Chapter 7: Alternating Current

Exercises — Exercises

Q1 A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply. (a) What is the rms value of current in the circuit? (b) What is the net power consumed over a full cycle?

SOLUTION

Given: R = 100 Ω, V_{rms} = 220 V, f = 50 Hz.

(a) rms current:
$I = V_{rms} / R = 220\ V / 100\ \Omega = 2.2\ A$

(b) Net power consumed over a full cycle:
For a purely resistive circuit, the power factor $\cos \varphi = 1$.
$P = V_{rms} \times I_{rms} = 220\ V \times 2.2\ A = 484\ W$
Equivalently, $P = I^2 R = (2.2)^2 \times 100 = 484\ W$.

Answer: (a) I = 2.2 A; (b) P = 484 W

Q2 (a) The peak voltage of an ac supply is 300 V. What is the rms voltage? (b) The rms value of current in an ac circuit is 10 A. What is the peak current?

SOLUTION

(a) Given: v_m = 300 V.
$V_{rms} = v_m / \sqrt{2} = 300 / 1.414 = 212.1\ V \approx 212\ V$

(b) Given: I_{rms} = 10 A.
$i_m = \sqrt{2} \times I_{rms} = 1.414 \times 10\ A = 14.1\ A$

Answer: (a) $V_{rms} \approx 212$ V; (b) $i_m \approx 14.1$ A

Q3 A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.

SOLUTION

Given: $L = 44\ mH = 44 \times 10^{-3}$ H, V_{rms} = 220 V, f = 50 Hz.

Inductive reactance:
$X_L = 2\pi f L = 2 \times 3.14159 \times 50 \times 44 \times 10^{-3}$
$X_L = 13.82\ \Omega$

rms current:

$I = V_{rms} / X_L = 220 / 13.82 = 15.92\ A \approx 15.9\ A$

Answer: $I \approx 15.9\ A$

Q4 A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.

SOLUTION

Given: $C = 60\ \mu F = 60 \times 10^{-6}\ F$, $V_{rms} = 110\ V$, $f = 60\ Hz$.

Capacitive reactance:

$X_C = 1 / (2\pi f C) = 1 / (2 \times 3.14159 \times 60 \times 60 \times 10^{-6})$

$X_C = 44.21\ \Omega$

rms current:

$I = V_{rms} / X_C = 110 / 44.21 = 2.49\ A \approx 2.49\ A$

Answer: $I \approx 2.49\ A$

Q5 In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.

SOLUTION

For the purely inductive circuit (Exercise 7.3):

The phase difference between voltage and current is $\varphi = \pi/2$.

Power factor = $\cos(\pi/2) = 0$.

Therefore, average power $P_L = V I \cos\varphi = 0$ W.

For the purely capacitive circuit (Exercise 7.4):

The phase difference between voltage and current is $\varphi = \pi/2$ (current leads voltage by $\pi/2$).

Power factor = $\cos(\pi/2) = 0$.

Therefore, average power $P_C = V I \cos\varphi = 0$ W.

Explanation: In a pure inductor, the current lags the voltage by $\pi/2$. In a pure capacitor, the current leads the voltage by $\pi/2$. In both cases, energy stored in the element during one half-cycle is completely returned to the source in the next half-cycle. There is no net dissipation; the currents flowing are called wattless currents.

Answer: Net power absorbed = 0 W in both circuits. (Power factor = 0 for both pure L and pure C.)

Q6 A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

SOLUTION

Given: C = 30 μF = 30 × 10^{-6} F, L = 27 mH = 27 × 10^{-3} H.

The angular frequency of free (LC) oscillations is:
$\omega_0 = 1 / \sqrt{(LC)}$
$\omega_0 = 1 / \sqrt{(27 \times 10^{-3} \times 30 \times 10^{-6})}$
$\omega_0 = 1 / \sqrt{(8.1 \times 10^{-7})}$
$\omega_0 = 1 / (9 \times 10^{-4})$
$\omega_0 = 1111.1$ rad/s ≈ 1.11 × 10^3 rad/s

Answer: $\omega_0 \approx 1.11 \times 10^3$ rad/s

Q7 A series LCR circuit with R = 20 Ω, L = 1.5 H and C = 35 μF is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

SOLUTION

Given: R = 20 Ω, L = 1.5 H, C = 35 μF = 35 × 10^{-6} F, V_{rms} = 200 V.

When the supply frequency equals the natural (resonant) frequency:
$\omega_0 = 1 / \sqrt{(LC)} = 1 / \sqrt{(1.5 \times 35 \times 10^{-6})} = 1 / \sqrt{(5.25 \times 10^{-5})} \approx 138$ rad/s

At resonance, $X_L = X_C$, so the impedance is purely resistive: Z = R = 20 Ω.
Power factor cos φ = 1.

rms current at resonance:
$I = V_{rms} / R = 200 / 20 = 10$ A

Average power transferred:
$P = I^2 R = (10)^2 \times 20 = 2000$ W = 2.0 kW

Answer: P = 2000 W = 2.0 kW

Q8 Figure 7.17 shows a series LCR circuit connected to a variable frequency 230 V source. L = 5.0 H, C = 80 μF, R = 40 Ω. (a) Determine the source frequency which drives the circuit in resonance. (b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency. (c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

SOLUTION

Given: L = 5.0 H, C = 80 μF = 80 × 10^{-6} F, R = 40 Ω, V_{rms} = 230 V.

(a) Resonant angular frequency:
$\omega_0 = 1 / \sqrt{(LC)} = 1 / \sqrt{(5.0 \times 80 \times 10^{-6})}$
$\omega_0 = 1 / \sqrt{(4 \times 10^{-4})} = 1 / 0.02 = 50$ rad/s

(b) At resonance, $X_L = X_C$, so impedance equals resistance:
Z = R = 40 Ω

rms current: $I = V_{rms} / Z = 230 / 40 = 5.75$ A
Amplitude (peak current): $i_m = \sqrt{2} \times I = 1.414 \times 5.75 = 8.13$ A

(c) rms potential drops at resonance:

Across the resistor:
$V_R = I \times R = 5.75 \times 40 = 230$ V

Inductive reactance at resonance:
$X_L = \omega_0 L = 50 \times 5.0 = 250$ Ω

Capacitive reactance at resonance:
$X_C = 1 / (\omega_0 C) = 1 / (50 \times 80 \times 10^{-6}) = 1 / 0.004 = 250$ Ω

Across the inductor:
$V_L = I \times X_L = 5.75 \times 250 = 1437.5$ V ≈ 1437 V

Across the capacitor:
$V_C = I \times X_C = 5.75 \times 250 = 1437.5$ V ≈ 1437 V

Potential drop across the LC combination:
$V_{LC} = V_L - V_C = 1437.5 - 1437.5 = 0$ V

This is zero because at resonance $X_L = X_C$, so V_L and V_C are equal in magnitude but exactly π out of phase (opposite directions in the phasor diagram), and they cancel completely.

Answer: (a) $\omega_0 = 50$ rad/s; (b) $Z = 40\ \Omega$, $i_m \approx 8.13$ A; (c) $V_R = 230$ V, $V_L = V_C \approx 1437.5$ V, $V_{LC} = 0$ V