Chapter 13: Nuclei

Exercises — Exercises

Q1 Obtain the binding energy (in MeV) of a nitrogen nucleus ($^{14}_{7}$N), given m($^{14}_{7}$N) = 14.00307 u.

SOLUTION

Given: m($^{14}_{7}$N) = 14.00307 u, Z = 7 (protons), A = 14, N = 7 (neutrons).
Standard data: m_H = 1.007825 u, m_n = 1.008665 u, 1 u = 931.5 MeV/c².

Step 1 — Mass defect (using atomic masses; electron masses cancel automatically):
Delta_M = Z × m_H + (A - Z) × m_n - m($^{14}_{7}$N)
Delta_M = 7 × 1.007825 + 7 × 1.008665 - 14.00307
Delta_M = 7.054775 + 7.060655 - 14.00307
Delta_M = 14.115430 - 14.00307 = 0.112360 u

Step 2 — Binding energy:
E_b = Delta_M × 931.5 MeV/u
E_b = 0.112360 × 931.5 = 104.66 MeV

Binding energy per nucleon = 104.66 / 14 = 7.48 MeV

Answer: Binding energy of $^{14}_{7}$N nucleus = 104.66 MeV (≈ 7.48 MeV per nucleon)

Q2 Obtain the binding energy of the nuclei 56_26 Fe and 209_83 Bi in units of MeV from the following data: m(56_26 Fe) = 55.934939 u, m(209_83 Bi) = 208.980388 u.

SOLUTION

Given: m_H = 1.007825 u, m_n = 1.008665 u, 1 u = 931.5 MeV/c².

--- Fe-56 (Z = 26, A = 56, N = 30) ---
Delta_M(Fe) = 26 × 1.007825 + 30 × 1.008665 - 55.934939
= 26.20345 + 30.25995 - 55.934939
= 56.46340 - 55.934939 = 0.528461 u
E_b(Fe) = 0.528461 × 931.5 = 492.26 MeV
Binding energy per nucleon = 492.26 / 56 = 8.79 MeV

--- Bi-209 (Z = 83, A = 209, N = 126) ---
Delta_M(Bi) = 83 × 1.007825 + 126 × 1.008665 - 208.980388

= 83.64948 + 127.09179 - 208.980388
= 210.74127 - 208.980388 = 1.760882 u
E_b(Bi) = 1.760882 × 931.5 = 1640.26 MeV
Binding energy per nucleon = 1640.26 / 209 = 7.85 MeV

Answer: E_b(56_26 Fe) = 492.26 MeV (8.79 MeV/nucleon); E_b(209_83 Bi) = 1640.26 MeV (7.85 MeV/nucleon)

Q3 A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of 63_29 Cu atoms (of mass 62.92960 u).

SOLUTION

Given: mass of coin = 3.0 g, m(63_29 Cu) = 62.92960 u.
Z = 29, A = 63, N = 34 (neutrons). m_H = 1.007825 u, m_n = 1.008665 u.

Step 1 — Mass defect per Cu-63 nucleus:
Delta_M = 29 × 1.007825 + 34 × 1.008665 - 62.92960
= 29.22693 + 34.29461 - 62.92960
= 63.52154 - 62.92960 = 0.59194 u

Step 2 — Binding energy per nucleus:
E_b = 0.59194 × 931.5 = 551.4 MeV

Step 3 — Number of Cu-63 atoms in 3.0 g coin:
N_A = 6.023 × 10^{23} mol^{-1}
n = (3.0 / 62.92960) × 6.023 × 10^{23} = 2.871 × 10^{22} atoms

Step 4 — Total nuclear energy required:
E_total = n × E_b = 2.871 × 10^{22} × 551.4 MeV
= 1.583 × 10^{25} MeV
= 1.583 × 10^{25} × 1.6 × 10^{-13} J
= 2.53 × 10^{12} J

Answer: Nuclear energy required = 2.53 × 10^{12} J (≈ 1.58 × 10^{25} MeV)

Q4 Obtain approximately the ratio of the nuclear radii of the gold isotope 197_79 Au and the silver isotope 107_47 Ag.

SOLUTION

Given: A(Au) = 197, A(Ag) = 107. Nuclear radius formula: R = R_0 × A^(1/3).

Step 1 — Write the ratio:
R(Au) / R(Ag) = [R_0 × (197)^(1/3)] / [R_0 × (107)^(1/3)]
= (197/107)^(1/3)

Step 2 — Compute:
(197/107)^(1/3) = (1.8411)^(1/3)
= 1.2256

Rounding: R(Au)/R(Ag) ≈ 1.23

Answer: R(197_79 Au) / R(107_47 Ag) = (197/107)^(1/3) ≈ 1.23

Q5 The Q value of a nuclear reaction A + b -> C + d is defined by Q = [m_A + m_b - m_C - m_d]c^2 where the masses refer to the respective nuclei. Determine the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.
(i) 1_1 H + 3_1 H -> 2_1 H + 2_1 H
(ii) 12_6 C + 12_6 C -> 20_10 Ne + 4_2 He
Atomic masses: m(2_1 H) = 2.014102 u, m(3_1 H) = 3.016049 u, m(12_6 C) = 12.000000 u, m(20_10 Ne) = 19.992439 u.

SOLUTION

Standard data: m_H = m(1_1 H) = 1.007825 u, m(4_2 He) = 4.002603 u, 1 u = 931.5 MeV/c^2.
Using atomic masses throughout (electron masses cancel on both sides).

--- (i) 1_1 H + 3_1 H -> 2_1 H + 2_1 H ---
Q = [m(1_1 H) + m(3_1 H) - 2 × m(2_1 H)] × 931.5 MeV/u
= [1.007825 + 3.016049 - 2 × 2.014102] × 931.5
= [4.023874 - 4.028204] × 931.5
= (-0.004330 u) × 931.5
= -4.034 MeV
Q < 0 => Endothermic reaction (energy must be supplied).

--- (ii) 12_6 C + 12_6 C -> 20_10 Ne + 4_2 He ---

Q = [2 × m(12₆ C) - m(20_10 Ne) - m(4₂ He)] × 931.5 MeV/u
= [2 × 12.000000 - 19.992439 - 4.002603] × 931.5
= [24.000000 - 23.995042] × 931.5
= (0.004958 u) × 931.5
= 4.618 MeV
Q > 0 => Exothermic reaction (energy is released).

Answer: (i) Q = -4.034 MeV (endothermic); (ii) Q = +4.618 MeV (exothermic)

Q6 Suppose, we think of fission of a 56_26 Fe nucleus into two equal fragments, 28_13 Al. Is the fission energetically possible? Argue by working out Q of the process. Given m(56_26 Fe) = 55.93494 u and m(28_13 Al) = 27.98191 u.

SOLUTION

Given: m(56_26 Fe) = 55.93494 u, m(28_13 Al) = 27.98191 u.
Reaction: 56_26 Fe -> 28_13 Al + 28_13 Al

Step 1 — Q-value calculation:
Q = [m(56_26 Fe) - 2 × m(28_13 Al)] × 931.5 MeV/u
= [55.93494 - 2 × 27.98191] × 931.5
= [55.93494 - 55.96382] × 931.5
= (-0.02888 u) × 931.5
= -26.90 MeV

Step 2 — Interpretation:
Q < 0 means the reaction is endothermic — energy must be supplied to the Fe nucleus to split it into two Al fragments. The fission of Fe-56 is therefore NOT energetically possible spontaneously.

This is consistent with the binding energy curve: Fe-56 lies near the peak (~8.75 MeV/nucleon). Splitting it into lighter Al-28 nuclei (which have a lower binding energy per nucleon) would increase the total mass, requiring external energy input.

Answer: Q = -26.90 MeV < 0. Fission of 56_26 Fe into two 28_13 Al fragments is energetically NOT possible (endothermic).

Q7 The fission properties of 239_94 Pu are very similar to those of 235_92 U. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure 239_94 Pu undergo fission?

SOLUTION

Given: energy per fission = 180 MeV, mass of Pu-239 = 1 kg, molar mass = 239 g/mol.
$N_A = 6.023 \times 10^{23}$ mol^{-1}.

Step 1 — Number of Pu-239 atoms in 1 kg:
$n = (1000 \text{ g} / 239 \text{ g/mol}) \times 6.023 \times 10^{23}$
$= 4.181 \text{ mol} \times 6.023 \times 10^{23}$
$= 2.520 \times 10^{24}$ atoms

Step 2 — Total energy released:
$E_{total} = 2.520 \times 10^{24} \times 180$ MeV
$= 4.536 \times 10^{26}$ MeV

Step 3 — Convert to joules (for reference):
$E_{total} = 4.536 \times 10^{26} \times 1.6 \times 10^{-13}$ J
$= 7.26 \times 10^{13}$ J

Answer: Total energy released = 4.54×10^{26} MeV (= 7.26×10^{13} J)

Q8 How long can an electric lamp of 100 W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as $2_1H + 2_1H \rightarrow 3_2He + n + 3.27$ MeV.

SOLUTION

Given: P = 100 W, mass of deuterium = 2.0 kg, molar mass of D = 2.014102 g/mol.
Energy per fusion reaction = 3.27 MeV.

Step 1 — Number of D-2 atoms in 2.0 kg:
$n_D = (2000 \text{ g} / 2.014102 \text{ g/mol}) \times 6.023 \times 10^{23}$
$= 993.0 \text{ mol} \times 6.023 \times 10^{23}$
$= 5.981 \times 10^{26}$ atoms

Step 2 — Number of fusion reactions (each uses 2 deuterium atoms):
$n_{reactions} = 5.981 \times 10^{26} / 2 = 2.990 \times 10^{26}$

Step 3 — Total energy released:
$E_{total} = 2.990 \times 10^{26} \times 3.27$ MeV

= 9.779 × 10^{26} MeV
= 9.779 × 10^{26} × 1.6 × 10^{-13} J
= 1.565 × 10^{14} J

Step 4 — Time for 100 W lamp:
t = E_total / P = 1.565 × 10^{14} J / 100 W
= 1.565 × 10^{12} s

Step 5 — Convert to years (1 year = 3.154 × 10^{7} s):
t = 1.565 × 10^{12} / 3.154 × 10^{7}
= 4.96 × 10^{4} years ≈ 4.9 × 10^{4} years

Answer: The lamp can glow for ≈ 4.9 × 10^{4} years (total energy = 1.56 × 10^{14} J)

Q9 Calculate the height of the potential barrier for a head-on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

SOLUTION

Given: radius of each deuteron r = 2.0 fm = 2.0 × 10^{-15} m.
Charge of each deuteron = e = 1.6 × 10^{-19} C (each has Z = 1).
k = 1/(4pi*epsilon$_0$) = 9 × 10^{9} N m^2 C^{-2}.

Step 1 — Distance between centres when they just touch:
d = r + r = 2 × 2.0 fm = 4.0 fm = 4.0 × 10^{-15} m

Step 2 — Coulomb potential energy at this separation:
U = k × e × e / d
= (9 × 10^{9} × (1.6 × 10^{-19})^2) / (4.0 × 10^{-15})
= (9 × 10^{9} × 2.56 × 10^{-38}) / (4.0 × 10^{-15})
= (2.304 × 10^{-28}) / (4.0 × 10^{-15})
= 5.76 × 10^{-14} J

Step 3 — Convert to MeV:
U = 5.76 × 10^{-14} / 1.6 × 10^{-13} MeV
= 0.360 MeV = 360 keV

Answer: Height of Coulomb potential barrier = 360 keV (= 5.76 × 10^{-14} J)

Q10 From the relation R = R_0 A^(1/3), where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

SOLUTION

Given: R = R_0 × A^(1/3), where R_0 = 1.2 × 10^{-15} m (= 1.2 fm).
The mass of a nucleus of mass number A is approximately m = A × u, where u = 1.66 × 10^{-27} kg.

Step 1 — Volume of nucleus (assume spherical):
V = (4/3) × pi × R^3
= (4/3) × pi × (R_0 A^(1/3))^3
= (4/3) × pi × R_0^3 × A

Step 2 — Nuclear density:
rho = m / V
= (A × u) / [(4/3) pi R_0^3 A]
= u / [(4/3) pi R_0^3]

The factor A cancels completely from numerator and denominator.
Therefore rho is independent of A — it is the same for all nuclei.

Step 3 — Numerical value:
rho = (1.66 × 10^{-27} kg) / [(4/3) × pi × (1.2 × 10^{-15} m)^3]
= (1.66 × 10^{-27}) / (7.24 × 10^{-45})
= 2.29 × 10^{17} kg m^{-3}

This is approximately 10^{14} times the density of water, and is the same for all nuclear species.

Answer: Nuclear density rho = u / [(4/3)pi R_0^3] = 2.29 × 10^{17} kg m^{-3}, independent of A (A cancels out exactly).