Chapter 8: Electromagnetic Waves

Exercises — Exercises

Q1 Figure 8.5 shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A.
(a) Calculate the capacitance and the rate of change of potential difference between the plates.
(b) Obtain the displacement current across the plates.
(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

SOLUTION

Given: radius r = 12 cm = 0.12 m, separation d = 5.0 cm = 0.05 m, charging current I = 0.15 A
Constants: $\varepsilon_o = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

(a) Area of each plate:
$A = \pi r^2 = \pi \times (0.12)^2 = 4.52 \times 10^{-2}\ m^2$

Capacitance of parallel plate capacitor:
$C = \varepsilon_o A / d = (8.85 \times 10^{-12})(4.52 \times 10^{-2}) / (0.05)$
$C = 8.01 \times 10^{-12}\ F \approx 8.01\ pF$

The charging current $I = C\ (dV/dt)$, so the rate of change of potential difference:
$dV/dt = I / C = 0.15 / (8.01 \times 10^{-12})$
$dV/dt = 1.87 \times 10^{10}\ V/s$

(b) The displacement current id equals the conduction current I that charges the capacitor. By the Ampere–Maxwell law, displacement current through the capacitor gap = conduction current in the external circuit:
$id = \varepsilon_o\ (d\Phi_E/dt) = \varepsilon_o A\ (dE/dt) = (dQ/dt) = I$
id = 0.15 A

(c) Yes, Kirchhoff's junction rule is valid at each plate if we account for both conduction current and displacement current. At a plate: the conduction current I arriving from the wire equals the displacement current id = 0.15 A leaving through the gap between the plates. Total current (conduction + displacement) is continuous, so the junction rule holds when both types of current are included.

Answer: (a) C ≈ 8.01 pF; dV/dt ≈ 1.87 × 10^{10} V/s. (b) Displacement current id = 0.15 A. (c) Yes, valid when displacement current is included along with conduction current.

Q2 A parallel plate capacitor (Fig. 8.6) made of circular plates each of radius R = 6.0 cm has a capacitance C = 100 pF. The capacitor is connected to a 230 V ac supply with a (angular) frequency of 300 rad s^{-1}.
(a) What is the rms value of the conduction current?
(b) Is the conduction current equal to the displacement current?
(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.

SOLUTION

Given: R = 6.0 cm = 0.06 m, C = 100 pF = 10^{-10} F, V_{rms} = 230 V, ω = 300 rad s^{-1}

Constants: $\mu_o = 4\pi \times 10^{-7}$ T m A^{-1}

(a) Capacitive reactance: $X_C = 1/(\omega C) = 1/(300 \times 10^{-10}) = 3.33 \times 10^7\ \Omega$
rms conduction current:
$I_{rms} = V_{rms} / X_C = V_{rms} \times \omega \times C = 230 \times 300 \times 10^{-10}$
$I_{rms} = 6.9 \times 10^{-6}$ A = 6.9 μA

(b) Yes. For an ideal capacitor, displacement current $i_d = \varepsilon_o (d\Phi_E/dt)$. Since the same charge Q(t) that accumulates on the plates produces the electric flux $\Phi_E = Q/\varepsilon_o$, we have $i_d = dQ/dt = I_{conduction}$ at every instant. So the displacement current equals the conduction current.

(c) The point at r = 3.0 cm = 0.03 m lies inside the plates (R = 6.0 cm). The displacement current is distributed uniformly over the plate area. At radius r < R, the enclosed displacement current fraction is (r^2/R^2) of the total.

Peak (amplitude) of conduction/displacement current:
$I_o = I_{rms} \times \sqrt{2} = 6.9 \times 10^{-6} \times 1.414 = 9.76 \times 10^{-6}$ A

Applying Ampere–Maxwell law on a circle of radius r (inside the capacitor gap):
$B \times 2\pi r = \mu_o \times i_{d,enclosed} = \mu_o \times I_o \times (r^2/R^2)$
$B_o = \mu_o I_o r / (2\pi R^2)$
$B_o = (4\pi \times 10^{-7} \times 9.76 \times 10^{-6} \times 0.03) / (2\pi \times (0.06)^2)$
$B_o = (4\pi \times 10^{-7} \times 9.76 \times 10^{-6} \times 0.03) / (2\pi \times 3.6 \times 10^{-3})$
$B_o = (2 \times 10^{-7} \times 9.76 \times 10^{-6} \times 0.03) / (3.6 \times 10^{-3})$
$B_o = 1.63 \times 10^{-11}$ T

Answer: (a) I_rms = 6.9 μA. (b) Yes, displacement current = conduction current at every instant. (c) Amplitude of B at r = 3.0 cm is $B_0 \approx 1.63 \times 10^{-11}$ T.

Q3 What physical quantity is the same for X-rays of wavelength 10^{-10} m, red light of wavelength 6800 Å and radiowaves of wavelength 500 m?

SOLUTION

X-rays ($\lambda = 10^{-10}$ m), red light (λ = 6800 Å = 6.8×10^{-7} m), and radiowaves (λ = 500 m) are all electromagnetic waves — members of the electromagnetic spectrum.

Despite having vastly different wavelengths and frequencies, all electromagnetic waves travel through vacuum with the same speed:
$c = 3 \times 10^8$ m/s

Verification of frequencies:
X-rays: $f = c/\lambda = 3 \times 10^8 / 10^{-10} = 3 \times 10^{18}$ Hz
Red light: $f = 3 \times 10^8 / 6.8 \times 10^{-7} = 4.4 \times 10^{14}$ Hz
Radiowaves: $f = 3 \times 10^8 / 500 = 6 \times 10^5$ Hz

The frequencies differ enormously, but the speed in vacuum is the same for all.

Answer: The speed in vacuum: $c = 3 \times 10^8$ m/s is the same for all three — X-rays, red light, and radiowaves are all electromagnetic waves traveling at the same speed in vacuum.

Q4 A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

SOLUTION

For an electromagnetic wave traveling along the z-direction:

The electric field vector E and magnetic field vector B must each be perpendicular to the direction of propagation (z-axis) and perpendicular to each other. Therefore, E and B both lie in the xy-plane and are mutually perpendicular.

For example: E along x-axis (î direction), B along y-axis (ĵ direction), so that E × B points along z (the propagation direction).

Wavelength calculation:
Given: frequency v = 30 MHz = 30×10^6 Hz, $c = 3 \times 10^8$ m/s

$\lambda = c / v = (3 \times 10^8) / (30 \times 10^6)$

$\lambda = 10$ m

Answer: E and B lie in the xy-plane, perpendicular to each other and to z (the propagation direction). Wavelength $\lambda = c/v = 3 \times 10^8 / 30 \times 10^6 = 10$ m.

Q5 A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?

SOLUTION

Given: frequency band $f_1 = 7.5$ MHz $= 7.5 \times 10^6$ Hz to $f_2 = 12$ MHz $= 12 \times 10^6$ Hz

$c = 3 \times 10^8$ m/s

Wavelength corresponding to f_1 (lower frequency, longer wavelength):

$\lambda_1 = c / f_1 = (3 \times 10^8) / (7.5 \times 10^6) = 40$ m

Wavelength corresponding to f_2 (higher frequency, shorter wavelength):

$\lambda_2 = c / f_2 = (3 \times 10^8) / (12 \times 10^6) = 25$ m

So the wavelength band is 25 m to 40 m.

Answer: Wavelength band: 25 m to 40 m (corresponding to frequencies 12 MHz and 7.5 MHz respectively).

Q6 A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. What is the frequency of the electromagnetic waves produced by the oscillator?

SOLUTION

An accelerating (oscillating) charge radiates electromagnetic waves. The frequency of the emitted electromagnetic wave equals the frequency of oscillation of the charge — this is a fundamental result of Maxwell's theory.

Given: oscillation frequency = 10^9 Hz

Therefore, frequency of the electromagnetic waves produced = 10^9 Hz

This frequency (1 GHz) lies in the microwave region of the electromagnetic spectrum.

Answer: The frequency of the electromagnetic waves = 10^9 Hz (equal to the oscillation frequency of the charged particle).

Q7 The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is B_o = 510 nT. What is the amplitude of the electric field part of the wave?

SOLUTION

Given: $B_o = 510\ nT = 510 \times 10^{-9}\ T = 5.1 \times 10^{-7}\ T$
$c = 3 \times 10^8\ m/s$

In an electromagnetic wave, the electric and magnetic field amplitudes are related by:
$E_o = c \times B_o$

$E_o = (3 \times 10^8\ m/s) \times (5.1 \times 10^{-7}\ T)$
$E_o = 153\ V/m\ (= 153\ N/C)$

Answer: $E_o = c \times B_o = 3 \times 10^8 \times 5.1 \times 10^{-7} = 153\ V/m$.

Q8 Suppose that the electric field amplitude of an electromagnetic wave is E_o = 120 N/C and that its frequency is v = 50.0 MHz. (a) Determine B_o, ω, k, and λ. (b) Find expressions for E and B.

SOLUTION

Given: $E_o = 120\ N/C$, $v = 50.0\ MHz = 5.0 \times 10^7\ Hz$
$c = 3 \times 10^8\ m/s$

(a) Magnetic field amplitude:
$B_o = E_o / c = 120 / (3 \times 10^8) = 4.0 \times 10^{-7}\ T = 400\ nT$

Angular frequency:
$\omega = 2\pi v = 2\pi \times 5.0 \times 10^7 = 3.14 \times 10^8\ rad/s$

Wave number:
$k = \omega / c = (3.14 \times 10^8) / (3 \times 10^8) = 1.05\ rad/m$

Wavelength:
$\lambda = c / v = (3 \times 10^8) / (5.0 \times 10^7) = 6.0\ m$

(b) Taking propagation along the z-axis, E along the x-axis, and B along the y-axis (consistent with E × B in the z-direction):

$E = E_o \sin(kz - \omega t)\ \hat{i} = 120 \sin(1.05z - 3.14 \times 10^8\ t)\ \hat{i}\ V/m$
$B = B_o \sin(kz - \omega t)\ \hat{j} = 4.0 \times 10^{-7} \sin(1.05z - 3.14 \times 10^8\ t)\ \hat{j}\ T$

Answer: $B_o = 4.0 \times 10^{-7}\ T$; $\omega = 3.14 \times 10^8\ rad/s$; $k = 1.05\ rad/m$; $\lambda = 6.0\ m$. $E = 120 \sin(1.05z - 3.14 \times 10^8\ t)\ \hat{i}\ V/m$; $B = 4.0 \times 10^{-7} \sin(1.05z - 3.14 \times 10^8\ t)\ \hat{j}\ T$.

Q9 The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula E = hv (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

SOLUTION

Given: $E = h\nu$, where $h = 6.63 \times 10^{-34}$ J·s and 1 eV = 1.6×10^{-19} J
Use $c = 3 \times 10^{8}$ m/s, so $E = hc/\lambda$

Photon energies for representative frequencies of each EM band:

Radio waves ($\nu \sim 10^{6}$ Hz):
$E = 6.63 \times 10^{-34} \times 10^{6} = 6.63 \times 10^{-28}$ J ≈ 4×10^{-9} eV

Microwaves ($\nu \sim 10^{9}$ Hz):
$E \approx 6.63 \times 10^{-25}$ J ≈ 4×10^{-6} eV

Infrared ($\nu \sim 10^{12}$ Hz):
$E \approx 6.63 \times 10^{-22}$ J ≈ 4×10^{-3} eV

Visible light ($\nu \sim 6 \times 10^{14}$ Hz):
$E \approx 3.98 \times 10^{-19}$ J ≈ 2.5 eV

Ultraviolet ($\nu \sim 10^{16}$ Hz):
$E \approx 6.63 \times 10^{-18}$ J ≈ 41 eV

X-rays ($\nu \sim 10^{18}$ Hz):
$E \approx 6.63 \times 10^{-16}$ J ≈ 4×10^{3} eV = 4 keV

Gamma rays ($\nu \sim 10^{21}$ Hz):
$E \approx 6.63 \times 10^{-13}$ J ≈ 4×10^{6} eV = 4 MeV

Connection to sources:
The photon energy scale matches the energy transitions in the source. Radio/microwave photons (~10^{-9} to 10^{-6} eV) correspond to rotational/spin transitions of molecules in circuits. Infrared photons (~10^{-3} eV) match vibrational transitions of molecules (thermal radiation of hot bodies). Visible and UV photons (1–40 eV) match electronic transitions in the outer shells of atoms. X-ray photons (keV range) correspond to transitions of inner-shell electrons in heavy atoms. Gamma-ray photons (MeV range) originate from nuclear transitions and radioactive decay. The energy scale of the photon is therefore directly tied to the characteristic energy scale of the physical process in the source.

Answer: Photon energies range from ~10^{-9} eV (radio) through ~2.5 eV (visible) to ~4 MeV (gamma rays). Higher photon energies match higher-energy source processes: electronic transitions (visible/UV), inner-shell electron transitions (X-rays), and nuclear transitions (gamma rays).

Q10 In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 V m^{-1}.
(a) What is the wavelength of the wave?
(b) What is the amplitude of the oscillating magnetic field?
(c) Show that the average energy density of the E field equals the average energy density of the B field. [$c = 3 \times 10^8$ m s^{-1}.]

SOLUTION

Given: $\nu = 2.0 \times 10^{10}$ Hz, $E_o = 48$ V m^{-1}, $c = 3 \times 10^8$ m/s
Constants: $\varepsilon_o = 8.85 \times 10^{-12}$ F m^{-1}, $\mu_o = 4\pi \times 10^{-7}$ T m A^{-1}

(a) Wavelength:
$\lambda = c / \nu = (3 \times 10^8) / (2.0 \times 10^{10})$
$\lambda = 1.5 \times 10^{-2}$ m = 1.5 cm

(b) Amplitude of magnetic field:
$B_o = E_o / c = 48 / (3 \times 10^8)$
$B_o = 1.6 \times 10^{-7}$ T

(c) Average energy density of the electric field:
$\langle u_E \rangle = (1/2)\, \varepsilon_o E_o^2$
(The time-average of $\sin^2(kz - \omega t) = 1/2$)

Average energy density of the magnetic field:
$\langle u_B \rangle = (1/2)\, B_o^2 / \mu_o$

To show $\langle u_E \rangle = \langle u_B \rangle$, use the relation $E_o = c\, B_o$, so $E_o^2 = c^2 B_o^2$:
$\langle u_E \rangle = (1/2)\, \varepsilon_o E_o^2 = (1/2)\, \varepsilon_o c^2 B_o^2$

Since $c = 1/\sqrt{(\mu_o \varepsilon_o)}$, we have $c^2 = 1/(\mu_o \varepsilon_o)$, hence $\varepsilon_o c^2 = 1/\mu_o$:
$\langle u_E \rangle = (1/2)\, (1/\mu_o)\, B_o^2 = (1/2)\, B_o^2/\mu_o = \langle u_B \rangle$

Therefore $\langle u_E \rangle = \langle u_B \rangle$ (proved)

Numerical check: $\langle u_E \rangle = 0.5 \times 8.85 \times 10^{-12} \times (48)^2 = 1.02 \times 10^{-8}$ J m^{-3}
$\langle u_B \rangle = 0.5 \times (1.6 \times 10^{-7})^2 / (4\pi \times 10^{-7}) = 1.02 \times 10^{-8}$ J m^{-3} ✓

Answer: (a) $\lambda = 1.5$ cm. (b) $B_o = 1.6 \times 10^{-7}$ T. (c) $\langle u_E \rangle = \frac{1}{2}\varepsilon_o E_o^2 = \frac{1}{2}\varepsilon_o c^2 B_o^2 = \frac{1}{2}B_o^2/\mu_o = \langle u_B \rangle$, since $c^2 = 1/(\mu_o \varepsilon_o)$.