Chapter 9: Ray Optics and Optical Instruments

Exercises — Exercises

Q1 A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

SOLUTION

Given: object size h = 2.5 cm, u = -27 cm (object in front of mirror), R = 36 cm
Focal length: f = -R/2 = -18 cm (concave mirror, negative sign).

Using the mirror equation: 1/v + 1/u = 1/f
1/v = 1/f - 1/u = 1/(-18) - 1/(-27) = -1/18 + 1/27 = (-3 + 2)/54 = -1/54
v = -54 cm

The screen must be placed 54 cm in front of the mirror (on the same side as the object).

Magnification: m = -v/u = -(-54)/(-27) = -2
Image size: h' = m × h = (-2) × 2.5 = -5.0 cm

Nature: The image is real (v is negative, same side as object), inverted (m is negative), and magnified (|m| = 2). Its size is 5.0 cm.

Effect of moving candle closer: As the candle moves closer to the mirror (u decreases in magnitude toward f), v increases (image moves farther from mirror). So the screen must be moved away from the mirror. When the object is between F and P (inside the focal length), the image becomes virtual and cannot be captured on a screen.

Answer: Screen at 54 cm from mirror; image is real, inverted, magnified (size = 5.0 cm). Moving candle closer requires moving screen farther away.

Q2 A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

SOLUTION

Given: h = 4.5 cm, u = -12 cm, f = +15 cm (convex mirror).

Mirror equation: 1/v + 1/u = 1/f
1/v = 1/f - 1/u = 1/15 - 1/(-12) = 1/15 + 1/12 = (4 + 5)/60 = 9/60
v = +6.67 cm (approximately 20/3 cm)

The image is formed 6.67 cm behind the mirror (virtual image).

Magnification: m = -v/u = -(6.67)/(-12) = +0.556
Image height: h' = m × h = 0.556 × 4.5 = +2.5 cm

The image is virtual, erect, and diminished (size = 2.5 cm).

As the needle is moved farther from the mirror (u increases in magnitude), the virtual image moves toward the focus F (which is behind the mirror at 15 cm). The image always remains virtual, erect, and diminished. Even when the object is at infinity, the image is at the focus, 15 cm behind the mirror.

Answer: Image at 6.67 cm behind the mirror (virtual, erect, diminished); magnification = +0.56. As needle moves farther, image approaches but never crosses the focal point.

Q3 A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

SOLUTION

Given: real depth = 12.5 cm, apparent depth = 9.4 cm.

Refractive index of water: n = real depth / apparent depth = 12.5 / 9.4 = 1.33

For the liquid of n = 1.63:
New apparent depth = real depth / n = 12.5 / 1.63 = 7.67 cm

The microscope was focused at 9.4 cm above the bottom. New focus point is at 7.67 cm above the bottom.

Distance the microscope must be moved down = 9.4 - 7.67 = 1.73 cm

The microscope must be moved 1.73 cm downward to focus on the needle again.

Answer: n_water = 1.33; microscope must be moved down by 1.73 cm.

Q4 Figures 9.27(a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. 9.27(c)].

SOLUTION

From Figure 9.27(a): ray in air at 60° refracts into glass at 35°.
By Snell's law: n_glass = sin 60° / sin 35° = 0.8660 / 0.5736 = 1.510

From Figure 9.27(b): ray in air at 60° refracts into water at 47°.
n_water = sin 60° / sin 47° = 0.8660 / 0.7314 = 1.184

For Figure 9.27(c): ray in water incident at 45° on a water-glass interface.
Snell's law: n_water × sin 45° = n_glass × sin r
sin r = (n_water / n_glass) × sin 45° = (1.184 / 1.510) × 0.7071
sin r = 0.7841 × 0.7071 = 0.5545
r = 33.7° ≈ 34°

Answer: Angle of refraction in glass ≈ 34°.

Q5 A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)

SOLUTION

Given: depth h = 80 cm = 0.80 m, n_water = 1.33.

Critical angle for water-air interface:
sin i_c = 1/n = 1/1.33 => i_c = 48.75°

Light can emerge only within the cone of half-angle i_c. The radius of the bright circle on the surface:
r = h × tan i_c = 80 × tan 48.75°
tan i_c = sin i_c / cos i_c = (1/1.33) / sqrt(1 - 1/1.33²) = 0.7519 / 0.6593 = 1.140

r = 80 × 1.140 = 91.23 cm

Area: A = pi × r² = pi × (91.23)² = 26,149 cm² = 2.61 m²

Answer: Area = 2.61 m²

Q6 A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

SOLUTION

Given: A = 60°, D_m = 40°.

Refractive index of prism (in air):
n = sin[(A + D_m)/2] / sin[A/2] = sin[(60 + 40)/2] / sin[60/2]
n = sin 50° / sin 30° = 0.7660 / 0.5000 = 1.532

Prism in water: n_glass/n_water = 1.532/1.33 = 1.152
Let D_m' be the new minimum deviation angle.
sin[(60 + D_m')/2] / sin 30° = 1.152
sin[(60 + D_m')/2] = 1.152 × 0.5000 = 0.5760
(60 + D_m')/2 = arcsin(0.5760) = 35.17°
60 + D_m' = 70.34°
D_m' = 10.34° ≈ 10.3°

Answer: n = 1.532; new angle of minimum deviation in water = 10.3°.

Q7 Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?

SOLUTION

Given: n = 1.55, f = 20 cm, R1 = R and R2 = -R (both faces same radius, double-convex).

Lens maker's formula:
1/f = (n - 1)[1/R1 - 1/R2] = (1.55 - 1)[1/R - 1/(-R)]
1/20 = 0.55 × (2/R)
1/20 = 1.10/R

R = 20 × 1.10 = 22 cm

Answer: Radius of curvature R = 22 cm.

Q8 A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm, and (b) a concave lens of focal length 16 cm?

SOLUTION

The converging beam would meet at P which is 12 cm beyond the lens. For the lens, P acts as a virtual object on the transmission side, so u = +12 cm.

Lens formula: 1/v - 1/u = 1/f (with sign convention u = +12 for virtual object)
=> 1/v = 1/f + 1/u

(a) Convex lens, f = +20 cm:
1/v = 1/20 + 1/12 = 3/60 + 5/60 = 8/60
v = 60/8 = 7.5 cm
The beam converges 7.5 cm beyond the lens (real convergence point).

(b) Concave lens, f = -16 cm:
1/v = 1/(-16) + 1/12 = -3/48 + 4/48 = 1/48
v = 48 cm
The beam converges 48 cm beyond the lens.

Answer: (a) Converges at 7.5 cm beyond the lens. (b) Converges at 48 cm beyond the lens.

Q9 An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

SOLUTION

Given: h = 3.0 cm, u = -14 cm, f = -21 cm (concave/diverging lens).

Lens formula: 1/v - 1/u = 1/f
1/v = 1/f + 1/u = 1/(-21) + 1/(-14) = -2/42 - 3/42 = -5/42
v = -42/5 = -8.4 cm

Magnification: m = v/u = (-8.4)/(-14) = +0.6

Image size: h' = m × h = 0.6 × 3.0 = 1.8 cm

The image is virtual (v negative), erect (m positive), diminished (|m| < 1), and located 8.4 cm on the same side as the object.

As the object is moved farther away: v approaches -f (image moves toward focus at -21 cm, but never reaches it). The image always remains virtual, erect, and diminished, becoming progressively smaller.

Answer: Image is virtual, erect, diminished (1.8 cm) at 8.4 cm from lens on the same side as object. Moving object farther: image approaches the focus, remaining virtual and diminished.

Q10 What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

SOLUTION

Given: f1 = +30 cm (convex), f2 = -20 cm (concave).

For lenses in contact: 1/f = 1/f1 + 1/f2
1/f = 1/30 + 1/(-20) = 2/60 - 3/60 = -1/60
f = -60 cm

The focal length is -60 cm. Since f is negative, the combination is a diverging lens.

Answer: f = -60 cm; the combination acts as a diverging (concave) lens.

Q11 A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm), and (b) at infinity? What is the magnifying power of the microscope in each case?

SOLUTION

Given: f_o = 2.0 cm, f_e = 6.25 cm, tube length (objective to eyepiece) = 15 cm, D = 25 cm.

(a) Final image at D = 25 cm (near point):
For eyepiece: v_e = -25 cm, f_e = 6.25 cm

1/u_e = 1/v_e - 1/f_e = 1/(-25) - 1/6.25 = -0.04 - 0.16 = -0.20
u_e = -5.0 cm (object 5 cm from eyepiece)

Image by objective: v_o = 15 - |u_e| = 15 - 5 = 10 cm
1/u_o = 1/v_o - 1/f_o = 1/10 - 1/2 = 0.10 - 0.50 = -0.40
u_o = -2.5 cm (object 2.5 cm from objective)

Magnification: m_e = 1 + D/f_e = 1 + 25/6.25 = 5.0
m_o = v_o/u_o = 10/(-2.5) = -4.0
Total m = m_o × m_e = -4.0 × 5.0 = -20
Magnifying power = 20 (magnitude).

(b) Final image at infinity:
Object for eyepiece at its focus: |u_e| = f_e = 6.25 cm
v_o = 15 - 6.25 = 8.75 cm
1/u_o = 1/8.75 - 1/2 = 0.1143 - 0.5 = -0.3857
u_o = -2.59 cm (object 2.59 cm from objective)

m_e = D/f_e = 25/6.25 = 4.0
m_o = 8.75/(-2.59) = -3.38
Total m = (-3.38) × 4.0 = -13.5
Magnifying power = 13.5 (magnitude).

Answer: (a) Object at 2.5 cm from objective; magnifying power = 20. (b) Object at 2.59 cm from objective; magnifying power = 13.5.

Q12 A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5 cm can bring an object placed at 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope.

SOLUTION

Given: f_o = 8.0 mm = 0.8 cm, f_e = 2.5 cm, u_o = -9.0 mm = -0.9 cm, D = 25 cm.

Image by objective:
1/v_o = 1/f_o + 1/u_o = 1/0.8 + 1/(-0.9) = 1.25 - 1.111 = 0.139
v_o = 7.2 cm

For final image at D = 25 cm, eyepiece object distance:
1/u_e = 1/v_e - 1/f_e = 1/(-25) - 1/2.5 = -0.04 - 0.4 = -0.44
u_e = -2.27 cm

Separation between lenses = $v_o + |u_e| = 7.2 + 2.27 = 9.47$ cm

Magnification:
$m_o = v_o / u_o = 7.2 / (-0.9) = -8.0$
$m_e = 1 + D/f_e = 1 + 25/2.5 = 11.0$
Total $m = m_o \times m_e = -8.0 \times 11.0 = -88$
Magnifying power = 88.

Answer: Separation = 9.47 cm; magnifying power = 88.

Q13 A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

SOLUTION

Given: $f_o = 144$ cm, $f_e = 6.0$ cm (telescope in normal adjustment, image at infinity).

Magnifying power: $m = f_o / f_e = 144 / 6.0 = 24$

Separation between lenses (tube length): $L = f_o + f_e = 144 + 6.0 = 150$ cm

Answer: Magnifying power = 24; separation = 150 cm.

Q14 (a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope? (b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^6 m, and the radius of lunar orbit is 3.8×10^8 m.

SOLUTION

Given: $f_o = 15$ m, $f_e = 1.0$ cm $= 0.01$ m.

(a) Angular magnification (normal adjustment):
$m = f_o / f_e = 15 / 0.01 = 1500$

(b) Diameter of moon's image by objective:
Angular diameter of moon: $\theta = d_{moon} / R_{orbit} = (3.48 \times 10^6) / (3.8 \times 10^8) = 9.16 \times 10^{-3}$ rad

Diameter of image = $\theta \times f_o = 9.16 \times 10^{-3} \times 15 = 0.1374$ m $= 13.74$ cm

Answer: (a) Angular magnification = 1500. (b) Diameter of moon's image = 13.74

cm.

Q15 Use the mirror equation to deduce that: (a) an object placed between f and 2f of a concave mirror produces a real image beyond 2f. (b) a convex mirror always produces a virtual image independent of the location of the object. (c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole. (d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

SOLUTION

(a) Concave mirror: f < 0. Object between f and 2f means 2f < u < f (both negative), i.e., |f| < |u| < 2|f|.
Mirror equation: 1/v = 1/f - 1/u
Since u is between -2f and -f: 1/u lies between -1/f and -1/(2f).

1/v = 1/f - 1/u. With f < 0 and	u	< 2	f	: 1/v = 1/f - 1/u < 1/(2f) (more negative).
This gives v < 2f (i.e.,	v	> 2	f	), so the real image forms beyond 2f. Also v is negative (same side as object) => real image.

(b) Convex mirror: f > 0. Object is always at u < 0 (real object).
1/v = 1/f - 1/u = 1/f + 1/|u| > 0 (both terms positive).
So v > 0 always, meaning image is always behind the mirror => virtual, regardless of u.

(c) Convex mirror: v > 0 and f > 0, u < 0.
1/v = 1/f + 1/|u|. Since 1/|u| > 0, we have 1/v > 1/f => v < f.
So the virtual image is always between the pole (v = 0) and the focus (v = f), i.e., 0 < v < f.
Magnification: m = -v/u = v/|u|. Since v < f < |u| (for any real object beyond the mirror), m < 1.
Image is always diminished.

(d) Concave mirror: f < 0. Object between pole and focus: 0 > u > f, i.e.,	u	<	f	.
1/v = 1/f - 1/u. Since	u	<	f	, 1/u (in magnitude) > 1/f => -1/u > -1/f (less negative).
So 1/v = 1/f - 1/u: with both f and u negative, 1/f < 0 and -1/u > 0, and	-1/u	>	1/f	.

Therefore 1/v > 0 => v > 0 (image is behind the mirror) => virtual.
Magnification: m = -v/u = -v/(negative) = v/|u| > 0. Since v > 0 is large and |u| < |f| is small, |m| > 1 => enlarged.
The image is virtual and enlarged.

Answer: All four results follow algebraically from the mirror equation with appropriate sign analysis as shown above.

Q16 A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

SOLUTION

Given: thickness of slab t = 15 cm, n = 1.5.

The apparent shift (amount by which the object appears raised) due to refraction through a parallel slab:
Shift = t × (1 - 1/n) = 15 × (1 - 1/1.5) = 15 × (1 - 0.667) = 15 × 0.333 = 5.0 cm

The pin appears to be raised by 5.0 cm.

The answer does NOT depend on the location of the slab (i.e., how far the slab is from the pin or from the eye), as long as the slab is parallel to the table. The shift depends only on the thickness and refractive index of the slab.

Answer: Pin appears raised by 5.0 cm. The answer does not depend on the location of the slab.

Q17 (a) Figure 9.28 shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure. (b) What is the answer if there is no outer covering of the pipe?

SOLUTION

Given: $n_{core} = 1.68$, $n_{cladding} = 1.44$.

(a) Critical angle at core-cladding interface:

$\sin i_c = n_{cladding} / n_{core} = 1.44 / 1.68 = 0.8571$

$i_c = 59.0°$

For TIR inside, the ray at the flat entry face (air-core) must refract at angle r such that:

$90° - r \geq i_c \Rightarrow r \leq 31.0°$

Maximum angle of incidence alpha at the flat face (angle with axis of pipe):

$\sin \alpha = n_{core} \times \sin r_{max} = n_{core} \times \cos i_c$

$\sin \alpha = \sqrt{n_{core}^2 - n_{cladding}^2} = \sqrt{1.68^2 - 1.44^2}$

$= \sqrt{2.8224 - 2.0736} = \sqrt{0.7488} = 0.8653$

$\alpha_{max} = 59.9° \approx 60°$

All rays incident at angles 0° to ~60° with the axis of the pipe undergo TIR.

(b) No outer covering ($n_{cladding} = 1.0$ = air):

$\sin \alpha_{max} = \sqrt{n_{core}^2 - 1} = \sqrt{1.68^2 - 1} = \sqrt{1.8224} = 1.350$

Since $\sin \alpha > 1$ is not possible, all angles of incidence (0° to 90°) are accepted.

Any ray entering the pipe (regardless of angle with axis) will undergo TIR at the core-air boundary.

Answer: (a) Rays incident at 0° to ~60° with the pipe axis undergo TIR. (b) With no cladding (air), all incident rays (0° to 90°) undergo TIR inside the pipe.

Q18 The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

SOLUTION

The object and screen are 3 m apart. Let the lens be at distance a from the object, so image distance = (3 - a) m.

Lens formula: $1/f = 1/v - 1/u = 1/(3-a) + 1/a = (a + 3 - a) / [a(3-a)] = 3 / [a(3-a)]$
$f = a(3 - a)/3$

For a real image, f is maximized when $df/da = 0$:
$df/da = (3 - 2a)/3 = 0 \Rightarrow a = 1.5$ m

$f_{max} = 1.5 \times (3 - 1.5)/3 = 1.5 \times 1.5/3 = 0.75$ m

Answer: Maximum possible focal length = 0.75 m.

Q19 A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.

SOLUTION

Given: D = distance between object and screen = 90 cm, d = separation between two lens positions = 20 cm.

Using the standard result for two-position method:
$f = (D^2 - d^2) / (4D)$
$f = (90^2 - 20^2) / (4 \times 90)$
$f = (8100 - 400) / 360$
$f = 7700 / 360$
$f = 21.4$ cm

Answer: Focal length = 21.4 cm.

Q20 (a) Determine the 'effective focal length' of the combination of the two lenses in Exercise 9.10, if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all? (b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40 cm. Determine the magnification produced by the two-lens system, and the size of the image.

SOLUTION

(a) Convex lens f1 = +30 cm (left), Concave lens f2 = -20 cm (right), separation d = 8 cm.

Beam from left (convex first):
Parallel beam => image by convex at v1 = f1 = 30 cm to its right.
Object for concave: 30 - 8 = 22 cm to its right (virtual object, u2 = +22 cm).
1/v2 = 1/f2 + 1/u2 = 1/(-20) + 1/22 = (-22 + 20)/440 = -2/440
v2 = -220 cm (virtual image, 220 cm to the left of concave lens).

Beam from right (concave first):
Parallel beam => virtual image by concave at -20 cm (20 cm to the left of concave lens).
This virtual image is (20 - 8) = 12 cm to the left of the convex lens => real object at u_conv = -12 cm.
1/v - 1/(-12) = 1/30 => 1/v = 1/30 - 1/12 = (2-5)/60 = -3/60
v = -20 cm

The effective focal length is different for the two directions (approximately -220 cm from left, and -20 cm from right), so the answer depends on which side the light is incident. The concept of a single effective focal length is NOT useful for this separated lens system — it behaves like a thick lens.

(b) Object 1.5 cm at 40 cm from convex lens (on the left side).
Image by convex lens (f1 = +30, u1 = -40):
1/v1 - 1/(-40) = 1/30 => 1/v1 = 1/30 - 1/40 = (4-3)/120 = 1/120
v1 = 120 cm

Image by concave (f2 = -20, object at 120-8 = 112 cm to right = virtual, u2 = +112):
1/v2 = 1/(-20) + 1/112 = -112/2240 + 20/2240 = -92/2240
v2 = -2240/92 = -24.35 cm (virtual image, 24.35 cm to the left of concave lens).

Total magnification: m1 = v1/u1 = 120/(-40) = -3
m2 = v2/u2 = (-24.35)/(112) = -0.217
m = m1 × m2 = (-3) × (-0.217) = +0.652

Size of image: h' = m × h = 0.652 × 1.5 = 0.98 cm

Answer: (a) Effective focal length ~ -220 cm (light from left) or -20 cm (light from right); depends on direction; the concept is not uniquely useful here. (b) Magnification = 0.65; image size = 0.98 cm.

Q21 At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

SOLUTION

Given: A = 60°, n = 1.524.

Critical angle for glass-air:
sin i_c = 1/n = 1/1.524 => i_c = 41.01°

For TIR to just occur at the second face, the angle of incidence there equals i_c:
r2 = i_c = 41.01°

At the prism: r1 + r2 = A => r1 = 60° - 41.01° = 18.99°

Applying Snell's law at the first face (air-glass):
sin i = n × sin r1 = 1.524 × sin 18.99° = 1.524 × 0.3254 = 0.4959
i = arcsin(0.4959) = 29.75° ≈ 29.8°

Answer: Angle of incidence at first face = 29.8°.

Q22 A card sheet divided into squares each of size 1 mm² is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 9 cm) held close to the eye. (a) What is the magnification produced by the lens? How much is the area of each square in the virtual image? (b) What is the angular magnification (magnifying power) of the lens? (c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

SOLUTION

Given: f = 9 cm, object at u = -9 cm = -f (object at focal point).

(a) When u = -f, the image is at infinity (v = infinity).

Linear magnification = v/u = infinity/(-9) = infinity.
Area of each square in the virtual image = infinity (image at infinity has infinite linear size).

(b) Angular magnification (magnifying power) when image is at infinity:
m_ang = D/f = 25/9 = 2.78

(c) No, the linear magnification (infinite) is NOT equal to the angular magnification (2.78).
When the image is at infinity, the concept of linear magnification breaks down. Angular magnification is the relevant and physically meaningful quantity for optical instruments — it compares the angle subtended at the eye with and without the lens.

Answer: (a) Linear magnification = infinity; area in image = infinity. (b) Angular magnification = 2.78. (c) They are not equal; angular magnification is the meaningful quantity here.

Q23 (a) At what distance should the lens be held from the card sheet in Exercise 9.22 in order to view the squares distinctly with the maximum possible magnifying power? (b) What is the magnification in this case? (c) Is the magnification equal to the magnifying power in this case? Explain.

SOLUTION

Maximum magnifying power occurs when the image is at the near point D = 25 cm.

(a) v = -25 cm, f = 9 cm:
1/u = 1/v - 1/f = 1/(-25) - 1/9 = -9/(225) - 25/(225) = -34/225
u = -225/34 = -6.62 cm
Hold the lens 6.62 cm from the card sheet.

(b) Linear magnification: m = v/u = (-25)/(-6.62) = +3.78

(c) Maximum angular magnification: m_ang = 1 + D/f = 1 + 25/9 = 3.78

Yes! In this case (image at near point), the linear magnification equals the angular magnification, both = 3.78.
This is because the image distance equals D = 25 cm, so the ratio v/u gives the same factor as the angular magnification formula 1 + D/f.

Answer: (a) 6.62 cm from the card. (b) Linear magnification = 3.78. (c) Yes, equal to the angular magnification (3.78) in this case.

Q24 What should be the distance between the object in Exercise 9.23 and the magnifying glass if the virtual image of each square in the figure is to have an area of 6.25 mm². Would you be able to see the squares distinctly with your eyes very close to the magnifier?

SOLUTION

Given: image area = 6.25 mm², object area = 1 mm², f = 9 cm.

Linear magnification required: m = sqrt(6.25/1) = 2.5
Since image is virtual and erect: m = v/u = 2.5 (both v and u negative, so v/u > 0).
=> v = 2.5u (both negative, so |v| = 2.5|u|)

Using lens formula: 1/v - 1/u = 1/f
1/(2.5u) - 1/u = 1/9
(1 - 2.5)/(2.5u) = -1.5/(2.5u) = 1/9
u = -1.5 × 9/2.5 = -13.5/2.5 = -5.4 cm

v = 2.5 × (-5.4) = -13.5 cm

The virtual image is at 13.5 cm from the lens. Since 13.5 cm < 25 cm (near point), the eye cannot focus on an image closer than 25 cm. Therefore, you would NOT be able to see the squares distinctly — the image is within the near point and causes strain/blurred vision for a normal eye.

Answer: Object at 5.4 cm from lens; image at 13.5 cm (virtual). You cannot see the squares distinctly — the image is closer than the near point (25 cm).

Q25 Answer the following questions: (a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification? (b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back? (c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power? (d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths? (e) When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?

SOLUTION

(a) The magnifying glass allows the object to be placed much closer to the eye than the near point D (= 25 cm). Without the lens, the object must be at 25 cm to be seen clearly, subtending a small angle. With the lens, the object at $u < f$ subtends the SAME angle at the lens as the virtual image does, but both angles are LARGER than the angle the object would subtend at 25 cm without the lens. Angular magnification means: the lens enables the object to be brought closer, so the angle it subtends (as seen through the lens as its virtual image) is larger than what the naked eye can achieve.

(b) Yes, angular magnification decreases if the eye is moved back from the lens. The angle subtended by the image at the eye decreases as the eye moves farther from the lens. For maximum magnification, the eye should be positioned very close to the lens.

(c) As the focal length f decreases, the lens surfaces must have smaller and smaller radii of curvature. Very small focal lengths are difficult to manufacture. More importantly, with very short f, the lens suffers from severe aberrations (spherical aberration, chromatic aberration) which degrade image quality. Practical limits are around $f \sim 1$ cm for a single lens.

(d) For a compound microscope, the total magnification is $m = m_o \times m_e = (L/f_o) \times (D/f_e)$. Both f_o and f_e must be short to achieve large magnification. Short f_o gives a large, magnified intermediate image; short f_e gives large angular magnification of that image.

(e) The exit pupil (eye point) of the eyepiece is a few mm beyond the eye lens. When the eye is exactly at the eyepiece, it cannot collect the full cone of light from the image — part of the light misses the eye's pupil. By positioning the eye a short distance (~1–2 cm, or the 'eye relief' distance) away, the eye pupil coincides with the exit pupil and captures the entire field of view with maximum light.

Answer: Conceptual answers as above: (a) magnifier brings object closer than near point, increasing angular subtense; (b) yes, decreases if eye moves back; (c) manufacturing difficulty + aberrations limit small f; (d) both short f needed for large total m; (e) eye must be at exit pupil ~1-2 cm from eyepiece.

Q26 An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope?

SOLUTION

Given: m = 30, f_o = 1.25 cm, f_e = 5 cm, D = 25 cm (normal near point).

For image at infinity: $m = (L/f_o) \times (D/f_e)$
$m_e = D/f_e = 25/5 = 5$
$m_o = m/m_e = 30/5 = 6$

For objective: $m_o = v_o/u_o = L/f_o$, so $L = m_o \times f_o = 6 \times 1.25 = 7.5$ cm
$v_o = L + f_o = 7.5 + 1.25 = 8.75$ cm

Object distance for objective:
$1/u_o = 1/v_o - 1/f_o = 1/8.75 - 1/1.25 = 0.1143 - 0.800 = -0.686$
$u_o = -1.46$ cm
Object must be placed 1.46 cm from the objective.

Separation between lenses = $v_o + f_e = 8.75 + 5.0 = 13.75$ cm

Setup: Place object 1.46 cm in front of objective. Objective and eyepiece separated by 13.75 cm. Final image at infinity (for relaxed eye viewing).

Answer: Object at 1.46 cm from objective; lens separation = 13.75 cm. This gives magnifying power = 30.

Q27 A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when (a) the telescope is in normal adjustment (i.e., when the final image is at infinity)? (b) the final image is formed at the least distance of distinct vision (25 cm)?

SOLUTION

Given: $f_o = 140$ cm, $f_e = 5.0$ cm, $D = 25$ cm.

(a) Normal adjustment (image at infinity):
$m = f_o / f_e = 140/5 = 28$

(b) Final image at $D = 25$ cm:
$m = (f_o/f_e) \times (1 + f_e/D) = (140/5) \times (1 + 5/25) = 28 \times 1.2 = 33.6$
Equivalently: $m = f_o \times (1/f_e + 1/D) = 140 \times (0.2 + 0.04) = 140 \times 0.24 = 33.6$

Answer: (a) $m = 28$ (normal adjustment). (b) $m = 33.6$ (image at near point).

Q28 (a) For the telescope described in Exercise 9.27(a), what is the separation between the objective lens and the eyepiece? (b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens? (c) What is the height of the final image of the tower if it is formed at 25 cm?

SOLUTION

$f_o = 140$ cm, $f_e = 5$ cm.

(a) Separation (normal adjustment) $= f_o + f_e = 140 + 5 = 145$ cm.

(b) Height of tower's image by objective:
Angular size of tower: $\theta = h/d = 100\text{ m} / 3000\text{ m} = 1/30$ rad
Image height (at focal plane of objective): $h' = \theta \times f_o = (100/3000) \times 140 = 4.67$ cm

(c) Height of final image at $D = 25$ cm:
Eyepiece magnifies the intermediate image (4.67 cm) to give the final image.
For eyepiece with $v_e = -25$ cm, $f_e = 5$ cm:
$1/u_e = 1/v_e - 1/f_e = 1/(-25) - 1/5 = -1/25 - 5/25 = -6/25$
$u_e = -25/6 = -4.17$ cm
$m_e = v_e/u_e = (-25)/(-4.17) = 6.0$
Final image height $= m_e \times h' = 6.0 \times 4.67 = 28.0$ cm

Answer: (a) 145 cm. (b) 4.67 cm. (c) 28.0 cm.

Q29 A Cassegrain telescope uses two mirrors as shown in Fig. 9.26. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be?

SOLUTION

Given: separation d = 20 mm, R_large = 220 mm => f_large = 110 mm (concave primary), R_small = 140 mm => f_small = 70 mm (convex secondary).

Step 1: Large concave mirror focuses parallel beam from infinity at its focal point, 110 mm from the primary.

Step 2: The small convex mirror is 20 mm in front of the primary, intercepting the converging beam. The convergence point is 110 - 20 = 90 mm behind the small mirror (virtual object for the small mirror).

For the small convex mirror: f_2 = +70 mm (convex), virtual object u = +90 mm (on the reflected/back side).
Mirror equation: 1/v + 1/u = 1/f
1/v = 1/f - 1/u = 1/70 - 1/90 = (9 - 7)/630 = 2/630
v = 315 mm

v is positive, so the final image forms 315 mm behind the small mirror — i.e., on the far side of the small mirror, in the direction of the primary mirror hole.

Distance from the primary (large) mirror = 315 - 20 = 295 mm.

Answer: Final image forms 315 mm from the small mirror (295 mm behind the large mirror, accessible through the hole in the primary).

Q30 Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in Fig. 9.29. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?

SOLUTION

Given: mirror deflection theta = 3.5°, screen distance D = 1.5 m.

When the mirror rotates by theta, the reflected beam rotates by 2theta (law of reflection).

Total angle of rotation of reflected beam = 2 × 3.5° = 7.0°

Displacement of spot on screen:
d = D × tan(2theta) = 1.5 × tan(7.0°) = 1.5 × 0.1228 = 0.1842 m

d = 0.184 m = 18.4 cm

(Using small-angle approximation: d ≈ D × 2theta_rad = 1.5 × 2 × (3.5 × pi/180) = 1.5 × 0.1222 = 0.183 m)

Answer: Displacement of reflected spot = 0.184 m (18.4 cm).

Q31 Figure 9.30 shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?

SOLUTION

When the inverted image coincides with the object, the reflected rays retrace their path — this occurs when the object is at the focal point of the system.

Case 2 (no liquid): needle at 30.0 cm = effective focal length of (equiconvex lens + plane mirror + equiconvex lens) system.
The system acts as two lenses in contact (mirror contributes P = 0):
1/f_eff = 2 × (1/f_lens) => f_lens = 2 × 30.0 = 60 cm

For equiconvex lens (n = 1.50, R1 = R, R2 = -R):
1/f_lens = (n - 1)(1/R1 - 1/R2) = 0.5 × (2/R) = 1/R
=> R = f_lens = 60 cm

Case 1 (with liquid): f_eff = 45.0 cm.
System: lens + liquid plano-concave + plane mirror + liquid + lens.
1/f_eff = 2/f_lens + 2/f_liquid (plane mirror: P = 0).
1/45 = 2/60 + 2/f_liquid
2/f_liquid = 1/45 - 2/60 = 4/180 - 6/180 = -2/180 = -1/90
f_liquid = -180 cm

The liquid layer acts as a plano-concave lens (f = -180 cm). Its curved surface has R = 60 cm, flat surface has R = infinity.
Lens maker's formula for liquid layer:

$1/f_{liquid} = (n_{liq} - 1) \times [1/(-60) - 1/\infty] = -(n_{liq} - 1)/60$

$-1/180 = -(n_{liq} - 1)/60$

$n_{liq} - 1 = 60/180 = 1/3$

$n_{liq} = 4/3 = 1.33$

Answer: Refractive index of liquid = 4/3 = 1.33.