Chapter 11: Dual Nature of Radiation and Matter

Exercises — Exercises

Q1 Find the (a) maximum frequency, and (b) minimum wavelength of X-rays produced by 30 kV electrons.

SOLUTION

Given: Accelerating potential $V = 30\ \text{kV} = 30 \times 10^3\ \text{V}$
Constants: $h = 6.63 \times 10^{-34}\ \text{J s}$, $c = 3 \times 10^8\ \text{m/s}$, $e = 1.6 \times 10^{-19}\ \text{C}$

When an electron is accelerated through potential V, it gains kinetic energy eV. This energy is entirely converted into photon energy at maximum (shortest wavelength, highest frequency).

(a) Maximum frequency:
$eV = h \times \nu_{max}$
$\nu_{max} = eV / h = (1.6 \times 10^{-19} \times 30 \times 10^3) / (6.63 \times 10^{-34})$
$\nu_{max} = (4.8 \times 10^{-15}) / (6.63 \times 10^{-34})$
$\nu_{max} = 7.24 \times 10^{18}\ \text{Hz}$

(b) Minimum wavelength:
$\lambda_{min} = c / \nu_{max} = hc / (eV)$
$\lambda_{min} = (6.63 \times 10^{-34} \times 3 \times 10^8) / (1.6 \times 10^{-19} \times 30 \times 10^3)$
$\lambda_{min} = (1.989 \times 10^{-25}) / (4.8 \times 10^{-15})$
$\lambda_{min} = 4.14 \times 10^{-11}\ \text{m} = 0.0414\ \text{nm}$

Answer: (a) $\nu_{max} = 7.24 \times 10^{18}$ Hz; (b) $\lambda_{min} = 4.14 \times 10^{-11}$ m (0.0414 nm)

Q2 The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the (a) maximum kinetic energy of the emitted electrons, (b) stopping potential, and (c) maximum speed of the emitted photoelectrons?

SOLUTION

Given: Work function $\phi_0 = 2.14\ \text{eV} = 2.14 \times 1.6 \times 10^{-19}\ \text{J} = 3.424 \times 10^{-19}\ \text{J}$
Frequency of incident light $\nu = 6 \times 10^{14}\ \text{Hz}$
Constants: $h = 6.63 \times 10^{-34}\ \text{J s}$, $e = 1.6 \times 10^{-19}\ \text{C}$, $m_e = 9.11 \times 10^{-31}\ \text{kg}$

Energy of incident photon:
$E = h \times \nu = 6.63 \times 10^{-34} \times 6 \times 10^{14} = 3.978 \times 10^{-19}\ \text{J} = 2.49\ \text{eV}$

(a) Maximum kinetic energy (Einstein's photoelectric equation):
$K_{max} = h\nu - \phi_o = 3.978 \times 10^{-19} - 3.424 \times 10^{-19}$
$K_{max} = 5.54 \times 10^{-20}$ J = 0.346 eV

(b) Stopping potential:
$K_{max} = eV_o \Rightarrow V_o = K_{max} / e = 5.54 \times 10^{-20} / 1.6 \times 10^{-19}$
$V_o = 0.346$ V

(c) Maximum speed:
$K_{max} = (1/2) m_e v_{max}^2 \Rightarrow v_{max} = \sqrt{2 K_{max} / m_e}$
$v_{max} = \sqrt{2 \times 5.54 \times 10^{-20} / 9.11 \times 10^{-31}}$
$v_{max} = \sqrt{1.217 \times 10^{11}} = 3.49 \times 10^5$ m/s

Answer: (a) $K_{max} = 5.54 \times 10^{-20}$ J (0.346 eV); (b) $V_o = 0.346$ V; (c) $v_{max} = 3.49 \times 10^5$ m/s

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Q3 The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

SOLUTION

Given: Stopping (cut-off) potential $V_o = 1.5$ V
Constant: $e = 1.6 \times 10^{-19}$ C

The stopping potential equals the maximum kinetic energy of emitted photoelectrons:
$K_{max} = e \times V_o$
$K_{max} = 1.6 \times 10^{-19} \times 1.5$
$K_{max} = 2.4 \times 10^{-19}$ J = 1.5 eV

Answer: $K_{max} = 2.4 \times 10^{-19}$ J = 1.5 eV

Q4 Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW. (a) Find the energy and momentum of each photon in the light beam, (b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and (c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

SOLUTION

Given: Wavelength lambda = 632.8 nm = 632.8×10^{-9} m, Power P = 9.42 mW = 9.42×10^{-3} W

Constants: h = 6.63×10^{-34} J s, c = 3×10^{8} m/s, mass of H atom $m_H = 1.67 \times 10^{-27}$ kg

(a) Energy of each photon:

$E = hc / \lambda = (6.63 \times 10^{-34} \times 3 \times 10^{8}) / (632.8 \times 10^{-9})$

$E = 1.989 \times 10^{-25} / 6.328 \times 10^{-7} = 3.14 \times 10^{-19}$ J

Momentum of each photon:

$p = h / \lambda = 6.63 \times 10^{-34} / 632.8 \times 10^{-9} = 1.05 \times 10^{-27}$ kg m/s

(b) Number of photons per second:

Power $P = N \times E \Rightarrow N = P / E$

$N = 9.42 \times 10^{-3} / 3.14 \times 10^{-19} = 3.00 \times 10^{16}$ photons/s

(c) Speed of hydrogen atom with same momentum as photon:

$p_{photon} = m_H \times v \Rightarrow v = p_{photon} / m_H$

$v = 1.05 \times 10^{-27} / 1.67 \times 10^{-27} = 0.627$ m/s

Answer: (a) $E = 3.14 \times 10^{-19}$ J, $p = 1.05 \times 10^{-27}$ kg m/s; (b) $N = 3.00 \times 10^{16}$ photons/s; (c) $v = 0.627$ m/s

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Q5 In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s. Calculate the value of Planck's constant.

SOLUTION

Given: Slope of V_o vs nu graph = 4.12×10^{-15} V s

Constant: e = 1.6×10^{-19} C

From Einstein's photoelectric equation:

$e V_o = h \nu - \phi_o$

$\Rightarrow V_o = (h/e)*nu - phi_o/e$

This is a linear equation in nu with slope = h/e.
Therefore: $h/e = 4.12 \times 10^{-15}$ V s
$h = 4.12 \times 10^{-15} \times e = 4.12 \times 10^{-15} \times 1.6 \times 10^{-19}$
$h = 6.59 \times 10^{-34}$ J s

Answer: $h = 6.59 \times 10^{-34}$ J s

Q6 The threshold frequency for a certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.

SOLUTION

Given: Threshold frequency $nu_o = 3.3 \times 10^{14}$ Hz
Incident frequency $nu = 8.2 \times 10^{14}$ Hz
Constants: $h = 6.63 \times 10^{-34}$ J s, $e = 1.6 \times 10^{-19}$ C

The maximum kinetic energy of photoelectrons:
$K_{max} = h(nu - nu_o) = 6.63 \times 10^{-34} \times (8.2 - 3.3) \times 10^{14}$
$K_{max} = 6.63 \times 10^{-34} \times 4.9 \times 10^{14} = 3.249 \times 10^{-19}$ J

Cut-off (stopping) potential:
$V_o = K_{max} / e = 3.249 \times 10^{-19} / 1.6 \times 10^{-19}$
$V_o = 2.03$ V

Answer: Cut-off voltage $V_o = 2.03$ V

Q7 The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

SOLUTION

Given: Work function $phi_o = 4.2$ eV
Wavelength of incident radiation lambda = 330 nm = 330×10^{-9} m
Constants: $h = 6.63 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m/s, $e = 1.6 \times 10^{-19}$ C

Energy of incident photon:
$E = hc / lambda = (6.63 \times 10^{-34} \times 3 \times 10^8) / (330 \times 10^{-9})$
$E = 1.989 \times 10^{-25} / 3.30 \times 10^{-7} = 6.027 \times 10^{-19}$ J
$E = 6.027 \times 10^{-19} / 1.6 \times 10^{-19}$ eV = 3.77 eV

Since the photon energy (3.77 eV) is less than the work function (4.2 eV), the photon does not have sufficient energy to eject an electron from the metal.

No photoelectric emission will occur for 330 nm radiation on this metal.

Answer: Photon energy = 3.77 eV < work function 4.2 eV. No photoelectric emission will occur.

Q8 Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^{5} m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

SOLUTION

Given: Incident frequency $\nu = 7.21 \times 10^{14}$ Hz
Maximum speed of ejected electrons $v_{max} = 6.0 \times 10^{5}$ m/s
Constants: $h = 6.63 \times 10^{-34}$ J s, $m_e = 9.11 \times 10^{-31}$ kg

Maximum kinetic energy of photoelectrons:
$K_{max} = (1/2)\, m_e v_{max}^2 = (1/2) \times 9.11 \times 10^{-31} \times (6.0 \times 10^{5})^2$
$K_{max} = (1/2) \times 9.11 \times 10^{-31} \times 3.6 \times 10^{11}$
$K_{max} = 1.64 \times 10^{-19}$ J

Using Einstein's equation: $K_{max} = h\nu - \phi_0$
$\phi_0 = h\nu - K_{max} = 6.63 \times 10^{-34} \times 7.21 \times 10^{14} - 1.64 \times 10^{-19}$
$\phi_0 = 4.780 \times 10^{-19} - 1.64 \times 10^{-19} = 3.14 \times 10^{-19}$ J

Threshold frequency:
$\nu_0 = \phi_0 / h = 3.14 \times 10^{-19} / 6.63 \times 10^{-34}$
$\nu_0 = 4.74 \times 10^{14}$ Hz

Answer: Threshold frequency $\nu_0 = 4.74 \times 10^{14}$ Hz

Q9 Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

SOLUTION

Given: Wavelength lambda = 488 nm = 488×10^{-9} m
Stopping potential $V_0 = 0.38$ V
Constants: $h = 6.63 \times 10^{-34}$ J s, $c = 3 \times 10^{8}$ m/s, $e = 1.6 \times 10^{-19}$ C

Energy of incident photon:
$E = hc / \lambda = (6.63 \times 10^{-34} \times 3 \times 10^{8}) / (488 \times 10^{-9})$
$E = 4.076 \times 10^{-19}$ J = 2.547 eV

Maximum kinetic energy of photoelectrons:
$K_{max} = e \times V_0 = 1.6 \times 10^{-19} \times 0.38 = 6.08 \times 10^{-20}$ J = 0.38 eV

Work function from Einstein's equation:
$\phi_0 = E - K_{max} = 4.076 \times 10^{-19} - 6.08 \times 10^{-20}$
$\phi_0 = 3.468 \times 10^{-19}$ J = 2.17 eV

Answer: Work function $\phi_0 = 3.47 \times 10^{-19}$ J = 2.17 eV

Q10 What is the de Broglie wavelength of (a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s, (b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and (c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

SOLUTION

de Broglie relation: $\lambda = h / p = h / (m \times v)$
Constant: $h = 6.63 \times 10^{-34}$ J s

(a) Bullet: m = 0.040 kg, v = 1.0 km/s = 1.0×10^{3} m/s
$p = m \times v = 0.040 \times 1000 = 40$ kg m/s
$\lambda = h/p = 6.63 \times 10^{-34} / 40 = 1.66 \times 10^{-35}$ m

(b) Ball: m = 0.060 kg, v = 1.0 m/s
$p = m \times v = 0.060 \times 1.0 = 0.060$ kg m/s
$\lambda = h/p = 6.63 \times 10^{-34} / 0.060 = 1.10 \times 10^{-32}$ m

(c) Dust particle: $m = 1.0 \times 10^{-9}$ kg, v = 2.2 m/s
$p = m \times v = 1.0 \times 10^{-9} \times 2.2 = 2.2 \times 10^{-9}$ kg m/s
$\lambda = h/p = 6.63 \times 10^{-34} / 2.2 \times 10^{-9} = 3.01 \times 10^{-25}$ m

All three wavelengths are immeasurably small, explaining why macroscopic objects exhibit no observable wave behaviour.

Answer: (a) $\lambda = 1.66 \times 10^{-35}$ m; (b) $\lambda = 1.10 \times 10^{-32}$ m; (c) $\lambda = 3.01 \times 10^{-25}$ m

Q11 Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

SOLUTION

For a photon of electromagnetic radiation with wavelength lambda:

Energy of photon: E = hc / lambda
Momentum of photon (from relativistic energy-momentum relation with rest mass = 0): p = E / c
Therefore: p = hc / (lambda × c) = h / lambda

de Broglie wavelength of the photon:
lambda_dB = h / p = h / (h / lambda) = lambda

Thus the de Broglie wavelength lambda_dB equals the wavelength lambda of the electromagnetic radiation.

This result is self-consistent: the de Broglie relation lambda = h/p, when applied to a photon using p = h/lambda (from Maxwell's theory of electromagnetic radiation and Compton effect), returns the same wavelength lambda. The particle-wave duality of the photon is built into its own description.

Answer: For a photon: p = h/lambda (electromagnetic momentum), so lambda_dB = h/p = h/(h/lambda) = lambda. Hence the de Broglie wavelength of a photon equals its electromagnetic wavelength.