Chapter 2: Lines and Angles

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Exercise 2.1 — Ray

Q1 Rihan marked a point on a piece of paper. How many lines can he draw that pass through the point? Sheetal marked two points on a piece of paper. How many different lines can she draw that pass through both of the points? Can you help Rihan and Sheetal find their answers?

SOLUTION

A single point does not restrict the direction of a line — a line can pass through it in any direction.

Through one point, infinitely many (uncountably many) lines can be drawn.

Two distinct points fix a unique direction. Only one straight line can connect two given points.

Answer: Rihan can draw infinitely many lines through one point. Sheetal can draw exactly one line through two given points.

Q2 Name the line segments in Fig. 2.4. Which of the five marked points are on exactly one of the line segments? Which are on two of the line segments?

SOLUTION

From Fig. 2.4, the figure shows a zigzag path: L–M–P–Q–R, forming four line segments.

Line segments: LM, MP, PQ, QR.

Each endpoint of the whole path touches only one segment: L is only on LM; R is only on QR.

The interior junction points each touch two segments: M is on LM and MP; P is on MP and PQ; Q is on PQ and QR.

Answer: Line segments: LM, MP, PQ, QR. Points on exactly one segment: L and R. Points on two segments: M, P, and Q.

Q3 Name the rays shown in Fig. 2.5. Is T the starting point of each of these rays?

SOLUTION

From Fig. 2.5: T is a point from which a ray goes toward A (upward-left) and another goes through N toward B (rightward). N is a point on the ray TB.

The rays are: TA (starting at T, going toward A), TB (starting at T, passing through N and continuing toward B), TN (same ray as TB), and NB (starting at N, going toward B).

T is the starting point of TA, TB, and TN.

T is NOT the starting point of ray NB — the starting point of NB is N.

Answer: Rays shown: TA, TB, TN, and NB. T is the starting point of TA, TB, and TN, but not of NB (whose starting point is N).

Q4 In Fig. 2.6, name: (a) Five points, (b) A line, (c) Four rays, (d) Five line segments.

SOLUTION

From Fig. 2.6: the figure shows a line passing through D, O, E (with arrows both ways), a ray from O to B (upper right), and a ray from O to C (right).

(a) Five points: D, E, O, B, C.

(b) A line: line DE (also written as DO, OE, or DB — any two points on the full double-arrowed line). The line passes through D, O, and E.

(c) Four rays: OB, OC, OE, OD (all starting at O).

(d) Five line segments: DE, DO, OE, OB, OC. (Also EB, DB are possible.)

Answer: (a) D, E, O, B, C. (b) Line DE. (c) OB, OC, OE, OD. (d) DE, DO, OE, OB, OC.

Q5 Here is a ray OA (Fig. 2.7). It starts at O and passes through the point A. It also passes through the point B. (a) Can you also name it as OB? Why? (b) Can we write OA as AO? Why or why not?

SOLUTION

(a) From Fig. 2.7: O is the starting point, and the ray passes through both B and A (with A farther along). Since B lies on the same ray starting at O and going in the direction of A, the ray can equally be named OB.

Yes, the ray can be named OB because O is the starting point and B is another point on the same ray.

(b) A ray is named by its starting point first. OA has starting point O, going toward A. AO would mean a ray starting at A and going toward O — that is the opposite direction.

No, we cannot write OA as AO because they are different rays with different starting points and opposite directions.

Answer: (a) Yes — O is the starting point and B lies on ray OA, so it can also be named OB. (b) No — OA and AO are different rays starting at different points and going in opposite directions.

Exercise 2.2 — Angle

Q1 Can you find the angles in the given pictures? Draw the rays forming any one of the angles and name the vertex of the angle. (Pictures show a bicycle and a tiled pattern.)

SOLUTION

In the bicycle picture, the spokes of the wheel and the frame members meet at points, forming angles at those junctions.

For example, in the bicycle frame: the two frame rods meeting at point B (or A, C, D as labelled) form an angle. One such angle is ∠BDC with vertex D.

Draw rays DB and DC to represent the arms of this angle; the vertex is D.

Similarly, angles can be identified at each intersection in the tiled pattern.

Answer: One example angle from the bicycle: ∠BDC with vertex D, formed by rays DB and DC. (Other valid angles at any intersection are also acceptable.)

Q2 Name the angles marked in the given figure. (Figure shows two rays from vertex T — one toward P and one toward Q — with a curve at T, and a horizontal ray toward R.)

SOLUTION

From the figure: T is the vertex. Two rays from T are marked with a curve between them — one going toward P (upper-left) and one along the horizontal toward R (right). Another ray goes toward Q (upper-right).

The curve at T marks the angle between the two rays indicated.

The angles marked are ∠PTR (or ∠RTP) and ∠RTQ (or ∠QTR).

Answer: The angles marked in the figure are ∠PTR (between ray TP and ray TR) and ∠RTQ (between ray TR and ray TQ).

Q3 Mark any three points on your paper that are not on one line. Label them A, B, C. Draw all possible lines going through pairs of these points. How many lines do you get? Name them. How many angles can you name using A, B, C? Write them down, and mark each of them with a curve as in Fig. 2.9.

SOLUTION

With three non-collinear points A, B, C, we can draw a line through each pair of points.

Number of lines: 3. They are line AB, line BC, and line CA.

At each of the three points, the two lines meeting there form an angle.

Number of angles: 3. They are ∠BAC (or ∠CAB) at vertex A, ∠ABC (or ∠CBA) at vertex B, and ∠BCA (or ∠ACB) at vertex C.

Answer: 3 lines: AB, BC, CA. 3 angles: ∠BAC (at A), ∠ABC (at B), ∠BCA (at C).

Q4 Now mark any four points on your paper so that no three of them are on one line. Label them A, B, C, D. Draw all possible lines going through pairs of these points. How many lines do you get? Name them. How many angles can you name using A, B, C, D? Write them all down, and mark each of them with a curve as in Fig. 2.9.

SOLUTION

With 4 points (no 3 collinear), the number of lines through pairs = C(4,2) = 6.

The 6 lines are: AB, BC, CD, DA, AC, BD.

At each vertex, the lines meeting there form angles. For each vertex, we choose 2 of the remaining 3 points to name an angle.

Number of angles = 4 × C(3,2) = 4 × 3 = 12.

The 12 angles are:
At A: ∠BAC, ∠BAD, ∠CAD
At B: ∠ABC, ∠ABD, ∠CBD
At C: ∠BCA, ∠BCD, ∠ACD
At D: ∠ADB, ∠ADC, ∠BDC

Answer: 6 lines: AB, BC, CD, DA, AC, BD. 12 angles: ∠BAC, ∠BAD, ∠CAD (at A); ∠ABC, ∠ABD, ∠CBD (at B); ∠BCA, ∠BCD, ∠ACD (at C); ∠ADB, ∠ADC, ∠BDC (at D).

Exercise 2.3 — Comparing Angles

Q1 In each case, determine which angle is greater and why. (a) ∠AOB or ∠XOY, (b) ∠AOB or ∠XOB, (c) ∠XOB or ∠XOC. (Figure shows rays OA, OX, OY, OB, OC from vertex O, in that order from upper-left going clockwise to right, with Y between X and B, and C to the right of B on the horizontal.)

SOLUTION

(a) ∠AOB or ∠XOY:
Ray OX lies between OA and OB; ray OY also lies between OA and OB (Y is inside ∠AOB).
So ∠XOY is entirely contained within ∠AOB, meaning ∠AOB = ∠AOX + ∠XOY + ∠YOB.
Therefore ∠AOB is greater than ∠XOY.

(b) ∠AOB or ∠XOB:
Ray OX is between OA and OB. So ∠AOB = ∠AOX + ∠XOB.
Therefore ∠AOB is greater than ∠XOB.

(c) ∠XOB or ∠XOC:
From the figure, OB and OC are the same ray (C lies on OB). So ∠XOB = ∠XOC.
Neither is greater — they are equal.

Answer: (a) ∠AOB is greater. (b) ∠AOB is greater. (c) Neither — ∠XOB = ∠XOC (OB and OC are the same ray).

Q2 Which angle is greater: ∠XOY or ∠AOB? Give reasons. (Figure shows rays OX and OA going upward-left with different amounts of spread, and OY, OB going right from vertex O.)

SOLUTION

From the figure, the two angles are drawn separately — ∠XOY and ∠AOB do not share a common diagram, so the spread cannot be judged by eye alone.

To determine which is greater, one must superimpose the two angles (align their vertices and one arm each) or measure them with a protractor.

By visual inspection alone, we cannot conclude which is greater.

Answer: We cannot tell just by looking. Superimposition or measurement with a protractor is needed to determine which of ∠XOY or ∠AOB is greater.

Exercise 2.4 — Special Types of Angles

Q1 How many right angles do the windows of your classroom contain? Do you see other right angles in your classroom?

SOLUTION

A rectangular window has four corners. Each corner is a right angle (90°).

So each window contains 4 right angles.

Other right angles in a classroom: corners of the blackboard, corners of the door, corners of books, corners of tiles on the floor, edges where walls meet the floor or ceiling.

Answer: Each rectangular window contains 4 right angles. Other right angles in the classroom include corners of the blackboard, door, books, and floor tiles.

Q2 Join A to other grid points in the figure by a straight line to get a straight angle. What are all the different ways of doing it? (A and B are grid points; B is to the right of A on the same row.)

SOLUTION

A straight angle is formed when two rays from A point in exactly opposite directions, i.e., the three points (the far point, A, and the chosen grid point) are collinear with A between them.

Since B is to the right of A, the ray AB goes to the right. To form a straight angle, the other ray from A must go to the left — i.e., join A to any grid point directly to its left on the same horizontal line.

In the first grid (A and B on the same horizontal row): join A to a point directly left of A on the same row to make a straight angle with ray AB.

In the second grid (A and B on a diagonal): the ray AB goes diagonally. The opposite ray goes in the mirror-opposite diagonal direction. Join A to the grid point in that opposite diagonal direction.

The key rule: the two points joined to A must be collinear with A between them.

Answer: Join A to any grid point that lies exactly opposite to B with respect to A (i.e., on the ray from B through A extended). In the horizontal case, join A to a point directly to its left on the same row.

Q3 Now join A to other grid points in the figure by a straight line to get a right angle. What are all the different ways of doing it? (Hint: Extend the line further as shown. To get a right angle at A, draw a line through it that divides the straight angle CAB into two equal parts.)

SOLUTION

A right angle at A is formed when the new ray from A is perpendicular to the ray AB.

The hint says: extend AB to get the straight angle CAB (where C is directly opposite B). A right angle at A means the new line bisects this straight angle, i.e., the new ray makes equal angles on both sides.

On a square grid, a line perpendicular to AB can be drawn by going 'up' or 'down' from A (if AB is horizontal). For example, if AB is horizontal, join A to the grid point directly above A or directly below A — either gives a right angle.

If AB is diagonal (e.g., going 1 right and 1 down per step), the perpendicular direction goes 1 right and 1 up per step (rotate 90°). Join A to the grid point in that perpendicular direction.

There are two perpendicular directions for any given line, giving two different ways.

Answer: Join A to grid points perpendicular to line AB. For a horizontal AB, join A to a point directly above or directly below A. For a diagonal AB, join A to a point in the rotated-90° direction. There are two perpendicular directions for each line.

Q4 Get a slanting crease on the paper. Now, try to get another crease that is perpendicular to the slanting crease. (a) How many right angles do you have now? Justify why the angles are exact right angles. (b) Describe how you folded the paper so that any other person who doesn't know the process can simply follow your description to get the right angle.

SOLUTION

(a) Two straight lines meeting at a point create 4 angles. When the lines are perpendicular, all 4 angles are right angles.

You now have 4 right angles.

Justification: The second fold was made so that the slanting crease folds exactly onto itself. This means the two sides of the second crease make equal angles

with the first crease. Since the two equal angles together form a straight angle (180°), each must be 90°. Hence all four angles at the intersection are right angles.

(b) Folding description: First fold the paper to get the slanting crease. Then, keeping the paper flat, fold it again so that the slanting crease (first fold line) lies exactly on top of itself — i.e., one part of the crease aligns with the other part. The new crease formed is perpendicular to the first.

Answer: (a) 4 right angles. Each is exactly 90° because the second crease bisects the straight angle on each side of the first crease (each half = 180°/2 = 90°). (b) Fold the paper so that the first (slanting) crease folds onto itself exactly; the new crease formed is the perpendicular.

Exercise 2.5 — Classifying Angles

Q1 Identify acute, right, obtuse and straight angles in the previous figures. (Referring to the three groups of angles shown in the 'Classifying Angles' section.)

SOLUTION

From the three-group classification figure:

First group (left panel): all angles that are less than a right angle — these are acute angles.

Second group (middle panel): angles that are exactly equal to a right angle (like the corner of an 'L') — these are right angles.

Third group (right panel): angles greater than a right angle but less than a straight angle — these are obtuse angles.

A straight angle (arms in a straight line, measuring 180°) is formed when a straight line is divided — it would appear as a flat open angle.

Answer: First group: acute angles. Second group: right angles. Third group: obtuse angles. A straight angle is one whose arms form a straight line (180°).

Q2 Do you know what the words acute and obtuse mean? Acute means sharp and obtuse means blunt. Why do you think these words have been chosen?

SOLUTION

Acute means 'sharp' in Latin. An acute angle has a small opening — its arms are close together, forming a sharp pointed shape like a sharp tip.

Obtuse means 'blunt' in Latin. An obtuse angle has a wide, blunt opening — its arms are spread far apart, like a blunt or rounded shape.

The words are chosen because the visual appearance of each angle matches the meaning of the word: acute angles look sharp and pointed, obtuse angles look wide and blunt.

Answer: Acute angles look sharp and pointed (small opening), matching the meaning 'sharp.' Obtuse angles have a wide, blunt opening, matching the meaning 'blunt.'

Q3 Find out the number of acute angles in each of the figures below. (Three figures: a single triangle, a triangle subdivided into 4 smaller triangles, and a triangle subdivided further with 9 smaller triangles.) What will be the next figure and how many acute angles will it have? Do you notice any pattern in the numbers?

SOLUTION

Figure 1 (1 triangle): 3 acute angles.

Figure 2 (1 large triangle divided into 4 small triangles): 12 acute angles.

Figure 3 (1 large triangle divided into 9 small triangles): 21 acute angles.

Pattern: 3, 12, 21 — each figure adds 9 more acute angles than the previous.

The next figure (16 small triangles) will have 21 + 9 = 30 acute angles.

Answer: Figure 1: 3 acute angles. Figure 2: 12 acute angles. Figure 3: 21 acute angles. The next figure will have 30 acute angles. The pattern increases by 9 each time.

Exercise 2.6 — Measuring Angles — Unlabelled Protractor

Q1 Write the measures of the following angles using the unlabelled protractor shown: (a) ∠KAL, (b) ∠WAL, (c) ∠TAK. (The vertex A coincides with the centre; L is at 0° (rightmost base), K is 30° from L, W is 50° from L, T is at 150° from L measured going left, or equivalently at 120° from K on the same side.)

SOLUTION

(a) ∠KAL: By counting the unit degree marks from AL (0°) to AK, the book states we count 30 units.
∠KAL = 30°.

(b) ∠WAL: Count from AL (0°) to AW on the unlabelled protractor.
∠WAL = 50°.

(c) ∠TAK: Count the units from AK to AT.
AK is at 30° from AL, and AT is at 150° from AL (on the left side).
∠TAK = 150° − 30° = 120°.

Answer: (a) ∠KAL = 30°. (b) ∠WAL = 50°. (c) ∠TAK = 120°.

Exercise 2.7 — Measuring Angles — Make Your Own Protractor

Q1 In Step 4 of making the protractor: fold the quarter-circle in half again. This new fold is 1/8 of the circle. What is the measure of this angle? (Fill in the blank: 1/8 of 360° = _____.)

SOLUTION

1/8 of a full turn = (1/8) × 360° = 45°.

The new crease is at 45° from the 0° edge, and the complementary crease is at 180° − 45° = 135°.

Answer: 45°. The two new creases mark 45° and 135°.

Q2 In Step 5, continuing with another half fold of the 45° section, we get an angle of measure _____ (fill in the blank).

SOLUTION

Halving the 45° angle: (1/2) × 45° = 22.5°.

The new crease is at 22.5° from the 0° edge.

Answer: 22.5°

Q3 Think! In Fig. 2.19, we have ∠AOB = ∠BOC = ∠COD = ∠DOE = ∠EOF = ∠FOG = ∠GOH = ∠HOI = _____. Why?

SOLUTION

The semicircle (straight angle = 180°) was folded repeatedly in halves until 8 equal parts were created.

Each fold halves the remaining angle. Starting from 180°:
After 1 fold: 90° each (2 parts)
After 2 folds: 45° each (4 parts)
After 3 folds: 22.5° each (8 parts)

So the straight angle of 180° is divided into 8 equal parts.
Each part = 180° ÷ 8 = 22.5°.

Answer: Each angle = 22.5°. The straight angle (180°) is divided into 8 equal parts by repeated halving, so each part = 180° ÷ 8 = 22.5°.

Q4 Find the degree measures of the following angles using your protractor. (Three angle figures: (i) ∠IHJ with H as vertex, I and J as the two arms; (ii) ∠GHK (or ∠IHJ) — a narrower angle; (iii) ∠IHJ — a wider angle with H as vertex.)

SOLUTION

These angles must be measured using a physical protractor placed on the printed figures.

From the answer key provided in the textbook:
Figure (i): ∠IHJ = 47°.
Figure (ii): ∠GHK = 23°.
Figure (iii): ∠IHJ = 108°.

Answer: ∠IHJ (first figure) = 47°. ∠GHK (second figure) = 23°. ∠IHJ (third figure) = 108°.

Q5 Find the degree measures for the angles given below. Check if your paper protractor can be used here! (Two figures: (i) a triangle with vertex at H, arms going to I and J; (ii) an obtuse angle at H with arms toward I and J.)

SOLUTION

These angles must be measured with a standard protractor placed on the figures.

From the textbook answer key:
Figure (i): ∠IHJ = 42°.
Figure (ii): ∠IHJ = 116°.

A paper protractor (made by folding) can only measure angles that are multiples of 22.5° exactly — it cannot accurately measure 42° or 116°.
So the paper protractor cannot be reliably used here for precise readings.

Answer: ∠IHJ (first) = 42°. ∠IHJ (second) = 116°. The paper protractor cannot be used here because these are not multiples of 22.5°.

Q6 How can you find the degree measure of the angle given below using a protractor? (The figure shows a reflex angle — the marked (shaded) angle is greater than 180°.)

SOLUTION

When the marked angle is a reflex angle (greater than 180°), place the protractor to measure the smaller (non-reflex) angle first.

Let the non-reflex angle measure = x°.

The reflex angle = 360° − x°.

From the example in the textbook: the non-reflex angle = 100°.
Reflex angle = 360° − 100° = 260°.

Answer: Measure the unmarked (non-reflex) angle with the protractor, then subtract from 360°. Example: if the non-reflex angle is 100°, the reflex angle = 360° − 100° = 260°.

Q7 Measure and write the degree measures for each of the following angles: (a), (b), (c), (d), (e), (f) — six angles shown as figures.

SOLUTION

These angles are drawn in the textbook and must be measured with a protractor placed on the printed figures.

From the textbook answer key:
(a) 80°
(b) 120°
(c) 60°
(d) 130°
(e) 130°
(f) 60°

Answer: (a) 80°, (b) 120°, (c) 60°, (d) 130°, (e) 130°, (f) 60°.

Q8 Find the degree measures of ∠BXE, ∠CXE, ∠AXB and ∠BXC. (Figure shows a protractor placed with centre at X; rays XA, XB, XC go to the left and above-left, ray XE goes to the right along the base.)

SOLUTION

From the protractor figure: X is the vertex, XE lies along the 0° baseline (right), and XA lies along the 180° direction (left). XB and XC are rays going up, with C closer to the vertical (nearer E's side) than B.

Reading the protractor: ray XB passes through 115° (measured from XE), so ∠BXE = 115°.

Ray XC passes through 85° from XE, so ∠CXE = 85°.

Since XA is at 180°: ∠AXB = 180° − 115° = 65°.

∠BXC = ∠BXE − ∠CXE = 115° − 85° = 30°.

Check: ∠AXB + ∠BXE = 65° + 115° = 180° (straight angle AXE), and ∠BXC + ∠CXE = 30° + 85° = 115° = ∠BXE.

Answer: ∠BXE = 115°, ∠CXE = 85°, ∠AXB = 65°, ∠BXC = 30°.

Q9 Find the degree measures of ∠PQR, ∠PQS and ∠PQT. (Figure shows vertex Q with ray QP going left and rays QR, QS, QT going upward-right at increasing angles.)

SOLUTION

From the figure: Q is the vertex, QP goes left (180° direction), and QR, QS, QT go to the right at different angles measured from QP.

Reading from the protractor on the figure (measuring from QP going clockwise to each ray):
∠PQR = 45°.
∠PQS = 100°.
∠PQT = 150°.

Verification: ∠PQR < ∠PQS < ∠PQT, consistent with R being closer to P than S, and S closer than T.

Answer: ∠PQR = 45°, ∠PQS = 100°, ∠PQT = 150°.

Exercise 2.8 — Where Are the Angles?

Q1 Angles in a clock: (a) The hands of a clock make different angles at different times. At 1 o'clock, the angle between the hands is 30°. Why? (b) What will be the angle at 2 o'clock? And at 4 o'clock? 6 o'clock?

SOLUTION

(a) A clock face is a full circle (360°) divided into 12 equal hour sections.
Angle between adjacent hour markings = 360° ÷ 12 = 30°.
At 1 o'clock, the hour hand points to 1 and the minute hand points to 12 — they are 1 hour-division apart.
So the angle = 1 × 30° = 30°.

(b) At 2 o'clock: hour hand at 2, minute hand at 12 — 2 divisions apart.
Angle = 2 × 30° = 60°.

At 4 o'clock: 4 divisions apart.
Angle = 4 × 30° = 120°.

At 6 o'clock: 6 divisions apart — the hands point in opposite directions.
Angle = 6 × 30° = 180° (a straight angle).

Answer: (a) The clock face is divided into 12 equal parts, each 30°. At 1 o'clock the hands are 1 division apart, so the angle is 30°. (b) 2 o'clock: 60°; 4 o'clock: 120°; 6 o'clock: 180°.

Q2 The angle of a door: Is it possible to express the amount by which a door is opened using an angle? What will be the vertex of the angle and what will be the arms of the angle?

SOLUTION

Yes, the amount by which a door is opened can be expressed as an angle.

The vertex of the angle is the hinge — the point where the door meets the wall (the pivot point).

One arm of the angle is the wall (the fixed reference).
The other arm is the door itself (the moving part).

As the door opens wider, the angle between the wall and the door increases.

Answer: Yes. Vertex = the hinge (where the door meets the wall). One arm = the wall; the other arm = the door. The angle increases as the door opens wider.

Q3 Vidya is enjoying her time on the swing. She notices that the greater the angle with which she starts the swinging, the greater is the speed she achieves on her swing. But where is the angle? Are you able to see any angle?

SOLUTION

Yes, an angle can be identified in the swing's motion.

The vertex of the angle is at the top of the swing chain (the pivot/support point).

One arm is the vertical position of the swing chain at rest (the rest position).
The other arm is the position of the swing chain when Vidya is at the highest point of her swing on one side.

The greater the angle at which she pulls back before releasing, the more energy she has and the faster she swings.

Answer: The angle is at the top pivot of the swing. One arm is the rest (vertical) position; the other arm is the pulled-back position of the chain. The greater this angle, the faster Vidya swings.

Q4 Here is a toy with slanting slabs attached to its sides; the greater the angles or slopes of the slabs, the faster the balls roll. Can angles be used to describe the slopes of the slabs? What are the arms of each angle? Which arm is visible and which is not?

SOLUTION

Yes, angles can be used to describe the slopes of the slabs.

Each slab makes an angle with the horizontal base of the toy.

One arm is the edge (surface) of the slanting slab — this arm is visible.
The other arm is an imaginary horizontal line at the base — this arm is not visible (it is a reference line).

The greater the angle between the slab and the horizontal, the steeper the slope and the faster the ball rolls.

Answer: Yes. One arm = the slanting slab edge (visible); other arm = the horizontal reference line (not visible). A larger angle means a steeper slope and faster rolling.

Q5 Observe the images below where there is an insect and its rotated version. Can angles be used to describe the amount of rotation? How? What will be the arms of the angle and the vertex? (Hint: Observe the horizontal line touching the insects.)

SOLUTION

Yes, the amount of rotation of the insect can be described by an angle.

The vertex of the angle is the point where the insect's body (or the reference line) pivots.

Following the hint: draw a horizontal line touching both insects. One arm is the horizontal line at the original position; the other arm is the horizontal line at the rotated position.

The angle between these two horizontal lines (the original and the rotated) gives the measure of the rotation.

Answer: Yes. Vertex = the pivot point of rotation. One arm = horizontal line at original position; other arm = horizontal line at rotated position. The angle between them describes the rotation.

Exercise 2.9 — Drawing Angles

Q1 In Fig. 2.23, list all the angles possible. Did you find them all? Now, guess the measures of all the angles. Then, measure the angles with a protractor. Record all your numbers in a table. See how close your guesses are to the actual measures. (Fig. 2.23 shows points A, P, R, B, C, D, L, S with lines joining them forming a figure.)

SOLUTION

From Fig. 2.23, identify all pairs of rays sharing a common vertex and list each angle.

Some angles visible in Fig. 2.23 include (reading from the answer key): ∠CAP, ∠ACD, ∠APL, ∠DLP, ∠RPL, ∠SLP, ∠PRS, ∠LSR, ∠BRS, ∠CLP.

To find all angles: at each intersection point, count the rays and form all possible angle pairs.

Measure each angle with a protractor placed on the figure. Record the guessed value and the measured value side by side to compare.

Answer: Angles in Fig. 2.23 include: ∠CAP, ∠ACD, ∠APL, ∠DLP, ∠RPL, ∠SLP, ∠PRS, ∠LSR, ∠BRS, ∠CLP (and others). Measure each with a protractor to find their degree values.

Exercise 2.10 — Types of Angles and their Measures

Q1 Use a protractor to find the measure of each angle. Then classify each angle as acute, obtuse, right, or reflex. (a) ∠PTR, (b) ∠PTQ, (c) ∠PTW, (d) ∠WTP. (Figure shows vertex T with rays TR, TQ going upper-right, TW going lower-right, and TP going left.)

SOLUTION

Measure each angle from the figure using a protractor:

(a) ∠PTR: ray TP goes left, ray TR goes upper-right. Measuring from the figure: ∠PTR = 30°.
30° is less than 90° → Acute angle.

(b) ∠PTQ: ray TP goes left, ray TQ is between TR and TW. ∠PTQ = 60°.
60° is less than 90° → Acute angle.

(c) ∠PTW: ray TP goes left, ray TW goes lower-right. ∠PTW = 102°.
102° is between 90° and 180° → Obtuse angle.

(d) ∠WTP: this is the reflex angle going from TW around the long way to TP.
∠WTP (reflex) = 360° − 102° = 258°.
258° is between 180° and 360° → Reflex angle.

Answer: (a) ∠PTR = 30° — Acute. (b) ∠PTQ = 60° — Acute. (c) ∠PTW = 102° — Obtuse. (d) ∠WTP = 258° — Reflex.

Exercise 2.11 — Types of Angles and their Measures — Let's Explore and Further Proble

Q1 In this figure, ∠TER = 80°. What is the measure of ∠BET? What is the measure of ∠SET? (Figure shows a straight line through B, E, R; ray ET goes upward making 80° with ER; ray ES is perpendicular to the line, making a right angle ∠BES = 90°.)

SOLUTION

B, E, R are collinear, so ∠BER = 180° (straight angle).

∠TER = 80° (given).

∠BET = ∠BER − ∠TER = 180° − 80° = 100°.

For ∠SET: from the figure, S is positioned so that ∠BES = 90° (ES is perpendicular to line BER).

∠SET = ∠BET − ∠BES = 100° − 90° = 10°.

Answer: ∠BET = 100°. ∠SET = 10°.

Q2 The Ashoka Chakra has 24 spokes. What is the degree measure of the angle between two spokes next to each other? What is the largest acute angle formed between two spokes?

SOLUTION

The Ashoka Chakra is a full circle (360°) with 24 equally spaced spokes.

Angle between two adjacent spokes = 360° ÷ 24 = 15°.

Angles between non-adjacent spokes are multiples of 15°: 15°, 30°, 45°, 60°, 75°, 90°, 105°, ...

An acute angle is less than 90°. The multiples of 15° less than 90° are: 15°, 30°, 45°, 60°, 75°.

The largest acute angle = 75° (= 5 × 15°).

Answer: Angle between adjacent spokes = 15°. Largest acute angle between two spokes = 75°.

Q3 Puzzle: I am an acute angle. If you double my measure, you get an acute angle. If you triple my measure, you will get an acute angle again. If you quadruple (four times) my measure, you will get an acute angle yet again! But if you multiply my measure by 5, you will get an obtuse angle measure. What are the possibilities for my measure?

SOLUTION

Let the angle measure be x (a whole number of degrees).

Conditions:
x is acute: $0° < x < 90°$
2x is acute: $2x < 90° \rightarrow x < 45°$
3x is acute: $3x < 90° \rightarrow x < 30°$
4x is acute: $4x < 90° \rightarrow x < 22.5°$, so $x \leq 22°$
5x is obtuse: $90° < 5x < 180° \rightarrow 18° < x < 36°$

Combining $x \leq 22°$ and $x > 18°$: x can be 19°, 20°, 21°, or 22°.

Verify x = 19: $4 \times 19 = 76°$ (acute ✓), $5 \times 19 = 95°$ (obtuse ✓).
Verify x = 22: $4 \times 22 = 88°$ (acute ✓), $5 \times 22 = 110°$ (obtuse ✓).

Answer: The possible measures are 19°, 20°, 21°, and 22°.