Chapter 12: Atoms

Exercises — Exercises

Q1 Choose the correct alternative from the clues given at the end of the each statement:
(a) The size of the atom in Thomson's model is ..... the atomic size in Rutherford's model. (much greater than / no different from / much less than.)
(b) In the ground state of ..... electrons are in stable equilibrium, while in ..... electrons always experience a net force. (Thomson's model / Rutherford's model.)
(c) A classical atom based on ..... is doomed to collapse. (Thomson's model / Rutherford's model.)
(d) An atom has a nearly continuous mass distribution in a ..... but has a highly non-uniform mass distribution in ..... (Thomson's model / Rutherford's model.)
(e) The positively charged part of the atom possesses most of the mass in ..... (Rutherford's model / both the models.)

SOLUTION

(a) no different from

In Thomson's model the atom is a sphere of radius ~10^{-10} m with embedded electrons. Rutherford's model also assigns the same overall atomic radius ~10^{-10} m to the atom (the nucleus is much smaller, but the atom itself is the same size). So the atomic size is no different in the two models.

(b) Thomson's model; Rutherford's model

In Thomson's model the electrons sit at equilibrium positions where the attractive pull from the positive sphere exactly cancels, giving stable equilibrium in the ground state. In Rutherford's model the electron moves in a circular orbit and always experiences a centripetal (net) force directed toward the nucleus.

(c) Rutherford's model

According to classical electromagnetic theory, an accelerating charge radiates energy. An electron in circular orbit (Rutherford's model) is continuously accelerating and would lose energy, spiral inward, and eventually collapse into the nucleus. Thomson's model has no such orbital acceleration issue.

(d) Thomson's model; Rutherford's model

Thomson's model distributes positive charge uniformly throughout the atomic volume, giving nearly continuous mass distribution. Rutherford's model concentrates almost all the mass in a tiny nucleus, making the mass distribution

highly non-uniform.

(e) both the models
In Thomson's model the uniform positive charge carries most of the atomic mass (electrons are light). In Rutherford's model the nucleus (which is positively charged) also carries most of the mass. Hence in both models the positively charged part possesses most of the mass.

Answer: (a) no different from; (b) Thomson's model, Rutherford's model; (c) Rutherford's model; (d) Thomson's model, Rutherford's model; (e) both the models

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Q2 Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?

SOLUTION

A hydrogen nucleus is a single proton, with charge +e and mass $m_p = 1.67 \times 10^{-27}$ kg.
An alpha particle has charge +2e and mass $m_{alpha} = 4\ m_p = 6.64 \times 10^{-27}$ kg.

Two key differences compared to the gold-foil experiment:

1. Charge of target nucleus: Gold has Z = 79 and charge 79e. Hydrogen has Z = 1 and charge e. The Coulomb repulsion is vastly smaller, so alpha particles experience far less deflection. Almost all alpha particles will pass straight through with negligible scattering.

2. Mass of target nucleus: The proton (mass m_p) is lighter than the alpha particle (mass 4 m_p). In a head-on collision, by conservation of momentum and kinetic energy, the alpha particle cannot be turned back by a lighter target (in an elastic collision between a heavier projectile and a lighter target, the projectile continues in the forward direction). Therefore, no backscattering (theta > 90 degrees) is expected.

Result: The alpha particles will pass through the hydrogen sheet almost undeflected. Large-angle scattering will be essentially absent, and backscattering will not occur. This contrasts sharply with the gold foil result where ~1 in 8000 alpha particles rebounded back.

Answer: No large-angle scattering and no backscattering. Almost all alpha particles pass through with minimal deflection, because the hydrogen nucleus (proton) has a very small charge (+e vs. +79e for gold) and is lighter than the alpha particle, so it cannot deflect the heavier projectile backward.

Q3 A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?

SOLUTION

Given: Energy difference between levels, Delta E = 2.3 eV
Constants: h = 6.63 × 10^{-34} J s, 1 eV = 1.6 × 10^{-19} J

Converting energy to joules:
Delta E = 2.3 × 1.6 × 10^{-19} = 3.68 × 10^{-19} J

Using Bohr's frequency condition: h * nu = Delta E
nu = Delta E / h = (3.68 × 10^{-19}) / (6.63 × 10^{-34})
nu = 5.55 × 10^{14} Hz

Answer: 5.55 × 10^{14} Hz

Q4 The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of the electron in this state?

SOLUTION

Given: Total energy in ground state, E_1 = -13.6 eV

For an electron in a circular orbit under Coulomb attraction, the virial theorem gives:
Kinetic energy K = -E_total
Potential energy U = 2 E_total

(These follow from: K = e^2/(8*pi*eps0*r) and U = -e^2/(4*pi*eps0*r) = -2K, so E = K + U = K - 2K = -K.)

Kinetic energy: K = -(-13.6) = +13.6 eV
Potential energy: U = 2 × (-13.6) = -27.2 eV

Check: K + U = 13.6 + (-27.2) = -13.6 eV = E_1. Correct.

Answer: Kinetic energy = +13.6 eV; Potential energy = -27.2 eV

Q5 A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of the photon.

SOLUTION

Given: Initial state $n_i = 1$, final state $n_f = 4$
Constants: $h = 6.63 \times 10^{-34}$ J s, $c = 3 \times 10^{8}$ m/s, 1 eV = 1.6×10^{-19} J

Energy of nth level: $E_n = -13.6 / n^2$ eV
$E_1 = -13.6$ eV
$E_4 = -13.6 / 16 = -0.85$ eV

Energy of photon absorbed:
$\Delta E = E_4 - E_1 = -0.85 - (-13.6) = 12.75$ eV
$\Delta E = 12.75 \times 1.6 \times 10^{-19} = 2.04 \times 10^{-18}$ J

Frequency of photon:
$\nu = \Delta E / h = (2.04 \times 10^{-18}) / (6.63 \times 10^{-34}) = 3.08 \times 10^{15}$ Hz

Wavelength of photon:
$\lambda = c / \nu = (3 \times 10^{8}) / (3.08 \times 10^{15}) = 9.74 \times 10^{-8}$ m = 97.4 nm

This wavelength lies in the ultraviolet region (Lyman series).

Answer: Wavelength = 9.74×10^{-8} m (97.4 nm, UV); Frequency = 3.08×10^{15} Hz

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Q6 (a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the n = 1, 2, and 3 levels. (b) Calculate the orbital period in each of these levels.

SOLUTION

Given: Bohr radius $a_0 = 5.3 \times 10^{-11}$ m
Constants: $e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ C²/(N m²), $h = 6.63 \times 10^{-34}$ J s

PART (a) — Electron speeds

From Bohr's quantisation condition: $m v_n r_n = n h/(2\pi)$
Combined with orbital radius $r_n = n^2 a_0$, the speed in level n is:
$v_n = v_1 / n$, where $v_1 = e^2 / (2 \varepsilon_0 h)$

$v_1 = (1.6 \times 10^{-19})^2 / (2 \times 8.85 \times 10^{-12} \times 6.63 \times 10^{-34})$
$v_1 = 2.56 \times 10^{-38} / 1.174 \times 10^{-44} = 2.18 \times 10^{6}$ m/s

$v_1 = 2.18 \times 10^{6}$ m/s
$v_2 = v_1 / 2 = 1.09 \times 10^{6}$ m/s

$v_3 = v_1 / 3 = 7.27 \times 10^5$ m/s

PART (b) — Orbital periods

T_n = 2*pi*r_n / v_n = 2*pi*(n² a_0) / (v_1/n) = (2*pi*a_0/v_1) * n^3

$T_1 = 2 * pi * (5.3 \times 10^{-11}) / (2.18 \times 10^6) = 1.53 \times 10^{-16}$ s
$T_2 = T_1 \times 2^3 = 1.53 \times 10^{-16} \times 8 = 1.22 \times 10^{-15}$ s
$T_3 = T_1 \times 3^3 = 1.53 \times 10^{-16} \times 27 = 4.14 \times 10^{-15}$ s

Answer: (a) $v_1 = 2.18 \times 10^6$ m/s, $v_2 = 1.09 \times 10^6$ m/s, $v_3 = 7.27 \times 10^5$ m/s; (b) $T_1 = 1.53 \times 10^{-16}$ s, $T_2 = 1.22 \times 10^{-15}$ s, $T_3 = 4.14 \times 10^{-15}$ s

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Q7 The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the n = 2 and n = 3 orbits?

SOLUTION

Given: Bohr radius $a_0 = r_1 = 5.3 \times 10^{-11}$ m

From Bohr's model, the radius of the nth orbit is:
r_n = n² a_0

For n = 2:
$r_2 = 2^2 \times 5.3 \times 10^{-11} = 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10}$ m

For n = 3:
$r_3 = 3^2 \times 5.3 \times 10^{-11} = 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10}$ m

Answer: $r_2 = 2.12 \times 10^{-10}$ m; $r_3 = 4.77 \times 10^{-10}$ m

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Q8 A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

SOLUTION

Given: Electron beam energy = 12.5 eV

At room temperature, hydrogen atoms are in the ground state (n = 1).

Energy required to excite from n=1 to higher levels:
n=1 to n=2: $E_2 - E_1 = (-3.4) - (-13.6) = 10.2$ eV
n=1 to n=3: $E_3 - E_1 = (-1.51) - (-13.6) = 12.09$ eV
n=1 to n=4: $E_4 - E_1 = (-0.85) - (-13.6) = 12.75$ eV

The 12.5 eV beam can supply up to 12.5 eV. It can excite atoms to n = 3 (needs

12.09 eV) but NOT to n = 4 (needs 12.75 eV).

So electrons in the beam excite hydrogen to a maximum of n = 3.

Possible transitions and wavelengths:

n = 3 to n = 1 (Lyman series):
Delta E = 12.09 eV, lambda = hc / (12.09 × 1.6 × 10^{-19}) = 102.8 nm (UV)

n = 3 to n = 2 (Balmer series):
Delta E = 1.89 eV, lambda = hc / (1.89 × 1.6 × 10^{-19}) = 657.7 nm (visible red)

n = 2 to n = 1 (Lyman series):
Delta E = 10.2 eV, lambda = hc / (10.2 × 1.6 × 10^{-19}) = 121.9 nm (UV)

Wavelengths emitted: 102.8 nm and 121.9 nm (Lyman series, UV) and 657.7 nm (Balmer series, visible).

Answer: Three wavelengths are emitted: 121.9 nm (Lyman, n=2→1), 102.8 nm (Lyman, n=3→1), and 657.7 nm (Balmer, n=3→2). Lyman series (UV) and Balmer series (visible) are both present.

Q9 In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5 × 10^{11} m with orbital speed 3 × 10^{4} m/s. (Mass of earth = 6.0 × 10^{24} kg.)

SOLUTION

Given: r = 1.5 × 10^{11} m, v = 3 × 10^{4} m/s, m = 6.0 × 10^{24} kg
Constants: h = 6.63 × 10^{-34} J s

From Bohr's quantisation condition:
L = m v r = n h / (2*pi)

Angular momentum of Earth:
L = m v r = 6.0 × 10^{24} × 3 × 10^{4} × 1.5 × 10^{11}
L = 2.70 × 10^{40} J s

Quantum number n:
n = 2*pi*L / h = (2 × 3.14159 × 2.70 × 10^{40}) / (6.63 × 10^{-34})
n = (1.696 × 10^{41}) / (6.63 × 10^{-34})
n = 2.56 × 10^{74}

This is an astronomically large quantum number, consistent with the correspondence principle — at such large n, quantum and classical mechanics

give the same predictions, which is why Earth's orbit looks continuous and classical.

Answer: $n \approx 2.56 \times 10^{74}$