Chapter 3: Current Electricity

Exercises — Exercises

Q1 The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω, what is the maximum current that can be drawn from the battery?

SOLUTION

Given: emf ε = 12 V, internal resistance r = 0.4 Ω.

Maximum current is drawn when external resistance R = 0 (short-circuit condition):
I_max = ε / r = 12 / 0.4 = 30 A.

Note: In practice, drawing this current would damage the battery. Batteries specify a maximum safe discharge current well below this theoretical limit.

Answer: Maximum current I_max = 30 A

Q2 A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?

SOLUTION

Given: ε = 10 V, r = 3 Ω, I = 0.5 A.

From Ohm's law for the full circuit: ε = I(R + r)
=> R + r = ε / I = 10 / 0.5 = 20 Ω
=> R = 20 - r = 20 - 3 = 17 Ω

Terminal voltage (= potential difference across R):
V = ε - I r = 10 - (0.5)(3) = 10 - 1.5 = 8.5 V

Alternatively, V = I R = 0.5 × 17 = 8.5 V. ✓

Answer: Resistance R = 17 Ω; Terminal voltage = 8.5 V

Q3 At room temperature (27.0 °C) the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that the temperature coefficient of the material of the resistor is 1.70×10^{-4} °C^{-1}?

SOLUTION

Given: $R_0 = 100\ \Omega$ at $T_0 = 27.0$ °C, $R_T = 117\ \Omega$, $\alpha = 1.70 \times 10^{-4}$ °C^{-1}.

Using the temperature-resistance relation:

$R_T = R_0 [1 + \alpha(T - T_0)]$

$\Rightarrow T - T_0 = (R_T/R_0 - 1) / \alpha$

$= (117/100 - 1) / (1.70 \times 10^{-4})$

$= 0.17 / (1.70 \times 10^{-4})$

$= 1000$ °C

$\Rightarrow T = T_0 + 1000 = 27.0 + 1000 = 1027$ °C

Answer: Temperature of the element T = 1027 °C

Q4 A negligibly small current is passed through a wire of length 15 m and uniform cross-section 6.0×10^{-7} m², and its resistance is measured to be 5.0 Ω. What is the resistivity of the material at the temperature of the experiment?

SOLUTION

Given: $l = 15$ m, $A = 6.0 \times 10^{-7}$ m², $R = 5.0\ \Omega$.

From the relation $R = \rho l/A$:

$\rho = R A / l$

$= (5.0 \times 6.0 \times 10^{-7}) / 15$

$= 30.0 \times 10^{-7} / 15$

$= 2.0 \times 10^{-7}\ \Omega \cdot m$

Answer: Resistivity $\rho = 2.0 \times 10^{-7}\ \Omega \cdot m$

Q5 A silver wire has a resistance of 2.1 Ω at 27.5 °C, and a resistance of 2.7 Ω at 100 °C. Determine the temperature coefficient of resistivity of silver.

SOLUTION

Given: $R_1 = 2.1\ \Omega$ at $T_1 = 27.5$ °C, $R_2 = 2.7\ \Omega$ at $T_2 = 100$ °C.

Using $R_2 = R_1 [1 + \alpha(T_2 - T_1)]$:

$\alpha = (R_2/R_1 - 1) / (T_2 - T_1)$

= (2.7/2.1 - 1) / (100 - 27.5)
= (1.2857 - 1) / 72.5
= 0.2857 / 72.5
≈ 3.94×10^{-3} °C^{-1}

Answer: Temperature coefficient α ≈ 3.94×10^{-3} °C^{-1}

Q6 A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0 °C? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is 1.70×10^{-4} °C^{-1}.

SOLUTION

Given: V = 230 V, I_initial = 3.2 A (at T_o = 27.0 °C), I_steady = 2.8 A, α = 1.70×10^{-4} °C^{-1}.

Initial resistance (at room temperature 27 °C):
R_o = V / I_initial = 230 / 3.2 = 71.875 Ω

Steady-state resistance:
R_steady = V / I_steady = 230 / 2.8 ≈ 82.14 Ω

Using R_steady = R_o [1 + α(T - T_o)]:
T - T_o = (R_steady/R_o - 1) / α
= (82.14/71.875 - 1) / (1.70×10^{-4})
= 0.14286 / (1.70×10^{-4})
≈ 840.3 °C

T = 27.0 + 840.3 ≈ 867 °C

Answer: Steady temperature of the nichrome element ≈ 867 °C

Q7 Determine the current in each branch of the network shown in Fig. 3.20. The network is a diamond with node B at top, A at left, C at right, and D at bottom. Resistors: A–B = 10 Ω, B–C = 5 Ω, B–D = 5 Ω (middle bridge), A–D = 5 Ω, C–D = 10 Ω. A 10 V battery in series with a 10 Ω resistor is connected across A and C along the bottom.

SOLUTION

Given: Five-resistor diamond network (B top, A left, C right, D bottom) with A–B = 10 Ω, B–C = 5 Ω, B–D = 5 Ω (vertical middle bridge), A–D = 5 Ω, C–D = 10 Ω, and a 10 V source with a 10 Ω series resistor connected across A and C.

Set node C as reference ($V_C = 0$). Let V_A, V_B, V_D be the unknown node voltages. The battery + 10 Ω branch drives a current from C into A, so the current arriving at A from that branch = $(10 - (V_A - V_C))/10 = (10 - V_A)/10$.

KCL at node A (current in from battery branch = current leaving into diamond):
$(10 - V_A)/10 = (V_A - V_B)/10 + (V_A - V_D)/5$
Multiplying by 10: $10 - V_A = (V_A - V_B) + 2(V_A - V_D)$
$\Rightarrow 4V_A - V_B - 2V_D = 10$ …(1)

KCL at node B:
$(V_A - V_B)/10 = (V_B - V_C)/5 + (V_B - V_D)/5$
Multiplying by 10: $V_A - V_B = 2V_B + 2(V_B - V_D)$
$\Rightarrow V_A - 5V_B + 2V_D = 0$ …(2)

KCL at node D:
$(V_A - V_D)/5 + (V_B - V_D)/5 = (V_D - V_C)/10$
Multiplying by 10: $2(V_A - V_D) + 2(V_B - V_D) = V_D$
$\Rightarrow 2V_A + 2V_B - 5V_D = 0$ …(3)

Solving equations (1), (2), (3) (with $V_C = 0$):
$V_A = 70/17 \approx 4.12$ V, $V_B = 30/17 \approx 1.76$ V, $V_D = 40/17 \approx 2.35$ V

Branch currents:
A→B (10 Ω): $I_{AB} = (V_A - V_B)/10 = (40/17)/10 = 4/17 \approx 0.235$ A
A→D (5 Ω): $I_{AD} = (V_A - V_D)/5 = (30/17)/5 = 6/17 \approx 0.353$ A
B→C (5 Ω): $I_{BC} = (V_B - V_C)/5 = (30/17)/5 = 6/17 \approx 0.353$ A
D→B (5 Ω): $I_{DB} = (V_D - V_B)/5 = (10/17)/5 = 2/17 \approx 0.118$ A (middle bridge, flows D→B)
D→C (10 Ω): $I_{DC} = (V_D - V_C)/10 = (40/17)/10 = 4/17 \approx 0.235$ A
Total (from battery, into A / out of C): $I = (10 - V_A)/10 = (100/17)/10 = 10/17 \approx$

0.588 A

KCL verification: At A: 10/17 = 4/17 + 6/17 ✓; At B: 4/17 + 2/17 = 6/17 ✓; At D: 6/17 = 2/17 + 4/17 ✓; At C: 6/17 + 4/17 = 10/17 ✓

Answer: A→B (10 Ω): 4/17 A ≈ 0.24 A; A→D (5 Ω): 6/17 A ≈ 0.35 A; B→C (5 Ω): 6/17 A ≈ 0.35 A; D→B middle bridge (5 Ω): 2/17 A ≈ 0.12 A; D→C (10 Ω): 4/17 A ≈ 0.24 A; Total current from battery: 10/17 A ≈ 0.59 A

Q8 A storage battery of emf 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?

SOLUTION

Given: Battery emf $\varepsilon_b = 8.0$ V, internal resistance $r = 0.5$ Ω, supply voltage $V_s = 120$ V, series resistance $R_s = 15.5$ Ω.

During charging, the supply voltage drives current AGAINST the battery's emf. The total EMF opposing the current = $\varepsilon_b = 8.0$ V.
Net driving voltage = $V_s - \varepsilon_b = 120 - 8.0 = 112$ V
Total resistance = $R_s + r = 15.5 + 0.5 = 16.0$ Ω

Charging current:
$I = (V_s - \varepsilon_b) / (R_s + r) = 112 / 16.0 = 7.0$ A

Terminal voltage of battery during charging:
During charging, current enters the positive terminal, so:
$V_{terminal} = \varepsilon_b + I r = 8.0 + 7.0 \times 0.5 = 8.0 + 3.5 = 11.5$ V

Purpose of the series resistor:
Without the series resistor, the charging current would be (120 – 8.0)/0.5 = 224 A — far too large, which would overheat and permanently damage the battery. The series resistor limits the current to a safe value (7 A) so that charging proceeds safely.

Answer: Terminal voltage during charging = 11.5 V. The series resistor limits the charging current to a safe value.

Q9 The number density of free electrons in a copper conductor estimated in Example 3.1 is 8.5×10^{28} m^{-3}. How long does an electron take to drift from one end of a wire 3.0 m long to its other end? The area of cross-section of the wire is 2.0×10^{-6} m^2 and it is carrying a current of 3.0 A.

SOLUTION

Given: $n = 8.5 \times 10^{28}$ m^{-3}, $L = 3.0$ m, $A = 2.0 \times 10^{-6}$ m^2, $I = 3.0$ A, $e = 1.6 \times 10^{-19}$ C.

Drift velocity of electrons:

$v_d = I / (n e A)$

$= 3.0 / (8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.0 \times 10^{-6})$

$= 3.0 / (2.72 \times 10^{4})$

$\approx 1.10 \times 10^{-4}$ m s^{-1}

Time to drift the full length of the wire:

$t = L / v_d = 3.0 / (1.10 \times 10^{-4})$

$\approx 2.72 \times 10^{4}$ s

≈ 7.6 hours

This extremely long time illustrates why the drift speed is so small — yet current is established almost instantly because the electric field propagates through the conductor at the speed of light.

Answer: Time for electron to drift 3.0 m $\approx 2.72 \times 10^{4}$ s (≈ 7.6 hours)