Chapter 3: Number Play

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Exercise 3.1 — Numbers can Tell us Things

Q1 Can the children rearrange themselves so that the children standing at the ends say '2'?

SOLUTION

A child says '2' if both neighbours are taller than them.
A child at the end of a line has only ONE neighbour (no one on the other side).
With only one neighbour, the maximum number a child at the end can say is 1, never 2.

Answer: No. End children have only one neighbour, so the maximum they can say is 1, never 2.

Q2 Can we arrange the children in a line so that all would say only 0s?

SOLUTION

A child says '0' if neither of its neighbours is taller — meaning both neighbours are the same height or shorter.
If all children are of the same height, then no neighbour is taller.
So every child says 0.

Answer: Yes. Arrange all children so they are all the same height — then every child says 0.

Q3 Can two children standing next to each other say the same number?

SOLUTION

Consider three children A, B, C in a line where A and B are adjacent.
If A is the tallest and B is the second tallest, A says 0 (no neighbour is taller) and B says 1 (only A is taller). These are different.
But consider an arrangement where A < B > C and B < D > E (two consecutive tall children): two adjacent shorter children can both say 1.
For example: short, tall, short, tall, short — the two short children in the middle each say 1 (both neighbours taller on one side each).
So yes, two adjacent children can say the same number.

Answer: Yes. Two adjacent children can say the same number (e.g., both can say 1 in certain arrangements).

Q4 There are 5 children in a group, all of different heights. Can they stand such that four of them say '1' and the last one says '0'? Why or why not?

SOLUTION

Arrange the 5 children in ascending order of height from left to right: shortest to tallest.
Child 1 (leftmost, shortest): only right neighbour (Child 2) is taller → says 1.
Child 2: left neighbour (Child 1) shorter, right neighbour (Child 3) taller → one taller neighbour → says 1.
Child 3: left (Child 2) shorter, right (Child 4) taller → says 1.
Child 4: left (Child 3) shorter, right (Child 5) taller → says 1.
Child 5 (tallest, rightmost): only left neighbour, who is shorter → says 0.

This gives the sequence 1, 1, 1, 1, 0 — four 1s and one 0.

Answer: Yes. Arrange the 5 children in ascending order of height. The four shorter children each say 1 and the tallest child (at the end) says 0.

Q5 For this group of 5 children, is the sequence 1, 1, 1, 1, 1 possible?

SOLUTION

For every child to say 1, every child must have exactly one taller neighbour.
The child at each end has only one neighbour. For an end child to say 1, that one neighbour must be taller.
So both end children require their single neighbour to be taller — meaning both ends have a taller child next to them.
The tallest child in the group has no one taller. Wherever the tallest child stands, no neighbour is taller → the tallest says 0, not 1.
Therefore the sequence 1, 1, 1, 1, 1 is impossible.

Answer: No. The tallest child always says 0 (no neighbour is taller), so all five cannot say 1.

Q6 Is the sequence 0, 1, 2, 1, 0 possible? Why or why not?

SOLUTION

Reading the sequence: the two end children say 0 (their single neighbour is shorter), the two middle-outer children say 1 (one taller neighbour each), and the centre child says 2 (both neighbours taller).
So the centre child must be the shortest, and the height must rise on both sides towards the ends, with the tallest children at the two ends.
Take heights (left to right) 4, 2, 1, 3, 5.
Position 1 (height 4): only neighbour 2 < 4 → says 0 ✓.
Position 2 (height 2): left 4 taller, right 1 shorter → one taller neighbour → says 1 ✓.
Position 3 (height 1): left 2 taller, right 3 taller → both taller → says 2 ✓.
Position 4 (height 3): left 1 shorter, right 5 taller → one taller neighbour → says 1 ✓.
Position 5 (height 5): only neighbour 3 < 5 → says 0 ✓.
This gives the sequence 0, 1, 2, 1, 0, so it is possible.

Answer: Yes. For example, with heights arranged as 4, 2, 1, 3, 5 (left to right), the sequence 0, 1, 2, 1, 0 is produced.

Q7 How would you rearrange the five children so that the maximum number of children say '2'?

SOLUTION

A child says '2' only if both neighbours are taller — the child must be a local minimum between two taller children.

End children can never say 2 (they have only one neighbour).

So at most the 3 interior positions (positions 2, 3, 4) could say 2.

For position 2 to say 2: children at positions 1 and 3 must both be taller than child at 2.

For position 4 to say 2: children at positions 3 and 5 must both be taller than child at 4.

Positions 2 and 4 saying 2 requires position 3 to be taller than both 2 and 4. But then position 3 is a local maximum — it says 0, not 2.

At most 2 children can say 2: arrange as tall, short, tall, short, medium — for example heights 4, 1, 5, 2, 3: pos2(1) says 2 ✓, pos4(2) says 2 ✓, others say 0 or 1.

Maximum = 2 children saying '2'.

Answer: At most 2 children can say '2'. Arrange them so the 2nd and 4th children are local minima, e.g., heights (tall, short, taller, short, medium) so both interior short children have two taller neighbours.

Exercise 3.2 — Supercells

Q1 Colour or mark the supercells in the table below.
6828 | 670 | 9435 | 3780 | 3708 | 7308 | 8000 | 5583 | 52

SOLUTION

A supercell is greater than all its adjacent (left and right) cells.
6828: right neighbour = 670. Is 6828 > 670? Yes. Left neighbour: none (first cell). → Supercell ✓
670: left = 6828 > 670. Not a supercell.
9435: left = 670 < 9435 ✓, right = 3780 < 9435 ✓. → Supercell ✓
3780: left = 9435 > 3780. Not a supercell.
3708: left = 3780 > 3708. Not a supercell.
7308: left = 3708 < 7308 ✓, right = 8000 > 7308. Not a supercell.
8000: left = 7308 < 8000 ✓, right = 5583 < 8000 ✓. → Supercell ✓
5583: left = 8000 > 5583. Not a supercell.
52: only left neighbour = 5583 > 52. Not a supercell.
Supercells: 6828, 9435, 8000.

Answer: The supercells are 6828 (position 1), 9435 (position 3), and 8000 (position 7).

Q2 Fill the table below with only 4-digit numbers such that the supercells are exactly the coloured cells.
The coloured (given) cells are at positions 1, 4 and 8: 5346 | ___ | ___ | 1258 | ___ | ___ | ___ | 9635 | ___

SOLUTION

The coloured cells (positions 1, 4 and 8, holding 5346, 1258 and 9635) must be the only supercells.
Position 1 (5346) is a supercell: it must be greater than its right neighbour (position 2).
Position 4 (1258) is a supercell: it must be greater than positions 3 and 5.
Position 8 (9635) is a supercell: it must be greater than positions 7 and 9.
So make every white cell smaller than the coloured cell next to it, and make sure no white cell beats both of its own neighbours.
One valid fill: 5346, 1000, 1000, 1258, 1100, 1200, 1300, 9635, 9000.
Check: 5346 > 1000 ✓; 1258 > 1000 and 1258 > 1100 ✓; 9635 > 1300 and 9635

> 9000 ✓; no white cell is a supercell.

Answer: One valid answer: 5346, 1000, 1000, 1258, 1100, 1200, 1300, 9635, 9000. (Many other valid fills are possible.)

Q3 Fill the table below such that we get as many supercells as possible. Use numbers between 100 and 1000 without repetitions.

SOLUTION

To maximise supercells in a 9-cell row, alternate high-low-high-low… pattern.
Place a large number, then a smaller one, then larger, and so on.
Example (alternating): 900, 100, 850, 110, 800, 120, 750, 130, 700.
Check: 900 > 100 ✓ (supercell); 100 < 900 and 100 < 850 (not supercell); 850 > 100 and 850 > 110 ✓ (supercell); 110 not supercell; 800 > 110 and 800 > 120 ✓; 120 not supercell; 750 > 120 and 750 > 130 ✓; 130 not supercell; 700 > 130 ✓ (last cell, supercell).
This gives 5 supercells: positions 1, 3, 5, 7, 9.

Answer: Maximum 5 supercells are possible in a 9-cell row. Fill alternately with high and low values (e.g., 900, 100, 850, 110, 800, 120, 750, 130, 700).

Q4 Out of the 9 numbers, how many supercells are there in the table above?

SOLUTION

Using the alternating high-low fill from Q3, the supercells are at positions 1, 3, 5, 7, 9.
That gives 5 supercells.

Answer: 5 supercells.

Q5 Can you fill a supercell table without repeating numbers such that there are no supercells? Why or why not?

SOLUTION

In any table with distinct numbers, one number must be the largest.
The largest number is greater than all its neighbours (since all other numbers are smaller).
Therefore the cell with the largest number is always a supercell.
It is impossible to fill a table with distinct numbers and have no supercells at all.

Answer: No. The cell with the largest number will always be a supercell (it is greater than all its neighbours), so it is impossible to have no supercells with distinct numbers.

Q6 Will the cell having the largest number in a table always be a supercell? Can the cell having the smallest number in a table be a supercell? Why or why not?

SOLUTION

Largest number: it is greater than every other number in the table, so it is certainly greater than all its neighbours. It is always a supercell.

Smallest number: all other numbers are larger than it. All its neighbours (which are other numbers in the table) are larger than it, not smaller.

For a cell to be a supercell it must be greater than all neighbours. The smallest number fails this — every neighbour is larger.

The smallest number can never be a supercell.

Answer: Yes, the largest number is always a supercell. No, the smallest number can never be a supercell — all its neighbours are larger.

Q7 Fill a table such that the cell having the second largest number is not a supercell.

SOLUTION

Place the largest and second largest numbers adjacent to each other.

Example (9 cells): 1, 2, 3, 4, 5, 6, 7, 9, 8.

Largest = 9 (position 8), second largest = 8 (position 9).

Position 8 (9): left neighbour = 7 < 9 ✓, right neighbour = 8 < 9 ✓ → supercell.

Position 9 (8): only left neighbour = 9 > 8 → NOT a supercell ✓.

So the second largest (8) is not a supercell because its neighbour 9 is larger.

Answer: Example: 1, 2, 3, 4, 5, 6, 7, 9, 8. The second largest number (8) is adjacent to 9, so 8 is not a supercell.

Q8 Fill a table such that the cell having the second largest number is not a supercell but the second smallest number is a supercell. Is it possible?

SOLUTION

We need: the second largest NOT a supercell, and the second smallest IS a supercell.

A cell is a supercell when it is greater than all its neighbours. The second smallest number (2) can be a supercell only if it has just one neighbour, and that neighbour is the smallest number (1) — so place 2 at an end, next to 1.

To stop the second largest from being a supercell, place it right next to the largest number.

Example: 2, 1, 9, 8, 3, 4, 5, 6, 7.

Position 1 (2): only right neighbour = 1 < 2 → supercell ✓.

Largest = 9 (position 3), second largest = 8 (position 4): left = 9 > 8 → not a supercell ✓.

So second smallest (2) IS a supercell and second largest (8) is NOT a supercell. Possible!

Answer: Yes, it is possible. Example: 2, 1, 9, 8, 3, 4, 5, 6, 7. The second smallest (2) is a supercell (neighbour is 1), and the second largest (8) is not a supercell (neighbour 9 is larger).

Exercise 3.2 — Supercells — Table 2

Q1 Complete Table 2 with 5-digit numbers whose digits are '1', '0', '6', '3', and '9' in some order. Only a coloured cell should have a number greater than all its neighbours. Fill in the blank: The biggest number in the table is ___.

SOLUTION

All numbers must use each of the digits 1, 0, 6, 3, 9 exactly once.
The coloured cells must be supercells (greater than all neighbours); white cells must not be supercells.
The greatest possible number using these digits is 96310.
Place 96310 (or a large permutation like 96301) in a coloured cell.
Fill remaining cells with smaller permutations ensuring only coloured cells are supercells.
One valid fill (from the textbook): Row 1: 96310, 96301, 36109, 39160; Row 2: 96103, 13609, 60319, 19306; Row 3: 13906, 10396, 60193, 60931; Row 4: 10369, 10963, 10936, 69031.
Biggest number: 96310.

Answer: The biggest number in the table is 96,310.

Q2 The smallest even number in the table is ___.

SOLUTION

An even number must end in an even digit, so here it must end in 0 or 6.
Ending in 0: the remaining digits 1, 3, 6, 9 fill the first four places; smallest arrangement is 1369, giving 13690.
Ending in 6: the remaining digits 1, 0, 3, 9 fill the first four places; smallest arrangement is 1039, giving 10396.
Compare: 10396 < 13690.
So the smallest even number is 10396.

Answer: The smallest even number in the table is 10,396.

Q3 The smallest number greater than 50,000 in the table is ___.

SOLUTION

A number greater than 50,000 must start with 6 or 9 (the digit 5 is not available, and starting with 1 or 3 gives a number below 50,000).

Starting with 9 gives numbers of at least 90,000 — larger.

So the smallest such numbers start with 6. In the completed Table 2, the value starting with 6 that is smallest is 60,193.

Therefore the smallest number greater than 50,000 in the table is 60,193.

Answer: The smallest number greater than 50,000 in the table is 60,193.

Exercise 3.3 — Patterns of Numbers on the Number Line

Q1 Identify the numbers marked on the number lines below, and label the remaining positions. Also put a circle around the smallest number and a box around the largest number in each sequence.

a. Number line showing 2010 and 2020

b. Number line showing 9996 and 9997

c. Number line showing 15,077, 15,078, and 15,083

d. Number line showing 86,705 and 87,705

SOLUTION

a. The gap between 2010 and 2020 is 10. Each tick = 5. Sequence: 1990, 1995, 2000, 2005, 2010, 2015, 2020, 2025, 2030, 2035. Smallest: 1990 (circle), Largest: 2035 (box).

b. The gap between 9996 and 9997 is 1. Each tick = 1. Sequence: 9993, 9994, 9995, 9996, 9997, 9998, 9999, 10000, 10001, 10002. Smallest: 9993 (circle), Largest: 10002 (box).

c. 15,077 and 15,078 are consecutive (step = 1). Sequence: 15077, 15078, 15079, 15080, 15081, 15082, 15083, 15084, 15085, 15086. Smallest: 15,077 (circle), Largest: 15,086 (box).

d. The gap between 86,705 and 87,705 is 1000. Each tick = 1000. Sequence: 83705, 84705, 85705, 86705, 87705, 88705, 89705, 90705, 91705, 92705. Smallest: 83,705 (circle), Largest: 92,705 (box).

Answer: a. Sequence (step 5): 1990, 1995, 2000, …, 2035; smallest = 1990, largest = 2035. b. Sequence (step 1): 9993, 9994, …, 10002; smallest = 9993, largest = 10002. c. Sequence (step 1): 15077, 15078, …, 15086; smallest = 15,077, largest = 15,086. d. Sequence (step 1000): 83705, 84705, …, 92705; smallest = 83,705, largest = 92,705.

Exercise 3.4 — Playing with Digits

Q1 Find out how many numbers have two digits, three digits, four digits, and five digits. (1-digit numbers: 9, from 1–9)

SOLUTION

1-digit numbers: 1 to 9 → 9 numbers.
2-digit numbers: 10 to 99 → 99 − 10 + 1 = 90 numbers.
3-digit numbers: 100 to 999 → 999 − 100 + 1 = 900 numbers.
4-digit numbers: 1000 to 9999 → 9999 − 1000 + 1 = 9000 numbers.
5-digit numbers: 10000 to 99999 → 99999 − 10000 + 1 = 90000 numbers.

Answer: 2-digit: 90, 3-digit: 900, 4-digit: 9000, 5-digit: 90,000.

Q2 Digit sum 14.
a. Write other numbers whose digits add up to 14.
b. What is the smallest number whose digit sum is 14?
c. What is the largest 5-digit number whose digit sum is 14?
d. How big a number can you form having the digit sum of 14? Can you make an even bigger number?

SOLUTION

a. Examples: 68 (6+8=14), 77 (7+7=14), 86, 95, 176 (1+7+6=14), 248 (2+4+8=14), 356, 455, 545, 653, 950, 1490, 5900.
b. To get the smallest number, use as few digits as possible and make the number as small as can be. The smallest 2-digit number with digit sum 14: digits must sum to 14 with largest digit in units place. 59 (5+9=14) is smaller than 68, 77, 86, 95. So smallest = 59.
c. Largest 5-digit number with digit sum 14: put the largest possible digit first. Start with 9: remaining sum = 5 for 4 digits. Put 5 next, then 0, 0, 0: number = 95000 (9+5+0+0+0=14). This is the largest.
d. Add zeros between non-zero digits to make very large numbers: 95000, 950000, 9500000, ... The more zeros we add, the larger the number. We can always make a bigger number by inserting more zeros.

Answer: a. E.g., 59, 68, 77, 86, 95, 176, 248, 356. b. 59. c. 95000. d. By adding zeros: 95,000 → 9,50,000 → 9,500,000 → … (can always be made bigger).

Q3 Calculate the digit sums of 3-digit numbers whose digits are consecutive (for example, 345). Do you see a pattern? Will this pattern continue?

SOLUTION

123: 1+2+3 = 6
234: 2+3+4 = 9
345: 3+4+5 = 12
456: 4+5+6 = 15
567: 5+6+7 = 18
678: 6+7+8 = 21
789: 7+8+9 = 24
Pattern: the digit sums are 6, 9, 12, 15, 18, 21, 24 — multiples of 3, increasing by 3 each time.
Reason: if the three consecutive digits are n, n+1, n+2, their sum = 3n+3 = 3(n+1), which is always a multiple of 3.
The pattern of multiples of 3 continues, but the sequence of consecutive-digit 3-digit numbers ends at 789.

Answer: Digit sums: 6, 9, 12, 15, 18, 21, 24. Pattern: all are multiples of 3 (increasing by 3). It continues as long as consecutive digits exist (last is 789 with sum 24).

Exercise 3.5 — Pretty Palindromic Patterns

Q1 Write all possible 3-digit palindromes using the digits 1, 2, 3 (digits can repeat).

SOLUTION

A 3-digit palindrome has the form ABA where first and last digits are the same.
A can be 1, 2, or 3; B can be 1, 2, or 3.
Listing all (3 choices for A × 3 choices for B = 9 palindromes):
A=1: 111, 121, 131
A=2: 212, 222, 232
A=3: 313, 323, 333

Answer: 111, 121, 131, 212, 222, 232, 313, 323, 333 — nine 3-digit palindromes in all.

Q2 Puzzle time: I am a 5-digit palindrome. I am an odd number. My 't' (tens) digit is double of my 'u' (units) digit. My 'h' (hundreds) digit is double of my 't' (tens) digit. Who am I?

SOLUTION

Let u = units digit. Then t = 2u and h = 2t = 4u.
Since h must be a single digit: 4u ≤ 9, so u ≤ 2.
Since the number is odd, u must be odd: u = 1.
Then t = 2×1 = 2, h = 2×2 = 4.
A 5-digit palindrome has form: tth, th, h, t, u → but the place values are: ten-thousands (tth), thousands (th), hundreds (h), tens (t), units (u).
Palindrome condition: ten-thousands digit = units digit = 1, thousands digit = tens digit = 2.
So the number is: 1, 2, 4, 2, 1 → 12421.
Check: 12421 reversed = 12421 ✓ (palindrome). Odd ✓. t=2, u=1, t=2u ✓. h=4, t=2, h=2t ✓.

Answer: 12421 (twelve thousand four hundred twenty-one).

Exercise 3.6 — The Magic Number of Kaprekar

Q1 Carry out the Kaprekar steps with a few 3-digit numbers. What number will start repeating?

SOLUTION

Kaprekar rule: from a 3-digit number, form the largest and smallest arrangements of its digits, subtract smaller from larger, repeat.

Example starting with 321:

321: largest = 321, smallest = 123. Difference = 321 − 123 = 198.

198: largest = 981, smallest = 189. Difference = 981 − 189 = 792.

792: largest = 972, smallest = 279. Difference = 972 − 279 = 693.

693: largest = 963, smallest = 369. Difference = 963 − 369 = 594.

594: largest = 954, smallest = 459. Difference = 954 − 459 = 495.

495: largest = 954, smallest = 459. Difference = 954 − 459 = 495.

495 keeps repeating. The 3-digit Kaprekar constant is 495.

Answer: 495 is the repeating number (3-digit Kaprekar constant). All 3-digit numbers eventually reach 495 and stay there.

Exercise 3.7 — Clock and Calendar Numbers

Q1 Pratibha uses the digits '4', '7', '3' and '2', and makes the smallest and largest 4-digit numbers with them: 2347 and 7432. The difference is 7432 – 2347 = 5085. The sum is 9779. Choose 4 digits to make:
a. the difference between the largest and smallest numbers greater than 5085.
b. the difference between the largest and smallest numbers less than 5085.
c. the sum of the largest and smallest numbers greater than 9779.
d. the sum of the largest and smallest numbers less than 9779.

SOLUTION

a. Need difference > 5085. Choose digits with a large spread, e.g., 1, 3, 4, 9: largest = 9431, smallest = 1349. Difference = 9431 − 1349 = 8082 > 5085 ✓.
b. Need difference < 5085. Choose digits that are close together, e.g., 2, 3, 4, 5: largest = 5432, smallest = 2345. Difference = 5432 − 2345 = 3087 < 5085 ✓.
c. Need sum > 9779. Choose large digits, e.g., 6, 7, 8, 9: largest = 9876, smallest = 6789. Sum = 9876 + 6789 = 16665 > 9779 ✓.
d. Need sum < 9779. Choose small digits, e.g., 1, 2, 3, 4: largest = 4321, smallest = 1234. Sum = 4321 + 1234 = 5555 < 9779 ✓.

Answer: a. e.g., digits 1,3,4,9: 9431−1349 = 8082. b. e.g., digits 2,3,4,5: 5432−2345 = 3087. c. e.g., digits 6,7,8,9: 9876+6789 = 16665. d. e.g., digits 1,2,3,4: 4321+1234 = 5555.

Q2 What is the sum of the smallest and largest 5-digit palindrome? What is their difference?

SOLUTION

Smallest 5-digit palindrome: 10001 (form A B C B A with A=1, B=0, C=0).
Largest 5-digit palindrome: 99999 (all nines).
Sum = 10001 + 99999 = 110000.
Difference = 99999 − 10001 = 89998.

Answer: Sum = 1,10,000. Difference = 89,998.

Q3 The time now is 10:01. How many minutes until the clock shows the next palindromic time? What about the one after that?

SOLUTION

Current time: 10:01. Written as digits: 1, 0, 0, 1 — reads the same forwards and backwards → 10:01 is itself a palindrome.
Next palindromic time: think of times where the digits mirror: 11:11.
Time from 10:01 to 11:11 = 1 hour 10 minutes = 70 minutes.
The palindrome after 11:11: 12:21.
Time from 10:01 to 12:21 = 2 hours 20 minutes = 140 minutes.

Answer: 70 minutes until 11:11 (the next palindromic time). 140 minutes until 12:21 (the one after that).

Q4 How many rounds does the number 5683 take to reach the Kaprekar constant?

SOLUTION

Apply Kaprekar steps (arrange digits to form largest and smallest, subtract):
Round 1: 5683 → digits 3,5,6,8: largest = 8653, smallest = 3568. C = 8653 − 3568 = 5085.
Round 2: 5085 → digits 0,5,5,8: largest = 8550, smallest = 0558 = 558. C = 8550 − 558 = 7992.
Round 3: 7992 → digits 2,7,9,9: largest = 9972, smallest = 2799. C = 9972 − 2799 = 7173.
Round 4: 7173 → digits 1,3,7,7: largest = 7731, smallest = 1377. C = 7731 − 1377 = 6354.
Round 5: 6354 → digits 3,4,5,6: largest = 6543, smallest = 3456. C = 6543 − 3456 = 3087.
Round 6: 3087 → digits 0,3,7,8: largest = 8730, smallest = 0378 = 378. C = 8730 − 378 = 8352.
Round 7: 8352 → digits 2,3,5,8: largest = 8532, smallest = 2358. C = 8532 − 2358 = 6174.
Reached the Kaprekar constant 6174 in 7 rounds.

Answer: 7 rounds. (5683 → 5085 → 7992 → 7173 → 6354 → 3087 → 8352 → 6174)

Exercise 3.8 — Mental Math

Q1 Using the numbers 40000, 7000, 300, 1500, 12000, 800 (each may be used multiple times), find expressions using addition and subtraction for: 45000, 5900, 17500, 21400.

SOLUTION

45000 = 40000 + 7000 − 1500 − 800 + 300
Verify: 40000 + 7000 = 47000; 47000 − 1500 = 45500; 45500 − 800 = 44700; 44700 + 300 = 45000 ✓

5900 = 7000 − 800 − 300
Verify: 7000 − 800 = 6200; 6200 − 300 = 5900 ✓

17500 = 12000 + 7000 − 1500
Verify: 12000 + 7000 = 19000; 19000 − 1500 = 17500 ✓

21400 = 40000 − 12000 − 7000 + 1500 − 800 − 300
Verify: 40000 − 12000 = 28000; 28000 − 7000 = 21000; 21000 + 1500 = 22500; 22500 − 800 = 21700; 21700 − 300 = 21400 ✓

Answer: 45000 = 40000 + 7000 − 1500 − 800 + 300; 5900 = 7000 − 800 − 300; 17500 = 12000 + 7000 − 1500; 21400 = 40000 − 12000 − 7000 + 1500 − 800 − 300.

Q2 Write an example for each of the scenarios below whenever possible:
(i) 5-digit + 5-digit giving a 5-digit sum more than 90,250
(ii) 5-digit + 3-digit giving a 6-digit sum
(iii) 4-digit + 4-digit giving a 6-digit sum
(iv) 5-digit + 5-digit giving a 6-digit sum
(v) 5-digit + 5-digit giving 18,500
(vi) 5-digit − 5-digit giving a difference less than 56,503
(vii) 5-digit − 3-digit giving a 4-digit difference
(viii) 5-digit − 4-digit giving a 4-digit difference
(ix) 5-digit − 5-digit giving a 3-digit difference
(x) 5-digit − 5-digit giving 91,500

SOLUTION

(i) 45000 + 45400 = 90400 > 90250 ✓
(ii) 99999 + 999 = 100998 (6 digits) ✓
(iii) Not possible. The sum of the two greatest 4-digit numbers: 9999 + 9999 =

19998 (5 digits, not 6).
(iv) 60000 + 40000 = 100000 (6 digits) ✓
(v) Not possible. The smallest 5-digit number is 10000. 10000 + 10000 = 20000 > 18500. So no two 5-digit numbers can sum to 18500.
(vi) 80000 − 50000 = 30000 < 56503 ✓
(vii) 10000 − 999 = 9001 (4 digits) ✓
(viii) 12000 − 2500 = 9500 (4 digits) ✓
(ix) 50999 − 50000 = 999 (3 digits) ✓
(x) Not possible. Maximum difference of 5-digit numbers = 99999 − 10000 = 89999 < 91500.

Answer: (i) 45000+45400=90400 ✓ (ii) 99999+999=100998 ✓ (iii) Not possible (max 9999+9999=19998) (iv) 60000+40000=100000 ✓ (v) Not possible (vi) 80000−50000=30000 ✓ (vii) 10000−999=9001 ✓ (viii) 12000−2500=9500 ✓ (ix) 50999−50000=999 ✓ (x) Not possible (max diff=89999)

Q3 Always, Sometimes, Never? Determine for each statement:
a. 5-digit number + 5-digit number gives a 5-digit number
b. 4-digit number + 2-digit number gives a 4-digit number
c. 4-digit number + 2-digit number gives a 6-digit number
d. 5-digit number − 5-digit number gives a 5-digit number
e. 5-digit number − 2-digit number gives a 3-digit number

SOLUTION

a. Sometimes true. E.g., 10000 + 10000 = 20000 (5 digits) ✓; but 50000 + 60000 = 110000 (6 digits) — not 5 digits.
b. Sometimes true. E.g., 1000 + 10 = 1010 (4 digits) ✓; but 9999 + 99 = 10098 (5 digits) — not 4 digits.
c. Never true. Maximum: 9999 + 99 = 10098 (5 digits). A 4-digit + 2-digit sum can be at most 5 digits, never 6.
d. Sometimes true. E.g., 99999 − 10000 = 89999 (5 digits) ✓; but 12000 − 10000 = 2000 (4 digits) — not 5 digits.
e. Never true. Smallest 5-digit number = 10000, largest 2-digit = 99. 10000 − 99 = 9901 (4 digits). A 5-digit number minus a 2-digit number is always at least 4 digits.

Answer: a. Sometimes true. b. Sometimes true. c. Never true. d. Sometimes true. e. Never true.

Exercise 3.9 — Playing with Number Patterns

Q1 Find out the sum of the numbers in each of the below figures.

a. Grid with 40s and 50s (3 rows of four 40s alternating with 2 rows of five 50s)

c. Grid with 32s (4 rows × 8 columns) and 64s (4 rows with 3 on left and 1 on right)

SOLUTION

a. Count the values: 3 rows × 4 cells of 40 = 12 cells of 40; 2 rows × 5 cells of 50 = 10 cells of 50.

Sum a = 12 × 40 + 10 × 50 = 480 + 500 = 980.

c. Top section: 4 rows × 8 columns = 32 cells, each containing 32.

Bottom section: 4 rows, each with 3 cells of 64 on the left and 1 cell of 64 on the right = 4 × 4 = 16 cells of 64.

Sum c = 32 × 32 + 16 × 64 = 1024 + 1024 = 2048.

Answer: a. Sum = 980. c. Sum = 2048.

Exercise 3.11 — Simple Estimation

Q1 Roshan wants to buy milk and 3 types of fruit to make fruit custard for 5 people. He estimates the cost to be 100. Do you agree with him? Why or why not?

SOLUTION

Cost of milk for 5 people: roughly 30–40 for half a litre.
Cost of 3 types of fruit (e.g., banana, apple, papaya): 20–30 each = 60–90 total.
Total: 90–130 approximately.
100 is a reasonable estimate if basic/affordable fruits and a small quantity of milk are used.
It may not be enough with expensive fruits or larger servings.

Answer: 100 is a reasonable estimate for simple fruit custard with affordable fruits and small portions, but may fall short with expensive fruits or larger quantities.

Q2 Estimate the distance between Gandhinagar (in Gujarat) to Kohima (in Nagaland).

SOLUTION

Gandhinagar is in western India (Gujarat); Kohima is in the far northeast (Nagaland).
India's width from west to east is roughly 3000 km.
Estimated straight-line distance ≈ 2500 km.

Answer: Approximately 2500 km.

Q3 Sheetal is in Grade 6 and says she has spent around 13,000 hours in school till date. Do you agree with her? Why or why not?

SOLUTION

A school day is about 6 hours.
A school year has about 200 working days.
Hours per year = 6 × 200 = 1200 hours.
Sheetal has been in school for about 8 years (Nursery through Grade 6).
Total hours ≈ 8 × 1200 = 9600 hours.
13,000 hours ÷ 1200 hours per year ≈ 10.8 years — which is too many.
Sheetal's estimate of 13,000 hours is too high.

Answer: No. A realistic estimate: 8 years × 200 school days × 6 hours = 9,600 hours, not 13,000.

Exercise 3.12 — Games and Winning Strategies

Q1 There is only one supercell (number greater than all its neighbours) in this grid:
16200 | 39344 | 29765
23609 | 62871 | 45306
19381 | 50319 | 38408
If you exchange two digits of one of the numbers, there will be 4 supercells. Figure out which digits to swap.

SOLUTION

Currently only 62871 is a supercell (it is greater than all its neighbours: 39344, 45306, 23609, 50319).
We want to make 62871 small so other cells can become supercells.
Swap the digits '6' and '1' in 62871 to get 12876.
New grid:

16200	39344	29765
23609	12876	45306
19381	50319	38408

Check supercells:
39344: neighbours = 12876, 16200, 29765. Is 39344 > all? 39344 > 12876 ✓, > 16200 ✓, > 29765 ✓ → Supercell ✓
23609: neighbours = 16200, 19381, 12876. 23609 > all → Supercell ✓
45306: neighbours = 29765, 38408, 12876. 45306 > all → Supercell ✓
50319: neighbours = 12876, 19381, 38408. 50319 > all → Supercell ✓
Total supercells = 4 ✓

Answer: Swap digits '6' and '1' in 62871, making it 12876. This creates 4 supercells: 39344, 23609, 45306, and 50319.

Q2 How many rounds does your year of birth take to reach the Kaprekar constant?

SOLUTION

The answer varies by birth year. As an example, take 2012:
2012: digits 0,1,2,2: largest = 2210, smallest = 0122 = 122. C = 2210 − 122 = 2088.
2088: digits 0,2,8,8: largest = 8820, smallest = 0288 = 288. C = 8820 − 288 =

8532.
8532: digits 2,3,5,8: largest = 8532, smallest = 2358. C = 8532 − 2358 = 6174.
Reached 6174 in 3 rounds.
Try your own birth year using the same steps!

Answer: It depends on your birth year. Try the Kaprekar steps on your own year. (Example: 2012 takes 3 rounds.)

Q3 We are the group of 5-digit numbers between 35,000 and 75,000 such that all of our digits are odd. Who is the largest number in our group? Who is the smallest? Who among us is the closest to 50,000?

SOLUTION

Odd digits: 1, 3, 5, 7, 9. Numbers strictly between 35000 and 75000 with all digits odd.
Largest: First digit as large as possible while keeping number < 75000. First digit = 7. Second digit must be odd and 7d < 75 → d ≤ 4; odd options ≤ 4: 1, 3. Choose 3 (larger). Remaining 3 digits: maximise → 9, 9, 9. Largest = 73999.
Smallest: First digit must give number > 35000. First digit = 3: second digit must be odd and 3d > 35 (as first two digits) → second digit ≥ 5. Smallest odd ≥ 5 is 5. Remaining 3 digits: minimise → 1, 1, 1. Smallest = 35111.
Closest to 50000: 50000 has digit 0 (even). Cannot be in the group. Nearest above: smallest 5-digit all-odd starting with 5 is 51111 (5+1+1+1+1). |51111 − 50000| = 1111. Nearest below 50000 with all odd digits: largest such < 50000 starting with first digit ≤ 4: must start with 3 (as 4 is even). Max = 39999. |39999 − 50000| = 10001. So 51111 is closer.

Answer: Largest: 73,999. Smallest: 35,111. Closest to 50,000: 51,111 (distance 1111).

Q4 Estimate the number of holidays you get in a year including weekends, festivals and vacation. Then, try to get an exact number and see how close your estimate is.

SOLUTION

Weekends: 52 weeks × 2 days = about 104 days (some schools have only Sunday off → about 52 days).
Festival/national holidays: roughly 15–20 days spread through the year.

Vacations (summer + winter + other breaks): roughly 45–60 days.
Estimated total: about 104 + 18 + 50 ≈ 172 days (or nearer 120 days if only Sundays are off).
The exact number varies by school and state; compare your estimate against your own school calendar.

Answer: About 150–180 holidays a year (weekends + festivals + vacations). The exact figure depends on your school's calendar — check it and compare with your estimate.

Q5 Estimate the number of liters a mug, a bucket and an overhead tank can hold.

SOLUTION

A typical mug holds about 1 litre (some are smaller, about half a litre).
A household bucket holds about 15–20 litres.
An overhead water tank is much larger — commonly about 500 to 1000 litres.
So the rough order is: mug ≈ 1 L, bucket ≈ 15–20 L, overhead tank ≈ 500–1000 L.

Answer: Approximately: mug ≈ 1 litre, bucket ≈ 15–20 litres, overhead tank ≈ 500–1000 litres.

Q6 Write one 5-digit number and two 3-digit numbers such that their sum is 18,670.

SOLUTION

Choose a 5-digit number close to 18,670 but leaving room for two 3-digit numbers.
Example: 18,670 − 300 − 370 = 18,000.
Check: 18000 + 300 + 370 = 18670 ✓.
18000 is a 5-digit number ✓; 300 and 370 are 3-digit numbers ✓.

Answer: Example: 18,000 + 300 + 370 = 18,670. (Many other valid answers exist.)

Q7 Choose a number between 210 and 390. Create a number pattern similar to those shown in Section 3.9 that will sum up to this number.

SOLUTION

Pick a target between 210 and 390, say 300.
Make a pattern using equal blocks that add to 300 — like the grids in Section 3.9.
Example: arrange five 60s in a plus/cross shape: 60 + 60 + 60 + 60 + 60 = 300.
Another example for 240: a 4 × 4 grid of 15s → 16 × 15 = 240.
Any arrangement of numbers whose total lands between 210 and 390 works — count the cells and multiply.

Answer: Example (target 300): a cross of five 60s, 60 × 5 = 300. (Any pattern summing to a value between 210 and 390 is valid.)

Q8 Recall the sequence of Powers of 2 from Chapter 1, Table 1. Why is the Collatz conjecture correct for all the starting numbers in this sequence?

SOLUTION

Powers of 2: 1, 2, 4, 8, 16, 32, 64, 128, ...
Every power of 2 is even.
Collatz rule for even numbers: divide by 2.
Starting from 2^n: divide by 2 → 2^(n−1) → 2^(n−2) → … → $2^1 = 2 \rightarrow 2^0 = 1$.
Each step halves the number, so we reach 1 after exactly n steps.
The Collatz conjecture holds for all powers of 2 because repeated halving always reaches 1.

Answer: Every power of 2 is even, so we keep dividing by 2 at every step. This gives 2^n → 2^(n−1) → … → 2 → 1. The sequence always reaches 1 in exactly n steps.

Q9 Check if the Collatz Conjecture holds for the starting number 100.

SOLUTION

Apply Collatz rule: even → ÷2, odd → ×3+1.
100 (even) → 50 → 25 (odd) → 76 → 38 → 19 (odd) → 58 → 29 (odd) → 88 → 44 → 22 → 11 (odd) → 34 → 17 (odd) → 52 → 26 → 13 (odd) → 40 → 20 → 10 → 5 (odd) → 16 → 8 → 4 → 2 → 1.
The sequence reaches 1. Collatz conjecture holds for 100.

Answer: Yes. The sequence: 100, 50, 25, 76, 38, 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. Reaches 1 ✓.

Q10 Starting with 0, players alternate adding numbers between 1 and 3. The first person to reach 22 wins. What is the winning strategy now?

SOLUTION

The target is 22, starting from 0, adding 1–3 per turn.
Work backwards: to win, you want to land on 22. From 22 you've won.
From 19, 20, or 21: the current player can reach 22 → current player wins.
From 18: any move (adding 1, 2, or 3) puts opponent at 19, 20, or 21 → opponent wins. So 18 is a losing position.
From 15, 16, or 17: can reach 18 → put opponent in losing position → winning.
From 14: any move puts opponent at 15, 16, or 17 → opponent wins. Losing position.
Losing positions: 18, 14, 10, 6, 2 (each 4 less than the next).
Starting position is 0. First player moves to 1, 2, or 3.
First player should say 2 (go to position 2, which is a losing position for the second player).
Then: whatever second player adds (1, 2, or 3), first player brings total to 6 → 10 → 14 → 18 → 22.
First player always wins by going first and saying 2.

Answer: The first player wins. Strategy: first player says 2, then always adds to bring the running total to 6, 10, 14, 18, and finally 22.