Chapter 6: Electromagnetic Induction

Exercises — Exercises

Q1 Predict the direction of induced current in the situations described by Figs. 6.15(a) to (f).

SOLUTION

We use Faraday's law and Lenz's law: the induced current opposes the change in magnetic flux that produces it.

(a) A solenoid (terminals p at left end, q at right end, r on the top external wire) has a bar magnet on its right with the S pole leading (S–N), moving LEFT (←) toward the solenoid.

The S pole approaches the coil's right (near) end, increasing the flux through it.

By Lenz's law the near (right) face must become an S pole to repel the approaching S pole.

For the right face to be an S pole, the current (viewed from the magnet) is clockwise.

Direction: current flows q → r → p in the external circuit.

(b) Two coils: left coil (p left end, q right end, r bottom wire) and right coil (x left end, y right end, z bottom wire). A bar magnet sits between them with S left and N right (S–N), moving LEFT (←) toward the left coil.

Left (pqr) coil: the S pole approaches its q end. Lenz: the near (q) face must become an S pole to repel it. Viewed from the magnet the current is clockwise => it flows p → r → q in the external circuit.

Right (xyz) coil: the N pole recedes from its x end. Lenz: the near (x) face must become an S pole to attract the receding N pole. This makes the current flow y → z → x in the external circuit.

(c) Two coils on a common axis. Left coil is connected to a battery via a tapping key that is JUST CLOSED. Current in the left coil grows from zero.

The increasing current in the left coil creates increasing flux through the right coil.

Lenz: induced current in the right coil opposes this increasing flux.

Direction: current flows from y to x in the right coil.

(d) Two coils on a common axis. A rheostat in the left coil circuit has its setting changed such that its resistance decreases, increasing the current in the left coil.

Increasing current => increasing flux through the right coil.

Lenz: induced current in right coil opposes the increasing flux.

Direction: current flows from x to y in the right coil.

(e) Two coaxial solenoids (right one has terminals x at left, y at right, r on the bottom wire). The tapping key in the left solenoid's circuit is JUST RELEASED, so its current falls rapidly to zero.

Decreasing current => decreasing flux through the right solenoid.

Lenz: the induced current in the right solenoid flows so as to maintain the flux (same sense as the decaying source current).

Direction: current flows x → y in the right solenoid.

(f) A horizontal circular loop has a long vertical straight wire passing through its centre (perpendicular to the loop's plane), carrying current I that is decreasing at a steady rate.

The magnetic field of the straight wire forms horizontal circles around it, so at every point of the loop this field lies IN the plane of the loop.

The field has no component perpendicular to the loop, so the flux through the loop is zero at all times.

Since the flux does not change, no emf is induced.

Direction: no induced current in the loop.

Answer: (a) q → r → p; (b) p → r → q in the pqr coil, y → z → x in the xyz coil; (c) y to x; (d) x to y; (e) x to y; (f) no induced current (the wire's field lies in the plane of the loop, so flux is zero)

Q2 Use Lenz's law to determine the direction of induced current in the situations described by Fig. 6.16: (a) A wire of irregular shape turning into a circular shape; (b) A circular loop being deformed into a narrow straight wire.

SOLUTION

The vertices are labelled a (bottom), b (left), c (top), d (right) in both cases. The path a → d → c → b → a is ANTICLOCKWISE; a → b → c → d → a is CLOCKWISE.

(a) Wire of irregular shape turning into a circular shape (Fig. 6.16a). Here the field is directed INTO the page (shown by ×).

For a fixed perimeter, the circular shape encloses the MAXIMUM possible area, so as the wire becomes circular the enclosed area INCREASES.

=> Magnetic flux into the page increases.

By Lenz's law the induced current must oppose this increase, so it creates a field OUT OF the page inside the loop.

=> By the right-hand rule the induced current is ANTICLOCKWISE.

Direction: a → d → c → b → a (anticlockwise).

(b) Circular loop being deformed into a narrow straight wire (Fig. 6.16b). Here the field is directed OUT OF the page (shown by dots).

As the circular loop is compressed into a narrow line, the enclosed area DECREASES toward zero.

=> Magnetic flux out of the page decreases.

By Lenz's law the induced current must oppose this decrease, so it creates a field OUT OF the page (same sense as B) to maintain the flux.

=> By the right-hand rule the induced current is ANTICLOCKWISE.

Direction: a → d → c → b → a (anticlockwise).

Answer: (a) Anticlockwise: a→d→c→b→a (opposes the increasing into-page flux); (b) Anticlockwise: a→d→c→b→a (opposes the decreasing out-of-page flux)

Q3 A long solenoid with 15 turns per cm has a small loop of area 2.0 cm² placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

SOLUTION

Given:

Number of turns per unit length: $n = 15$ turns/cm $= 15 \times 100 = 1500$ turns/m

Area of small loop: $A = 2.0\ cm^2 = 2.0 \times 10^{-4}\ m^2$

Change in current: $\Delta I = 4.0 - 2.0 = 2.0$ A

Time interval: $\Delta t = 0.1$ s

Constant used: $\mu_o = 4\pi \times 10^{-7}$ T·m/A

Step 1: Find the rate of change of current.

$dI/dt = \Delta I/\Delta t = 2.0 / 0.1 = 20$ A/s

Step 2: The magnetic field inside a long solenoid is $B = \mu_o n I$.

Rate of change of B: $dB/dt = \mu_o \times n \times (dI/dt)$

$dB/dt = 4\pi \times 10^{-7} \times 1500 \times 20$

$dB/dt = 4\pi \times 10^{-7} \times 30{,}000$

$dB/dt = 12\pi \times 10^{-3} \approx 3.77 \times 10^{-2}$ T/s

Step 3: The small loop is placed normal to the solenoid axis, so flux $\Phi = B \times A$.

Induced emf $\varepsilon = d\Phi/dt = A \times (dB/dt)$

$\varepsilon = (2.0 \times 10^{-4}) \times (12\pi \times 10^{-3})$

$\varepsilon = 24\pi \times 10^{-7}$

$\varepsilon \approx 7.54 \times 10^{-6}$ V

Answer: Induced emf $\varepsilon \approx 7.54 \times 10^{-6}$ V (≈ 7.5 μV)

Q4 A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s^{-1} in a direction normal to the (a) longer side, (b) shorter side of the loop? For how long does the induced voltage last in each case?

SOLUTION

Given:

Sides of loop: $l_1 = 8$ cm $= 0.08$ m (longer side), $l_2 = 2$ cm $= 0.02$ m (shorter side)

Magnetic field: $B = 0.3$ T (normal to loop)

Velocity: $v = 1$ cm/s $= 0.01$ m/s

The loop has a small cut, so it is an open loop — emf appears across the cut.

The motional emf is generated only by the side of the loop that is crossing the boundary of the magnetic field region.

(a) Velocity normal to the longer side (8 cm side):

The direction of motion is perpendicular to the 8 cm side,
meaning the loop moves in the direction of the 2 cm side.
The side crossing the field boundary has length $l_1 = 0.08$ m.
Induced emf: $\varepsilon = B \times l_1 \times v = 0.3 \times 0.08 \times 0.01$
$\varepsilon = 2.4 \times 10^{-4}$ V $= 0.24$ mV

The loop must travel a distance equal to the shorter side (2 cm) to completely exit the field.

Time taken: $t = l_2 / v = 0.02 / 0.01 = 2$ s

(b) Velocity normal to the shorter side (2 cm side):

The direction of motion is perpendicular to the 2 cm side,
meaning the loop moves in the direction of the 8 cm side.
The side crossing the field boundary has length $l_2 = 0.02$ m.
Induced emf: $\varepsilon = B \times l_2 \times v = 0.3 \times 0.02 \times 0.01$

$\varepsilon = 6 \times 10^{-5}$ V = 0.06 mV

The loop must travel a distance equal to the longer side (8 cm) to completely exit the field.

Time taken: $t = l_1 / v = 0.08 / 0.01 = 8$ s

Answer: (a) $\varepsilon = 2.4 \times 10^{-4}$ V for 2 s; (b) $\varepsilon = 6 \times 10^{-5}$ V for 8 s

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Q5 A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

SOLUTION

Given:

Length of rod: L = 1.0 m

Angular frequency: ω = 400 rad/s

Magnetic field: B = 0.5 T (parallel to the axis of rotation)

The rod rotates about one end. Each element dr of the rod at distance r from the pivot

moves with speed $v = \omega r$ perpendicular to B.

The motional emf across element dr: $d\varepsilon = B \times v \times dr = B \times \omega \times r \times dr$

Total emf from centre (r = 0) to ring (r = L):

$\varepsilon = \int_0^L B\,\omega\,r\,dr = B\,\omega \times [r^2/2]_0^L = (1/2)\,B\,\omega\,L^2$

Substituting values:

$\varepsilon = (1/2) \times 0.5 \times 400 \times (1.0)^2$

$\varepsilon = (1/2) \times 0.5 \times 400 \times 1.0$

$\varepsilon = 100$ V

Answer: emf = 100 V

Q6 A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1}, at right angles to the horizontal component of the earth's magnetic field, 0.30 × 10^{-4} Wb m^{-2}. (a) What is the instantaneous value of the emf induced in the wire? (b) What is the direction of the emf? (c) Which end of the wire is at the higher electrical potential?

SOLUTION

Given:

Length of wire: l = 10 m (extending east–west)

Falling speed: v = 5.0 m/s (vertically downward)

Horizontal component of Earth's field: $B_H = 0.30 \times 10^{-4}$ T (directed northward)

(a) Instantaneous value of induced emf:

The wire falls vertically, so v is perpendicular to B_H.

Using motional emf formula: $\varepsilon = B_H \times l \times v$

$\varepsilon = 0.30 \times 10^{-4} \times 10 \times 5.0$

$\varepsilon = 1.5 \times 10^{-3}$ V = 1.5 mV

(b) Direction of the emf:

Set up coordinates: wire along x-axis (east = +x), north = +y, up = +z.

Velocity: v = -z (downward); B_H = +y (northward).

Force on a positive charge in the wire: $F = q(v \times B_H)$

$v \times B_H = (-\hat{z}) \times (\hat{y}) = -(\hat{z} \times \hat{y}) = -(-\hat{x}) = +\hat{x}$ (eastward).

Positive charges are pushed toward the east end.

Conventional current flows from west to east inside the wire.

The emf acts from west to east (inside the conductor).

(c) Which end is at higher potential:

Positive charges accumulate at the east end.

The east end is at the higher electrical potential.

Answer: (a) $\varepsilon = 1.5 \times 10^{-3}$ V; (b) emf directed from west to east; (c) East end is at higher potential

Q7 Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V is induced, give an estimate of the self-inductance of the circuit.

SOLUTION

Given:

Initial current: $I_1 = 5.0$ A

Final current: $I_2 = 0.0$ A

Time interval: $\Delta t = 0.1$ s

Induced emf: $\varepsilon = 200$ V (average)

The self-induced emf is given by:

$\varepsilon = L \times |\Delta I/\Delta t|$

Rate of change of current:

$|\Delta I/\Delta t| = |I_2 - I_1| / \Delta t = (5.0 - 0.0) / 0.1 = 50$ A/s

Solving for self-inductance L:

$L = \varepsilon / (|\Delta I/\Delta t|)$

$L = 200 / 50$

$L = 4$ H

Answer: Self-inductance L = 4 H

Q8 A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

SOLUTION

Given:

Mutual inductance: $M = 1.5$ H

Change in current in one coil: $\Delta I = 20 - 0 = 20$ A

Time interval: $\Delta t = 0.5$ s

The flux linkage with the secondary coil (coil 1) due to current in coil 2 is:

$N_1\Phi_1 = M \times I_2$

When current changes by $\Delta I_2 = 20$ A, the change in flux linkage is:

$\Delta(N_1\Phi_1) = M \times \Delta I_2$

$\Delta(N_1\Phi_1) = 1.5 \times 20$

$\Delta(N_1\Phi_1) = 30$ Wb

Verification — the induced emf in the other coil:

$\varepsilon_1 = M \times (\Delta I_2/\Delta t) = 1.5 \times (20/0.5) = 60$ V

Answer: Change of flux linkage = 30 Wb