/600 = 13/150.

P(E1|A) = P(E1) P(A|E1) / P(A) = (1/6)(0.01) / (13/150)
= (1/600) / (13/150) = (1/600) × (150/13) = 150/7800 = 1/52

Answer: P(scooter driver | accident) = 1/52

Q8 A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

SOLUTION

Let E1 = item from machine A, E2 = item from machine B, D = item is defective.
P(E1) = 0.6, P(E2) = 0.4.
P(D|E1) = 0.02, P(D|E2) = 0.01.

P(D) = P(E1) P(D|E1) + P(E2) P(D|E2) = 0.6 × 0.02 + 0.4 × 0.01 = 0.012 + 0.004 = 0.016.

P(E2|D) = P(E2) P(D|E2) / P(D) = (0.4 × 0.01) / 0.016 = 0.004 / 0.016 = 1/4

Answer: P(machine B | defective) = 1/4

Q9 Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

SOLUTION

Let E1 = first group wins, E2 = second group wins, A = new product introduced.
P(E1) = 0.6, P(E2) = 0.4.
P(A|E1) = 0.7, P(A|E2) = 0.3.

P(A) = P(E1) P(A|E1) + P(E2) P(A|E2) = 0.6 × 0.7 + 0.4 × 0.3 = 0.42 + 0.12 = 0.54.

P(E2|A) = P(E2) P(A|E2) / P(A) = (0.4 × 0.3) / 0.54 = 0.12 / 0.54 = 12/54 = 2/9

Answer: P(second group | new product introduced) = 2/9

Q10 Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

SOLUTION

Let E1 = die shows 1,2,3 or 4, E2 = die shows 5 or 6, A = exactly one head.
P(E1) = 4/6 = 2/3, P(E2) = 2/6 = 1/3.

If E1: one coin toss. P(exactly 1 head | E1) = 1/2.

If E2: three coin tosses. P(exactly 1 head | E2) = C(3,1)(1/2)³ = 3/8.

P(A) = P(E1) P(A|E1) + P(E2) P(A|E2) = (2/3)(1/2) + (1/3)(3/8)
= 1/3 + 1/8 = 8/24 + 3/24 = 11/24.

P(E1|A) = P(E1) P(A|E1) / P(A) = (2/3)(1/2) / (11/24) = (1/3) / (11/24) = (1/3) × (24/11) = 8/11

Answer: P(die showed 1,2,3 or 4 | exactly one head) = 8/11

Q11 A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

SOLUTION

Let E1 = item produced by A, E2 = by B, E3 = by C, D = defective.
P(E1) = 0.5, P(E2) = 0.3, P(E3) = 0.2.
P(D|E1) = 0.01, P(D|E2) = 0.05, P(D|E3) = 0.07.

P(D) = 0.5 × 0.01 + 0.3 × 0.05 + 0.2 × 0.07
= 0.005 + 0.015 + 0.014 = 0.034.

P(E1|D) = P(E1) P(D|E1) / P(D) = (0.5 × 0.01) / 0.034 = 0.005 / 0.034 = 5/34

Answer: P(produced by A | defective) = 5/34

Q12 A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

SOLUTION

Let E1 = lost card is a diamond, E2 = lost card is not a diamond, A = two drawn cards are both diamonds.
P(E1) = 13/52 = 1/4, P(E2) = 39/52 = 3/4.

If E1 (lost card is diamond): 12 diamonds remain in 51 cards.
P(A|E1) = C(12,2)/C(51,2) = 66/1275 = 22/425.

If E2 (lost card is not diamond): 13 diamonds remain in 51 cards.
P(A|E2) = C(13,2)/C(51,2) = 78/1275 = 26/425.

P(A) = P(E1) P(A|E1) + P(E2) P(A|E2)
= (1/4)(22/425) + (3/4)(26/425)
= 22/1700 + 78/1700 = 100/1700 = 1/17.

P(E1|A) = P(E1) P(A|E1) / P(A) = (1/4)(22/425) / (1/17)
= (22/1700) × 17 = 22 × 17/1700 = 374/1700 = 11/50

Answer: P(lost card is a diamond | two drawn are diamonds) = 11/50

Q13 Probability that A speaks truth is 4/5. A coin is tossed. A reports that a head appears. The probability that actually there was head is
(A) 4/5
(B) 1/2
(C) 1/5
(D) 2/5

SOLUTION

Let S1 = head actually occurred, S2 = tail actually occurred, R = A reports head.
P(S1) = 1/2, P(S2) = 1/2.

P(R	S1) = 4/5 (A tells truth and says head).
P(R	S2) = 1/5 (A lies and says head when tail).

P(R) = P(S1) P(R|S1) + P(S2) P(R|S2) = (1/2)(4/5) + (1/2)(1/5) = 2/5 + 1/10 = 5/10 = 1/2.

P(S1|R) = P(S1) P(R|S1) / P(R) = (1/2)(4/5) / (1/2) = 4/5

Answer: (A) 4/5

Q14 If A and B are two events such that A ⊂ B and P(B) ≠ 0, then which of the following is correct?
(A) P(A|B) = P(B)/P(A)
(B) P(A|B) < P(A)
(C) P(A|B) ≥ P(A)
(D) None of these

SOLUTION

Since A ⊂ B, every outcome in A is also in B, so A ∩ B = A.

P(A|B) = P(A ∩ B)/P(B) = P(A)/P(B).

Since P(B) ≤ 1 (and P(B) > 0), we have P(A)/P(B) ≥ P(A).
Therefore P(A|B) ≥ P(A).

Answer: (C) P(A|B) ≥ P(A)

Misc — Miscellaneous Exercise

Q1 A and B are two events such that $P(A) \neq 0$. Find $P(B|A)$, if
(i) A is a subset of B
(ii) $A \cap B = \varphi$

SOLUTION

(i) If $A \subset B$, then $A \cap B = A$.
$P(B|A) = P(A \cap B)/P(A) = P(A)/P(A) = 1$.

(ii) If $A \cap B = \varphi$, then $P(A \cap B) = 0$.
$P(B|A) = P(A \cap B)/P(A) = 0/P(A) = 0$.

Answer: (i) $P(B|A) = 1$, (ii) $P(B|A) = 0$

Q2 A couple has two children,
(i) Find the probability that both children are males, if it is known that at least one of the children is male.
(ii) Find the probability that both children are females, if it is known that the elder child is a female.

SOLUTION

Sample space S = {(M,M),(M,F),(F,M),(F,F)}, each equally likely.

(i) A = both males = {(M,M)}, B = at least one male = {(M,M),(M,F),(F,M)}.
$A \cap B = \{(M,M)\}$, $P(A \cap B) = 1/4$, $P(B) = 3/4$.
$P(A|B) = (1/4)/(3/4) = 1/3$.

(ii) C = both females = {(F,F)}, D = elder (first) child is female = {(F,M),(F,F)}.
$C \cap D = \{(F,F)\}$, $P(C \cap D) = 1/4$, $P(D) = 2/4 = 1/2$.
$P(C|D) = (1/4)/(1/2) = 1/2$.

Answer: (i) 1/3, (ii) 1/2

Q3 Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal numbers of males and females.

SOLUTION

Let M = male, F = female, G = grey hair.
P(M) = 1/2, P(F) = 1/2.
P(G|M) = 5/100 = 1/20, P(G|F) = 0.25/100 = 1/400.

P(G) = P(M) P(G|M) + P(F) P(G|F) = (1/2)(1/20) + (1/2)(1/400)
= 1/40 + 1/800 = 20/800 + 1/800 = 21/800.

P(M|G) = P(M) P(G|M) / P(G) = (1/2)(1/20) / (21/800)
= (1/40) / (21/800) = (1/40) × (800/21) = 20/21

Answer: P(male | grey hair) = 20/21

Q4 Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

SOLUTION

Let X = number of right-handed people in a sample of 10.
X ~ Binomial(n=10, p=9/10).

P(X ≤ 6) = 1 − P(X ≥ 7) = 1 − [P(X=7) + P(X=8) + P(X=9) + P(X=10)].

P(X=k) = C(10,k) (9/10)^k (1/10)^(10−k).

P(X=7) = C(10,7)(9/10)⁷(1/10)³ = 120 × 9⁷/10¹⁰
P(X=8) = C(10,8)(9/10)⁸(1/10)² = 45 × 9⁸/10¹⁰
P(X=9) = C(10,9)(9/10)⁹(1/10)¹ = 10 × 9⁹/10¹⁰
P(X=10) = C(10,10)(9/10)¹⁰ = 9¹⁰/10¹⁰

9⁷ = 4782969, 9⁸ = 43046721, 9⁹ = 387420489, 9¹⁰ = 3486784401.

P(X≥7) = (120×4782969 + 45×43046721 + 10×387420489 + 3486784401)/10¹⁰
= (573956280 + 1937102445 + 3874204890 + 3486784401)/10¹⁰
= 9872048016/10¹⁰.

P(X ≤ 6) = 1 − 9872048016/10¹⁰ = 127951984/10¹⁰

Answer: P(at most 6 right-handed) = 1 − 9872048016/10¹⁰ = 127951984/10¹⁰

Q5 If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?

SOLUTION

A leap year has 366 days = 52 complete weeks + 2 extra days.

The 52 complete weeks always contribute 52 Tuesdays.
The 2 extra days can be any consecutive pair:
{(Mon,Tue), (Tue,Wed), (Wed,Thu), (Thu,Fri), (Fri,Sat), (Sat,Sun), (Sun,Mon)}
— 7 equally likely possibilities.

The leap year will contain 53 Tuesdays if the extra days include a Tuesday.
Favourable pairs: (Mon,Tue) and (Tue,Wed) — 2 out of 7.

P(53 Tuesdays) = 2/7

Answer: P(53 Tuesdays in a leap year) = 2/7

Q6 Suppose we have four boxes A, B, C and D containing coloured marbles as given below:
Box A: 1 Red, 6 White, 3 Black (total 10)
Box B: 6 Red, 2 White, 2 Black (total 10)
Box C: 8 Red, 1 White, 1 Black (total 10)
Box D: 0 Red, 6 White, 4 Black (total 10)
One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A, from box B, from box C?

SOLUTION

P(each box selected) = 1/4.
P(red|A) = 1/10, P(red|B) = 6/10 = 3/5, P(red|C) = 8/10 = 4/5, P(red|D) = 0.

P(red) = (1/4)(1/10) + (1/4)(6/10) + (1/4)(8/10) + (1/4)(0)
= (1/4)(1/10 + 6/10 + 8/10) = (1/4)(15/10) = (1/4)(3/2) = 3/8.

P(A|red) = P(A) P(red|A) / P(red) = (1/4)(1/10) / (3/8) = (1/40) / (3/8) = (1/40)(8/3) = 1/15.

P(B|red) = P(B) P(red|B) / P(red) = (1/4)(6/10) / (3/8) = (6/40) / (3/8) = (6/40)(8/3) = 48/120 = 2/5.

P(C|red) = P(C) P(red|C) / P(red) = (1/4)(8/10) / (3/8) = (8/40) / (3/8) = (8/40)(8/3) = 64/120 = 8/15.

Answer: P(box A | red) = 1/15, P(box B | red) = 2/5, P(box C | red) = 8/15

Q7 Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.

SOLUTION

Let M = patient followed meditation/yoga, D = patient took drug, H = heart attack. P(M) = P(D) = 1/2.

Base probability of heart attack = 40% = 0.4.

Meditation reduces risk by 30%: P(H	M) = 0.4 − 0.4 × 0.30 = 0.4 × 0.70 = 0.28.
Drug reduces risk by 25%: P(H	D) = 0.4 − 0.4 × 0.25 = 0.4 × 0.75 = 0.30.

P(H) = P(M) P(H|M) + P(D) P(H|D) = (1/2)(0.28) + (1/2)(0.30) = 0.14 + 0.15 = 0.29.

P(M|H) = P(M) P(H|M) / P(H) = (1/2)(0.28) / 0.29 = 0.14/0.29 = 14/29

Answer: P(meditation/yoga | heart attack) = 14/29

Q8 If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability 1/2.)

SOLUTION

A second order determinant is |a b; c d| = ad − bc.

Each of a, b, c, d is independently 0 or 1 with probability 1/2. Total cases = 2^4 = 16.

det > 0 requires ad − bc > 0, i.e., ad = 1 and bc = 0.
ad = 1: a = 1 and d = 1.
bc = 0: b = 0 or c = 0 (i.e., not both equal to 1).

Enumerate (a=1, d=1, bc combinations):
b=0, c=0: ad−bc = 1−0 = 1 > 0 ✓
b=0, c=1: ad−bc = 1−0 = 1 > 0 ✓
b=1, c=0: ad−bc = 1−0 = 1 > 0 ✓
b=1, c=1: ad−bc = 1−1 = 0 (not > 0) ✗

3 favourable outcomes out of 16.

P(det > 0) = 3/16

Answer: P(determinant is positive) = 3/16

Q9 An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:
P(A fails) = 0.2
P(B fails alone) = 0.15
P(A and B fail) = 0.15
Evaluate the following probabilities:
(i) P(A fails | B has failed)
(ii) P(A fails alone)

SOLUTION

Given: P(A fails) = 0.2, P(B fails alone) = 0.15, P(A and B fail) = P(A ∩ B) = 0.15.

P(B fails) = P(B fails alone) + P(A and B fail) = 0.15 + 0.15 = 0.30.

(i) P(A fails | B has failed) = P(A and B fail) / P(B fails) = 0.15 / 0.30 = 1/2

(ii) P(A fails alone) = P(A fails) − P(A and B fail) = 0.2 − 0.15 = 0.05

Answer: (i) P(A fails | B has failed) = 1/2, (ii) P(A fails alone) = 0.05

Q10 Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

SOLUTION

Let E1 = red ball transferred, E2 = black ball transferred, A = ball drawn from Bag II is red.
P(E1) = 3/7, P(E2) = 4/7.

If red transferred (E1): Bag II has 5 red + 5 black = 10 balls.
P(A|E1) = 5/10 = 1/2.

If black transferred (E2): Bag II has 4 red + 6 black = 10 balls.
P(A|E2) = 4/10 = 2/5.

P(A) = P(E1) P(A|E1) + P(E2) P(A|E2) = (3/7)(1/2) + (4/7)(2/5)
= 3/14 + 8/35 = 15/70 + 16/70 = 31/70.

P(E2|A) = P(E2) P(A|E2) / P(A) = (4/7)(2/5) / (31/70)
= (8/35) / (31/70) = (8/35) × (70/31) = 560/1085 = 16/31

Answer: P(transferred ball was black | drawn ball is red) = 16/31

Q11 If A and B are two events such that P(A) ≠ 0 and P(B|A) = 1, then
(A) A ⊂ B
(B) B ⊂ A
(C) B = φ
(D) A = φ

SOLUTION

P(B|A) = 1 => P(A ∩ B)/P(A) = 1 => P(A ∩ B) = P(A).
This means every outcome in A is also in B, i.e., A ⊂ B.

Answer: (A) A ⊂ B

Q12 If P(A|B) > P(A), then which of the following is correct:
(A) P(B|A) < P(B)
(B) P(A ∩ B) < P(A) · P(B)
(C) P(B|A) > P(B)
(D) P(B|A) = P(B)

SOLUTION

P(A|B) > P(A)
=> P(A ∩ B)/P(B) > P(A)
=> P(A ∩ B) > P(A) · P(B).

Now P(B	A) = P(A ∩ B)/P(A) > P(A) · P(B)/P(A) = P(B).
Therefore P(B	A) > P(B).

Answer: (C) P(B|A) > P(B)

Q13 If A and B are any two events such that P(A) + P(B) − P(A and B) = P(A), then
(A) P(B|A) = 1
(B) P(A|B) = 1
(C) P(B|A) = 0
(D) P(A|B) = 0

SOLUTION

P(A) + P(B) − P(A ∩ B) = P(A)
=> P(B) = P(A ∩ B).

P(A|B) = P(A ∩ B)/P(B) = P(B)/P(B) = 1.

Answer: (B) P(A|B) = 1