Chapter 10: Wave Optics

Exercises — Exercises

Q1 Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of (a) reflected, and (b) refracted light? Refractive index of water is 1.33.

SOLUTION

Given: $\lambda_{incident} = 589\ nm = 589 \times 10^{-9}\ m$, $n_{water} = 1.33$
Speed of light in air (vacuum): $c = 3.0 \times 10^{8}\ m/s$

(a) Reflected light — remains in the same medium (air):
Reflection does not change the medium. Wavelength, frequency and speed are all unchanged.
Wavelength: $\lambda = 589\ nm$
Frequency: $f = c / \lambda = (3.0 \times 10^{8}) / (589 \times 10^{-9}) = 5.09 \times 10^{14}\ Hz$
Speed: $v = 3.0 \times 10^{8}\ m/s$

(b) Refracted light — enters water (n = 1.33):
Frequency does not change on refraction (boundary condition): $f = 5.09 \times 10^{14}\ Hz$
Speed in water: $v = c / n = (3.0 \times 10^{8}) / 1.33 = 2.26 \times 10^{8}\ m/s$
Wavelength in water: $\lambda_w = v / f = \lambda_{air} / n = 589 / 1.33 = 443\ nm$

Answer: (a) Reflected: $\lambda = 589\ nm$, $f = 5.09 \times 10^{14}\ Hz$, $v = 3.0 \times 10^{8}\ m/s$. (b) Refracted: $\lambda = 443\ nm$, $f = 5.09 \times 10^{14}\ Hz$, $v = 2.26 \times 10^{8}\ m/s$

Q2 What is the shape of the wavefront in each of the following cases: (a) Light diverging from a point source. (b) Light emerging out of a convex lens when a point source is placed at its focus. (c) The portion of the wavefront of light from a distant star intercepted by the Earth.

SOLUTION

(a) Light diverging from a point source:
All points equidistant from the point source lie on a sphere. The wavefront is spherical (with the source at the centre).

(b) Light emerging from a convex lens when the point source is at its focus:
The lens converts the diverging spherical waves from the focal point into a parallel beam. The emerging wavefront is plane (flat).

(c) Portion of wavefront from a distant star intercepted by the Earth:
The star is so far away that the radius of curvature of its spherical wavefront is extremely large. The small portion intercepted by the Earth is effectively flat — the wavefront is plane.

Answer: (a) Spherical wavefront. (b) Plane wavefront. (c) Plane wavefront.

Q3 (a) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is 3.0×10^8 m s^{-1}) (b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

SOLUTION

(a) Given: $n_{glass} = 1.5$, $c = 3.0 \times 10^8$ m/s
Speed of light in glass:
$v = c / n = (3.0 \times 10^8) / 1.5 = 2.0 \times 10^8$ m/s

(b) No, the speed of light in glass is NOT independent of colour.
The refractive index of glass varies with wavelength — this is called dispersion.
Violet light has a shorter wavelength than red light. Glass has a higher refractive index for violet than for red ($n_{violet} > n_{red}$).
Since $v = c/n$, a higher refractive index means lower speed.
Therefore, violet light travels slower than red light in a glass prism.

Answer: (a) $v = 2.0 \times 10^8$ m/s. (b) No; violet light travels slower than red light in glass ($n_{violet} > n_{red} \Rightarrow v_{violet} < v_{red}$).

Q4 In a Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.

SOLUTION

Given: $d = 0.28$ mm $= 0.28 \times 10^{-3}$ m, $D = 1.4$ m
Distance from central fringe to 4th bright fringe: $x_4 = 1.2$ cm $= 1.2 \times 10^{-2}$ m

Position of nth bright fringe: $x_n = n \times \lambda \times D / d$
For $n = 4$: $x_4 = 4 \times \lambda \times D / d$

Solving for λ:

lambda = $x_4 \times d / (4 \times D)$
lambda = $(1.2 \times 10^{-2} \times 0.28 \times 10^{-3}) / (4 \times 1.4)$
lambda = $(3.36 \times 10^{-6}) / 5.6$
lambda = 6.0×10^{-7} m = 600 nm

Answer: lambda = 600 nm

Q5 In Young's double-slit experiment using monochromatic light of wavelength lambda, the intensity of light at a point on the screen where path difference is lambda, is K units. What is the intensity of light at a point where path difference is lambda/3?

SOLUTION

The intensity in a double-slit interference pattern is:
$I = 4 I_o \cos^2(\text{phi}/2)$
where phi = (2*pi / lambda) × (path difference) and I_o is the intensity due to each slit alone.

Step 1 — Find I_o in terms of K.
At path difference = lambda: phi = (2*pi/lambda) × lambda = 2*pi
$I = 4 I_o \cos^2(\pi) = 4 I_o \times 1 = 4 I_o = K$
=> $I_o = K/4$

Step 2 — Intensity at path difference = lambda/3.
phi = (2*pi/lambda) × (lambda/3) = 2*pi/3
$I = 4 I_o \cos^2(\pi/3) = 4 \times (K/4) \times (1/2)^2$
$I = K \times (1/4) = K/4$

Answer: I = K/4

Q6 A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment. (a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm. (b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

SOLUTION

Given: $\lambda_1 = 650\ \text{nm} = 650 \times 10^{-9}\ \text{m}$, $\lambda_2 = 520\ \text{nm} = 520 \times 10^{-9}\ \text{m}$
Let d = slit separation, D = distance to screen.

(a) Position of nth bright fringe: $x_n = n \times \lambda \times D / d$
Third bright fringe for $\lambda_1 = 650$ nm (n = 3):
$x_3 = 3 \times 650 \times 10^{-9} \times D / d$
$x_3 = (1.95 \times 10^{-6}) \times (D/d)\ \text{m}$
$x_3 = 1950\ D/d\ \text{nm}$

(b) The bright fringes of both wavelengths coincide when:
$n_1 \times \lambda_1 = n_2 \times \lambda_2$
$n_1 \times 650 = n_2 \times 520$
$n_1 / n_2 = 520 / 650 = 4 / 5$
The smallest integers satisfying this are $n_1 = 4$ and $n_2 = 5$.

Least coincidence distance:
$x = n_1 \times \lambda_1 \times D / d = 4 \times 650 \times 10^{-9} \times D / d$
$x = 2600 \times 10^{-9} \times (D/d)\ \text{m} = 2.6 \times 10^{-6} \times (D/d)\ \text{m}$

Verification: $5 \times 520 \times 10^{-9} \times D/d = 2600 \times 10^{-9} \times D/d$ (confirmed equal)

Answer: (a) $x_3 = 1.95 \times 10^{-6} \times (D/d)$ m (= 1.95 D/d micrometres). (b) $x = 2.6 \times 10^{-6} \times (D/d)$ m (= 2.6 D/d micrometres), occurring at $n_1 = 4$ for 650 nm and $n_2 = 5$ for 520 nm.