Chapter 13: Animal Jumps

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Exercise 13.1 — Find the Hidden Numbers — Factors and Multiples Introduction (p. 164)

Q1 Numbers are put in a box, multiplied by a number, and come out as 28, 36, 48, and 72.
(a) Can you guess the multiplier if you see the 4 numbers coming out of the box?
(b) Is there more than one possible multiplier?
(c) What numbers might have been put inside the box?

SOLUTION

Find all common factors of 28, 36, 48, and 72 — any of these could be the multiplier.

Factors of 28: 1, 2, 4, 7, 14, 28
Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
Factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Common factors of all four numbers: 1, 2, 4.

(a) Possible multipliers are 1, 2, or 4.
(b) Yes — more than one multiplier is possible (1, 2, and 4 all work).
(c) If multiplier = 4: inputs are 7, 9, 12, 18.
If multiplier = 2: inputs are 14, 18, 24, 36.
If multiplier = 1: inputs are 28, 36, 48, 72 (same as output).

Answer: (a) The multiplier could be 1, 2, or 4. (b) Yes, more than one multiplier is possible. (c) Multiplier 4 => inputs: 7, 9, 12, 18; multiplier 2 => inputs: 14, 18, 24, 36.

Exercise 13.2 — Fill in the Blanks — Factors of 12 (p. 165)

Q1 Fill in the blanks to complete the multiplication sentences for 12.

2 x ___ = 12

12 x ___ = 12

3 x ___ = 12

1 x ___ = 12

SOLUTION

2 x 6 = 12 (because 12 / 2 = 6)

12 x 1 = 12 (because 12 / 12 = 1)

3 x 4 = 12 (because 12 / 3 = 4)

1 x 12 = 12 (because 12 / 1 = 12)

Answer: 2 x 6 = 12; 12 x 1 = 12; 3 x 4 = 12; 1 x 12 = 12

Exercise 13.3 — Let Us Do — Arrays and Factors (p. 165)

Q1 Make different arrays for the following numbers. Identify the factors in each case.
(a) 10 (b) 14 (c) 13 (d) 20 (e) 25 (f) 32 (g) 37 (h) 46 (i) 54

SOLUTION

(a) 10: Arrays — 1x10, 2x5, 5x2, 10x1
Factors of 10: 1, 2, 5, 10

(b) 14: Arrays — 1x14, 2x7, 7x2, 14x1
Factors of 14: 1, 2, 7, 14

(c) 13: Arrays — only 1x13 and 13x1
Factors of 13: 1, 13
(13 is a prime number — only 2 factors.)

(d) 20: Arrays — 1x20, 2x10, 4x5, 5x4, 10x2, 20x1
Factors of 20: 1, 2, 4, 5, 10, 20

(e) 25: Arrays — 1x25, 5x5, 25x1
Factors of 25: 1, 5, 25

(f) 32: Arrays — 1x32, 2x16, 4x8, 8x4, 16x2, 32x1
Factors of 32: 1, 2, 4, 8, 16, 32

(g) 37: Arrays — only 1x37 and 37x1
Factors of 37: 1, 37
(37 is a prime number — only 2 factors.)

(h) 46: Arrays — 1x46, 2x23, 23x2, 46x1
Factors of 46: 1, 2, 23, 46

(i) 54: Arrays — 1x54, 2x27, 3x18, 6x9, 9x6, 18x3, 27x2, 54x1
Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

Numbers like 13 and 37 are called prime numbers because they have exactly two factors — 1 and the number itself. They cannot be arranged as a rectangular array with more than one row.

Answer: (a) 1,2,5,10 (b) 1,2,7,14 (c) 1,13 (prime) (d) 1,2,4,5,10,20 (e) 1,5,25 (f) 1,2,4,8,16,32 (g) 1,37 (prime) (h) 1,2,23,46 (i) 1,2,3,6,9,18,27,54

Exercise 13.4 — Let Us Do — Common Multiples (p. 166)

Q1 Find 5 common multiples of the following pairs of numbers.
(a) 2 and 3 (b) 5 and 8 (c) 2 and 4 (d) 3 and 9 (e) 5 and 10 (f) 8 and 12 (g) 6 and 8 (h) 9 and 12 (f) 6 and 9

SOLUTION

A common multiple is a number that appears in the times table of BOTH numbers.

(a) 2 and 3: 6, 12, 18, 24, 30
(Multiples of 2: 2,4,6,...; multiples of 3: 3,6,9,... — both touch 6,12,18,...)

(b) 5 and 8: 40, 80, 120, 160, 200
(Smallest common multiple = 5x8 = 40, since gcd(5,8)=1)

(c) 2 and 4: 4, 8, 12, 16, 20
(Every multiple of 4 is also a multiple of 2)

(d) 3 and 9: 9, 18, 27, 36, 45
(Every multiple of 9 is also a multiple of 3)

(e) 5 and 10: 10, 20, 30, 40, 50
(Every multiple of 10 is also a multiple of 5)

(f) 8 and 12: 24, 48, 72, 96, 120
(Smallest common multiple = 24, since 8x3=24 and 12x2=24)

(g) 6 and 8: 24, 48, 72, 96, 120
(6x4=24 and 8x3=24)

(h) 9 and 12: 36, 72, 108, 144, 180
(9x4=36 and 12x3=36)

(f) 6 and 9: 18, 36, 54, 72, 90
(6x3=18 and 9x2=18)

What do you notice? When one number is a multiple of the other (like 2 and 4, or 3 and 9), all multiples of the larger one are the common multiples. When neither divides the other, the common multiples are larger.

Answer: (a) 6,12,18,24,30 (b) 40,80,120,160,200 (c) 4,8,12,16,20 (d) 9,18,27,36,45 (e) 10,20,30,40,50 (f) 24,48,72,96,120 (g) 24,48,72,96,120 (h) 36,72,108,144,180 (f/9) 18,36,54,72,90

Q2 Food is available at the end of a cobbled road. Robby, the rabbit, takes a jump of 4 each time. Deeku, the deer, takes a jump of 6 each time. They both start at 0. Will both Robby and Deeku reach the food? Who will reach first? How do you know? Explain your answer.

SOLUTION

The cobbled road goes from 0 to 64 (food is at position 64).

Robby (jumps of 4) lands on: 4, 8, 12, 16, ..., 60, 64.
64 / 4 = 16 — YES, Robby reaches 64 in exactly 16 jumps.

Deeku (jumps of 6) lands on: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, ...
64 / 6 = 10 remainder 4 — NO, Deeku cannot reach 64 exactly.
64 is NOT a multiple of 6.

So only Robby the rabbit reaches the food. Deeku the deer never lands on 64.

Answer: Only Robby the rabbit reaches the food. 64 is a multiple of 4 (4 x 16 = 64), so Robby lands there in 16 jumps. 64 is not a multiple of 6 (64 / 6 = 10 r 4), so Deeku never lands on it.

Q3 Mowgli's friends live along a trail at marked places. Which of his friends will he be able to visit, if he jumps by 2 steps starting from 0?
Then: which friends will he meet jumping by 3 steps? By 5 steps? By 10 steps?

SOLUTION

Jumping by 2 from 0 means landing on 0, 2, 4, 6, 8, 10, ... (all even numbers).
The friends standing on these even places are: ant at 4, frog at 12, bird at 14, rabbit at 50.
So Mowgli can visit the ant, the frog, the bird, and the rabbit.
2 is a common factor of 4, 12, 14, and 50.

Jumping by 3: lands on 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, ...
Marked places on multiples of 3: 9 (spider), 21 (snake), 39 (deer), 57 (monkey).
3 is a common factor of 9, 21, 39, and 57.

Jumping by 5: lands on 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55.
5 is a common factor of 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55.

Jumping by 10: lands on 10, 20, 30, 40, 50.
10 is a common factor of 10, 20, 30, 40, 50.

Answer: Jump 2: meets friends at 4 (ant), 12 (frog), 14 (bird), 50 (rabbit). Jump 3: meets friends at 9, 21, 39, 57. Jump 5: lands on 5,10,15,20,25,30,35,40,45,50,55. Jump 10: lands on 10,20,30,40,50.

Q4 Find some common factors of 24 and 36.
(a) Can we jump by 2 steps at a time to reach both 24 and 36? Yes/No. 2 is/is not a common factor of 24 and 36.
(b) Can we jump by 3 steps at a time to reach both 24 and 36? Yes/No.
(c) Can we jump by 4 steps at a time to reach both 24 and 36? Yes/No.
(d) What other jumps can we take to reach both 24 and 36?
(e) How many common factors can you find for 24 and 36? List them.
(f) What about jumping by 1 step each time to reach both 24 and 36?

SOLUTION

(a) Jump by 2: 24/2=12 (yes), 36/2=18 (yes). YES — 2 IS a common factor of 24 and 36.

(b) Jump by 3: 24/3=8 (yes), 36/3=12 (yes). YES — 3 IS a common factor of 24 and 36.

(c) Jump by 4: 24/4=6 (yes), 36/4=9 (yes). YES — 4 IS a common factor of 24 and 36.

(d) Other jumps that reach both 24 and 36:
Jump 6: 24/6=4 (yes), 36/6=6 (yes). 6 is a common factor.
Jump 12: 24/12=2 (yes), 36/12=3 (yes). 12 is a common factor.
Jump 1: 1 divides every number. 1 is always a common factor.

(e) All common factors of 24 and 36: 1, 2, 3, 4, 6, 12.
Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
Common: 1, 2, 3, 4, 6, 12 — that is 6 common factors.

(f) Jumping by 1: we land on every number, so YES — we always reach 24 and 36.
1 is a common factor of every pair of numbers.

Answer: (a) Yes — 2 IS a common factor. (b) Yes — 3 IS. (c) Yes — 4 IS. (d) Also 6, 12, and 1. (e) Common factors of 24 and 36: 1, 2, 3, 4, 6, 12. (f) Yes — 1 always works.

Q5 What are the common factors of 12 and 13?

SOLUTION

Factors of 12: 1, 2, 3, 4, 6, 12
Factors of 13: 1, 13 (13 is a prime number)

Common factors = numbers that appear in both lists: only 1.
The only common factor of 12 and 13 is 1.

Note: The number 1 and the number itself are always factors of any number.

Answer: The only common factor of 12 and 13 is 1.

Q6 Find which of the following numbers can be reached by jumps of 4 steps.
0, 10, 16, 27, 36, 48
4 is the common factor of the numbers ___.

SOLUTION

Check which numbers are multiples of 4 (exactly divisible by 4):
0: 0/4 = 0 — yes (but 0 is the starting point)
10: 10/4 = 2 remainder 2 — NO
16: 16/4 = 4 — YES
27: 27/4 = 6 remainder 3 — NO
36: 36/4 = 9 — YES
48: 48/4 = 12 — YES

Numbers reachable by jumps of 4: 0, 16, 36, 48.
4 is a common factor of 16, 36, and 48.

Answer: Numbers reachable by jumps of 4: 0, 16, 36, 48. 4 is the common factor of 16, 36, and 48.

Q7 Find the common factors of the following pairs of numbers.
(a) 12 and 16 (b) 8 and 12 (c) 4 and 16 (d) 2 and 9 (e) 3 and 5 (f) 12 and 15 (f) 20 and 5 (g) 9 and 21 (h) 6 and 27

SOLUTION

(a) 12 and 16:
Factors of 12: 1,2,3,4,6,12 | Factors of 16: 1,2,4,8,16
Common factors: 1, 2, 4

(b) 8 and 12:
Factors of 8: 1,2,4,8 | Factors of 12: 1,2,3,4,6,12
Common factors: 1, 2, 4

(c) 4 and 16:
Factors of 4: 1,2,4 | Factors of 16: 1,2,4,8,16
Common factors: 1, 2, 4

(d) 2 and 9:
Factors of 2: 1,2 | Factors of 9: 1,3,9
Common factors: 1

(e) 3 and 5:
Factors of 3: 1,3 | Factors of 5: 1,5
Common factors: 1

(f) 12 and 15:
Factors of 12: 1,2,3,4,6,12 | Factors of 15: 1,3,5,15
Common factors: 1, 3

(f) 20 and 5:
Factors of 20: 1,2,4,5,10,20 | Factors of 5: 1,5
Common factors: 1, 5

(g) 9 and 21:
Factors of 9: 1,3,9 | Factors of 21: 1,3,7,21
Common factors: 1, 3

(h) 6 and 27:
Factors of 6: 1,2,3,6 | Factors of 27: 1,3,9,27
Common factors: 1, 3

What do you notice? Pairs where both numbers share no common factor except 1 (like 2 and 9, or 3 and 5) are called co-prime pairs.

Answer: (a) 1,2,4 (b) 1,2,4 (c) 1,2,4 (d) 1 (e) 1 (f/12,15) 1,3 (f/20,5) 1,5 (g) 1,3 (h) 1,3

Q8 State whether the following statements are true (T) or false (F).
(a) Factors of even numbers must be even.
(b) Multiples of odd numbers cannot be even.
(c) Factors of odd numbers cannot be even.
(d) One of the common multiples of two consecutive numbers is their product.
(e) The only common factor of any two consecutive numbers is 1.
(f) 0 cannot be a factor of any number.

SOLUTION

(a) FALSE.
Example: factors of 6 include 1 and 3, which are odd. Even numbers can have odd factors.

(b) FALSE.
Example: 3 x 2 = 6 (even), so 6 is a multiple of the odd number 3.

(c) TRUE.
An odd number cannot be divided exactly by any even number.
(If an even number divided an odd number, the result would not be a whole number.)

(d) TRUE.
For any two consecutive numbers, say n and (n+1), their product n x (n+1) is divisible by both n and (n+1).
Example: 4 and 5 are consecutive. 4 x 5 = 20. 20/4 = 5 and 20/5 = 4. So 20 is a common multiple.

(e) TRUE.
Consecutive numbers (like 8 and 9, or 15 and 16) share no common factor except 1.
Any common factor would have to divide their difference, which is 1 — so the only common factor is 1.

(f) TRUE.

Division by 0 is not defined. 0 cannot divide any number, so 0 is not a factor of any number.

Answer: (a) F (b) F (c) T (d) T (e) T (f) T

Q9 Sher Khan, the tiger, goes hunting every 3rd day. Bagheera, the panther, goes hunting every 5th day. If both of them start on the same day, on which days will they be hunting together?

SOLUTION

They hunt together on days that are multiples of BOTH 3 and 5.

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ...
Multiples of 5: 5, 10, 15, 20, 25, 30, 35, ...

Common multiples: 15, 30, 45, 60, 75, ...

They will hunt together on Day 15, Day 30, Day 45, Day 60, Day 75, and so on — every 15 days.

Answer: They hunt together on Day 15, Day 30, Day 45, Day 60, ... (every 15 days, since 15 is the smallest common multiple of 3 and 5).

Q10 (a) In the trail, Sher Khan's house is on number 25 and Baloo the bear is on number 30. Mowgli wants to meet Baloo but avoid Sher Khan's house. How long (in steps) could each jump be?
(b) What number of jumps (in steps) could he choose so that he can meet both Kaa, the snake, at 21 and Akela, the wolf, at 35?

SOLUTION

(a) Mowgli needs a jump size that is a factor of 30 (reaches Baloo) but NOT a factor of 25 (skips Sher Khan).
Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
Factors of 25: 1, 5, 25
Jump sizes that reach 30 but NOT 25 (i.e. not in both lists): 2, 3, 6, 10, 15, 30
(1 and 5 are removed because they are also factors of 25.)

(b) Mowgli needs a jump size that reaches BOTH 21 and 35.
Factors of 21: 1, 3, 7, 21
Factors of 35: 1, 5, 7, 35
Common factors of 21 and 35: 1 and 7.
So he can jump by 1 or by 7 steps.
Jump of 7: 7, 14, 21, 28, 35 — reaches both 21 and 35. ✓

Answer: (a) Jump sizes that reach 30 but avoid 25: 2, 3, 6, 10, 15, or 30 steps. (b) Jump of 1 or 7 steps reaches both 21 and 35. Jump of 7 is the most practical (21 = 7x3, 35 = 7x5).

Q11 Sort the following numbers into those that are—
(a) divisible by 2 only
(b) divisible by 5 only
(c) divisible by 10 only
(d) divisible by 2, 5, and 10.
Numbers: 90, 22, 38, 30, 75, 45, 66, 78, 62, 40, 84, 56, 25, 95, 55

SOLUTION

A number divisible by 10 is always divisible by both 2 and 5 (since 10 = 2 x 5).
So the 'divisible by 10 only' region in the Venn diagram will be empty.

Check each number:
22: div by 2? Yes (22/2=11). By 5? No. By 10? No. => divisible by 2 only
38: div by 2? Yes. By 5? No. By 10? No. => divisible by 2 only
56: div by 2? Yes. By 5? No. By 10? No. => divisible by 2 only
62: div by 2? Yes. By 5? No. By 10? No. => divisible by 2 only
66: div by 2? Yes. By 5? No. By 10? No. => divisible by 2 only
78: div by 2? Yes. By 5? No. By 10? No. => divisible by 2 only
84: div by 2? Yes. By 5? No. By 10? No. => divisible by 2 only

25: div by 5? Yes (25/5=5). By 2? No. By 10? No. => divisible by 5 only
45: div by 5? Yes. By 2? No. By 10? No. => divisible by 5 only
55: div by 5? Yes. By 2? No. By 10? No. => divisible by 5 only
75: div by 5? Yes. By 2? No. By 10? No. => divisible by 5 only
95: div by 5? Yes. By 2? No. By 10? No. => divisible by 5 only

30: div by 2? Yes. By 5? Yes. By 10? Yes (30/10=3). => divisible by 2, 5, and 10
40: div by 2? Yes. By 5? Yes. By 10? Yes (40/10=4). => divisible by 2, 5, and 10
90: div by 2? Yes. By 5? Yes. By 10? Yes (90/10=9). => divisible by 2, 5, and 10

(a) Divisible by 2 only: 22, 38, 56, 62, 66, 78, 84
(b) Divisible by 5 only: 25, 45, 55, 75, 95
(c) Divisible by 10 only: NONE (every multiple of 10 is also a multiple of both 2 and 5)
(d) Divisible by 2, 5, and 10: 30, 40, 90

Answer: (a) 22, 38, 56, 62, 66, 78, 84 (b) 25, 45, 55, 75, 95 (c) None — no number is divisible by 10 without also being divisible by 2 and 5 (d) 30, 40, 90