Chapter 2: Electrostatic Potential and Capacitance

Exercises — Exercises

Q1 Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

SOLUTION

Given: $q_1 = +5 \times 10^{-8}$ C, $q_2 = -3 \times 10^{-8}$ C, separation $d = 16$ cm $= 0.16$ m.
Take q_1 at the origin and q_2 at $x = 0.16$ m on the x-axis.

Case 1 — Point P between the charges at distance x from q_1:
$V = kq_1/x + kq_2/(d - x) = 0$
$\Rightarrow 5 \times 10^{-8} / x = 3 \times 10^{-8} / (0.16 - x)$
$\Rightarrow 5(0.16 - x) = 3x \Rightarrow 0.80 = 8x \Rightarrow x = 0.10$ m $= 10$ cm

Case 2 — Point P outside the charges, beyond q_2, at distance x from q_1 ($x > 0.16$ m):
$V = kq_1/x + kq_2/(x - d) = 0$
$\Rightarrow 5 \times 10^{-8} / x = 3 \times 10^{-8} / (x - 0.16)$
$\Rightarrow 5(x - 0.16) = 3x \Rightarrow 2x = 0.80 \Rightarrow x = 0.40$ m $= 40$ cm

(There is no solution on the side beyond q_1 because both terms would have the same sign and cannot cancel.)

Answer: Electric potential is zero at two points: 10 cm from the $+5 \times 10^{-8}$ C charge (between the charges), and 40 cm from the $+5 \times 10^{-8}$ C charge (on the side of the negative charge, beyond it).

Q2 A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.

SOLUTION

Given: Side of regular hexagon = 10 cm = 0.10 m; charge $q = 5$ μC $= 5 \times 10^{-6}$ C at each of 6 vertices.

In a regular hexagon, the distance from each vertex to the centre equals the side length.
$\Rightarrow r = 0.10$ m for every vertex.

Potential at centre due to one charge:

$V_1 = kq/r = (9 \times 10^9 \times 5 \times 10^{-6}) / 0.10 = 4.5 \times 10^5$ V

By superposition, total potential (all six charges identical and equidistant):
$V = 6 \times V_1 = 6 \times 4.5 \times 10^5 = 2.7 \times 10^6$ V

Answer: 2.7×10^6 V (2.7 MV)

Q3 Two charges 2 μC and –2 μC are placed at points A and B 6 cm apart. (a) Identify an equipotential surface of the system. (b) What is the direction of the electric field at every point on this surface?

SOLUTION

Given: q_A = +2 μC at A, q_B = –2 μC at B, AB = 6 cm. The two charges form an electric dipole.

(a) At any point P on the plane that perpendicularly bisects the line segment AB, P is equidistant from A and B (let this distance be r).
V at P = kq_A/r + kq_B/r = k(+2μC)/r + k(–2μC)/r = 0
=> V = 0 everywhere on this perpendicular bisecting plane.
The perpendicular bisecting plane of AB is therefore an equipotential surface with V = 0.

(b) The electric field at every point is perpendicular to the equipotential surface.
On the perpendicular bisector the net field points from A (+2μC) towards B (–2μC), i.e., parallel to AB (from the positive to the negative charge).
This direction is normal to the plane of the equipotential surface.

Answer: (a) The plane perpendicular to AB passing through its midpoint (perpendicular bisector plane of AB) is an equipotential surface with V = 0. (b) The electric field at every point on this surface is directed from A to B (from the positive to the negative charge), i.e., perpendicular to the equipotential surface.

Q4 A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly on its surface. What is the electric field (a) inside the sphere, (b) just outside the sphere, (c) at a point 18 cm from the centre of the sphere?

SOLUTION

Given: R = 12 cm = 0.12 m; Q = 1.6×10^{-7} C; k = 9×10^9 N m² C⁻².

(a) Inside a conductor the electrostatic field is zero in the static situation.
E (inside) = 0 V/m

(b) Just outside the surface the electric field is that of the entire charge concentrated at the centre:
$E = kQ/R^2 = (9 \times 10^9 \times 1.6 \times 10^{-7}) / (0.12)^2$
$= (1.44 \times 10^3) / (1.44 \times 10^{-2}) = 1.0 \times 10^5$ V/m
Direction: radially outward ($Q > 0$).

(c) At r = 18 cm = 0.18 m (outside the sphere):
$E = kQ/r^2 = (9 \times 10^9 \times 1.6 \times 10^{-7}) / (0.18)^2$
$= (1.44 \times 10^3) / (3.24 \times 10^{-2}) = 4.4 \times 10^4$ V/m
Direction: radially outward.

Answer: (a) E = 0 inside the sphere. (b) $E = 1.0 \times 10^5$ V/m just outside (radially outward). (c) $E = 4.4 \times 10^4$ V/m at 18 cm from the centre.

Q5 A parallel plate capacitor with air between the plates has a capacitance of 8 pF ($1\ pF = 10^{-12}$ F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

SOLUTION

Given: $C_o = 8$ pF (air-gap capacitor); new separation $d' = d/2$; dielectric constant $K = 6$.

Original capacitance: $C_o = \varepsilon_o A / d$

New capacitance: $C = K\varepsilon_o A / d' = K\, \varepsilon_o A / (d/2) = 2K\, \varepsilon_o A / d = 2K\, C_o$

$C = 2 \times 6 \times 8$ pF = 96 pF

Answer: 96 pF

Q6 Three capacitors each of capacitance 9 pF are connected in series. (a) What is the total capacitance of the combination? (b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

SOLUTION

Given: $C_1 = C_2 = C_3 = 9$ pF, connected in series; supply V = 120 V.

(a) For series combination: $1/C = 1/C_1 + 1/C_2 + 1/C_3 = 1/9 + 1/9 + 1/9 = 3/9$
$\Rightarrow C = 3$ pF

(b) In series, the same charge Q appears on every capacitor:

$Q = C \times V = 3\ pF \times 120\ V = 360\ pC$

Voltage across each capacitor:
$V_{each} = Q / C_{each} = 360\ pC / 9\ pF = 40\ V$

Check: $3 \times 40\ V = 120\ V$. Correct.

Answer: (a) Total capacitance = 3 pF. (b) Potential difference across each capacitor = 40 V.

Q7 Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel. (a) What is the total capacitance of the combination? (b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

SOLUTION

Given: $C_1 = 2\ pF$, $C_2 = 3\ pF$, $C_3 = 4\ pF$ in parallel; supply $V = 100\ V$.

(a) For parallel combination: $C = C_1 + C_2 + C_3 = 2 + 3 + 4 = 9\ pF$

(b) In parallel, each capacitor has the same voltage $V = 100\ V$.
$Q_1 = C_1 V = 2\ pF \times 100\ V = 200\ pC$
$Q_2 = C_2 V = 3\ pF \times 100\ V = 300\ pC$
$Q_3 = C_3 V = 4\ pF \times 100\ V = 400\ pC$

Answer: (a) Total capacitance = 9 pF. (b) $Q_1 = 200\ pC$, $Q_2 = 300\ pC$, $Q_3 = 400\ pC$.

Q8 In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3}\ m^2$ and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

SOLUTION

Given: $A = 6 \times 10^{-3}\ m^2$, $d = 3\ mm = 3 \times 10^{-3}\ m$, $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$; $V = 100\ V$.

Capacitance: $C = \varepsilon_0 A / d$
$= (8.85 \times 10^{-12} \times 6 \times 10^{-3}) / (3 \times 10^{-3})$
$= (8.85 \times 10^{-12} \times 2)$
$= 17.7 \times 10^{-12}\ F = 17.7\ pF$

Charge on each plate:
$Q = C \times V = 17.7 \times 10^{-12}\ F \times 100\ V = 1.77 \times 10^{-9}\ C = 1.77\ nC$

Answer: Capacitance C = 17.7 pF; charge on each plate Q = 1.77 × 10^{-9} C (1.77 nC).

Q9 Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates, (a) while the voltage supply remained connected, (b) after the supply was disconnected.

SOLUTION

Given: C_o = 17.7 pF (from Ex. 2.8), d = 3 mm, mica thickness = 3 mm (fills the entire gap), K = 6.
With mica fully filling the gap: C = K C_o = 6 × 17.7 pF = 106.2 pF

(a) Supply remains connected (V = 100 V is maintained):
The potential difference stays at 100 V.
Capacitance increases to 106.2 pF.
Charge drawn from supply: Q = C × V = 106.2 pF × 100 V = 1.06 × 10^{-8} C
(Charge increases by factor K = 6 from the original 1.77 nC.)
Electric field between plates: E = V/d = 100 V / 3 × 10^{-3} m = 3.33 × 10^{4} V/m (unchanged).
More charge flows from the supply to compensate the induced bound charges in mica.

(b) Supply disconnected before inserting mica (Q is fixed = 1.77 × 10^{-9} C):
Capacitance increases to 106.2 pF.
Voltage drops: V' = Q / C = 1.77 × 10^{-9} / 106.2 × 10^{-12} = 16.7 V
Electric field in mica: E = V'/d = 16.7 / 3 × 10^{-3} = 5.56 × 10^{3} V/m
(E is reduced by factor K from the original 3.33 × 10^{4} V/m.)
The inserted dielectric reduces both the voltage and the field while charge is conserved.

Answer: (a) With supply connected: capacitance increases to 106.2 pF, voltage stays 100 V, charge increases to ~1.06 × 10^{-8} C, electric field unchanged. (b) With supply disconnected: capacitance increases to 106.2 pF, charge stays 1.77 nC, voltage drops to ~16.7 V, electric field reduces to ~5.56 × 10^{3} V/m.

Q10 A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?

SOLUTION

Given: $C = 12\ pF = 12 \times 10^{-12}\ F$, $V = 50\ V$.

Energy stored: $U = (1/2)\ C\ V^2$
$= (1/2) \times 12 \times 10^{-12} \times (50)^2$
$= (1/2) \times 12 \times 10^{-12} \times 2500$
$= 6 \times 10^{-12} \times 2500$
$= 1.5 \times 10^{-8}\ J$

Answer: $U = 1.5 \times 10^{-8}\ J$

Q11 A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

SOLUTION

Given: $C_1 = C_2 = 600\ pF = 6 \times 10^{-10}\ F$; initial voltage $V_o = 200\ V$; C_2 is initially uncharged.

Initial charge: $Q = C_1 V_o = 6 \times 10^{-10} \times 200 = 1.2 \times 10^{-7}\ C$
Initial energy: $U_i = (1/2)\ C_1 V_o^2 = (1/2) \times 6 \times 10^{-10} \times (200)^2 = 1.2 \times 10^{-5}\ J$

After connecting the two capacitors (supply disconnected, so Q is conserved):
Total charge $Q = 1.2 \times 10^{-7}\ C$ is shared equally ($C_1 = C_2$).
Common voltage: $V' = Q / (C_1 + C_2) = 1.2 \times 10^{-7} / (1.2 \times 10^{-9}) = 100\ V$

Final energy: $U_f = (1/2)(C_1 + C_2)\ V'^2 = (1/2) \times 1.2 \times 10^{-9} \times (100)^2$
$= (1/2) \times 1.2 \times 10^{-9} \times 10^4 = 6 \times 10^{-6}\ J$

Energy lost: $\Delta U = U_i - U_f = 1.2 \times 10^{-5} - 6 \times 10^{-6} = 6 \times 10^{-6}\ J$

This energy is dissipated as heat (and electromagnetic radiation) in the connecting conductors during the transient current flow as charges redistribute.

Answer: Energy lost = $6 \times 10^{-6}\ J$ (6 μJ). This energy is dissipated as heat during the transient redistribution of charge.