Chapter 5: Prime Time

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Exercise 5.1 — Common Multiples and Common Factors

Q1 At what number is 'idli-vada' said for the 10th time? (In the Idli-Vada game, 'idli' is said for multiples of 3 and 'vada' for multiples of 5.)

SOLUTION

'Idli-vada' is said for numbers that are multiples of both 3 and 5.

The least common multiple (LCM) of 3 and 5 is 15.

So 'idli-vada' is said at: 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, …

Counting: 1st = 15, 2nd = 30, 3rd = 45, 4th = 60, 5th = 75, 6th = 90, 7th = 105, 8th = 120, 9th = 135, 10th = 150.

Answer: 150

Q2 If the game is played for the numbers 1 to 90, find out:
a. How many times would the children say 'idli' (including the times they say 'idli-vada')?
b. How many times would the children say 'vada' (including the times they say 'idli-vada')?
c. How many times would the children say 'idli-vada'?

SOLUTION

a. 'Idli' is said for every multiple of 3 from 1 to 90.
Multiples of 3 from 1 to 90: 3, 6, 9, …, 90.
Count = 90 ÷ 3 = 30.

b. 'Vada' is said for every multiple of 5 from 1 to 90.
Multiples of 5 from 1 to 90: 5, 10, 15, …, 90.
Count = 90 ÷ 5 = 18.

c. 'Idli-vada' is said for every multiple of 15 from 1 to 90.
Multiples of 15 from 1 to 90: 15, 30, 45, 60, 75, 90.
Count = 90 ÷ 15 = 6.

Answer: a. 30 times; b. 18 times; c. 6 times

Q3 What if the game was played till 900? How would your answers change?

SOLUTION

Scaling from 90 to 900 (10 times larger):

a. Multiples of 3 up to 900: 900 ÷ 3 = 300. 'Idli' is said 300 times.

b. Multiples of 5 up to 900: 900 ÷ 5 = 180. 'Vada' is said 180 times.

c. Multiples of 15 up to 900: 900 ÷ 15 = 60. 'Idli-vada' is said 60 times.

Each count is exactly 10 times the count for 90, as 900 = 10 × 90.

Answer: 'Idli' 300 times; 'Vada' 180 times; 'Idli-vada' 60 times

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Q4 Is the figure (the Venn diagram with circles for 'Multiples of 3' and 'Multiples of 5') somehow related to the 'idli-vada' game? Hint: Imagine playing the game till 30. Draw the figure if the game is played till 60.

SOLUTION

Yes, the figure is directly related to the idli-vada game.

The left circle contains numbers for which 'idli' is said (multiples of 3 only).
The right circle contains numbers for which 'vada' is said (multiples of 5 only).
The overlapping region (common multiples of 3 and 5) contains the numbers for which 'idli-vada' is said: 15, 30, 45, …

For the game played till 60, the common multiples in the overlap would be 15, 30, 45, 60.
Left circle (multiples of 3 only, not 5): 3, 6, 9, 12, 18, 21, 24, 27, 33, 36, 39, 42, 48, 51, 54, 57.
Right circle (multiples of 5 only, not 3): 5, 10, 20, 25, 35, 40, 50, 55.

Answer: Yes — the Venn diagram's two circles represent the 'idli' numbers (multiples of 3) and 'vada' numbers (multiples of 5); the overlapping region gives the 'idli-vada' numbers (common multiples of 3 and 5).

Exercise 5.2 — Common Multiples and Common Factors

Q1 Find all multiples of 40 that lie between 310 and 410.

SOLUTION

Multiples of 40: 40, 80, 120, 160, 200, 240, 280, 320, 360, 400, 440, …

Those lying strictly between 310 and 410 are: 320, 360, 400.

Answer: 320, 360, 400

Q2 Who am I?
a. I am a number less than 40. One of my factors is 7. The sum of my digits is 8.
b. I am a number less than 100. Two of my factors are 3 and 5. One of my digits is 1 more than the other.

SOLUTION

a. The number must be a multiple of 7 and less than 40.
Multiples of 7 less than 40: 7, 14, 21, 28, 35.
Check digit sums: 7→7, 14→5, 21→3, 28→10, 35→3+5=8. ✓
The number is 35.

b. The number must be a multiple of both 3 and 5, so a multiple of LCM(3,5) = 15.
Multiples of 15 less than 100: 15, 30, 45, 60, 75, 90.
Check 'one digit is 1 more than the other': |1−5|=4, |3−0|=3, |4−5|=1 ✓, |6−0|=6, |7−5|=2, |9−0|=9.
Only 45 satisfies: 5 = 4 + 1.
The number is 45.

Answer: a. 35; b. 45

Q3 A number for which the sum of all its factors is equal to twice the number is called a perfect number. The number 28 is a perfect number. Its factors are 1, 2, 4, 7, 14 and 28. Their sum is 56 which is twice 28. Find a perfect number between 1 and 10.

SOLUTION

Check each number from 1 to 10:

6: factors are 1, 2, 3, 6. Sum = 1 + 2 + 3 + 6 = 12 = 2 × 6. ✓

All other numbers from 1 to 10 do not satisfy this condition.

1: sum = 1 ≠ 2. 2: 1+2=3 ≠ 4. 3: 1+3=4 ≠ 6. 4: 1+2+4=7 ≠ 8.
5: 1+5=6 ≠ 10. 7: 1+7=8 ≠ 14. 8: 1+2+4+8=15 ≠ 16.
9: 1+3+9=13 ≠ 18. 10: 1+2+5+10=18 ≠ 20.

Answer: 6

Q4 Find the common factors of:
a. 20 and 28
b. 35 and 50
c. 4, 8 and 12
d. 5, 15 and 25

SOLUTION

a. Factors of 20: 1, 2, 4, 5, 10, 20.
Factors of 28: 1, 2, 4, 7, 14, 28.
Common factors: 1, 2, 4.

b. Factors of 35: 1, 5, 7, 35.
Factors of 50: 1, 2, 5, 10, 25, 50.
Common factors: 1, 5.

c. Factors of 4: 1, 2, 4.
Factors of 8: 1, 2, 4, 8.
Factors of 12: 1, 2, 3, 4, 6, 12.
Common factors of all three: 1, 2, 4.

d. Factors of 5: 1, 5.
Factors of 15: 1, 3, 5, 15.
Factors of 25: 1, 5, 25.
Common factors of all three: 1, 5.

Answer: a. 1, 2, 4; b. 1, 5; c. 1, 2, 4; d. 1, 5

Q5 Find any three numbers that are multiples of 25 but not multiples of 50.

SOLUTION

Multiples of 25: 25, 50, 75, 100, 125, 150, 175, …
Multiples of 50: 50, 100, 150, 200, …

Multiples of 25 that are NOT multiples of 50 are the odd multiples of 25: 25, 75, 125, 175, …

Three such numbers: 25, 75, 125.

Answer: 25, 75, 125 (any three odd multiples of 25)

Q6 Anshu and his friends play the 'idli-vada' game with two numbers, which are both smaller than 10. The first time anybody says 'idli-vada' is after the number 50. What could the two numbers be which are assigned 'idli' and 'vada'?

SOLUTION

The first 'idli-vada' is said at the LCM of the two numbers.
We need: LCM of two numbers (both < 10) > 50.

Check pairs with LCM > 50:
(7, 8): LCM = 56 > 50. ✓
(7, 9): LCM = 63 > 50. ✓
(8, 9): LCM = 72 > 50. ✓

The two numbers could be 7 and 8 (first idli-vada at 56) or 8 and 9 (first at 72).

Answer: 7 and 8 (first idli-vada at 56), or 8 and 9 (first at 72)

Q7 In the treasure hunting game, Grumpy has kept treasures on 28 and 70. What jump sizes will land on both the numbers?

SOLUTION

A jump size lands on both 28 and 70 if and only if it is a common factor of both.

Factors of 28: 1, 2, 4, 7, 14, 28.
Factors of 70: 1, 2, 5, 7, 10, 14, 35, 70.

Common factors: 1, 2, 7, 14.

Verify: jump 14 → lands on 14, 28 ✓, 42, 56, 70 ✓. Jump 7 → 7, 14, 21, 28 ✓, …, 70 ✓.

Answer: Jump sizes 1, 2, 7, and 14

Q8 In the diagram, Guna has erased all the numbers except the common multiples. The common multiples shown are 24, 48, and 72. Find out what those numbers could be and fill in the missing numbers in the empty regions.

SOLUTION

The common multiples are 24, 48, 72.
These are all multiples of 24, and 24 = 8 × 3.

LCM of the two numbers = 24.
Possible pairs: (3, 8) or (6, 8).

Using 3 and 8:
Multiples of 3 only (not 8): 3, 6, 9, 12, 15, 18, 21, 27, 30, …
Multiples of 8 only (not 3): 8, 16, 32, 40, 56, 64, …
Common multiples (of both): 24, 48, 72, …

Answer: The two numbers could be 3 and 8 (or 6 and 8); common multiples shown are 24, 48, 72

Q9 Find the smallest number that is a multiple of all the numbers from 1 to 10, except for 7.

SOLUTION

We need LCM(1, 2, 3, 4, 5, 6, 8, 9, 10).

Find LCM step by step:
LCM(1,2) = 2
LCM(2,3) = 6
LCM(6,4) = 12
LCM(12,5) = 60
LCM(60,6) = 60
LCM(60,8) = 120
LCM(120,9) = 360
LCM(360,10) = 360

Verify: 360 ÷ 1=360, ÷2=180, ÷3=120, ÷4=90, ÷5=72, ÷6=60, ÷8=45, ÷9=40, ÷10=36. All exact. ✓

Answer: 360

Q10 Find the smallest number that is a multiple of all the numbers from 1 to 10.

SOLUTION

We need LCM(1, 2, 3, 4, 5, 6, 7, 8, 9, 10).

From the previous question, LCM(1 to 10 except 7) = 360.
Now include 7: LCM(360, 7).

360 = $2^3 \times 3^2 \times 5$. Since 7 is prime and does not divide 360:
LCM(360, 7) = 360 × 7 = 2520.

Verify: 2520 ÷ 7 = 360. ✓ Also 2520 divisible by 1–10 (all verified as for 360, plus ÷7=360). ✓

Answer: 2520

Exercise 5.3 — Prime Numbers

Q1 We see that 2 is a prime and also an even number. Is there any other even prime?

SOLUTION

Any even number greater than 2 can be written as $2 \times k$ for some $k > 1$.
This means it has 2 and k as factors (in addition to 1 and itself), so it has more than two factors.
Therefore it is composite, not prime.

2 is the only even prime number.

Answer: No, 2 is the only even prime number

Q2 Look at the list of primes till 100. What is the smallest difference between two successive primes? What is the largest difference?

SOLUTION

Primes up to 100: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Consecutive differences: $3-2=1$, $5-3=2$, $7-5=2$, $11-7=4$, $13-11=2$, $17-13=4$, $19-17=2$, $23-19=4$, $29-23=6$, $31-29=2$, …, $97-89=8$.

Smallest difference: 1 (between 2 and 3).
Largest difference: 8 (between 89 and 97).

Answer: Smallest difference: 1 (between 2 and 3); Largest difference: 8 (between 89 and 97)

Q3 Are there an equal number of primes occurring in every row in the table (Sieve of Eratosthenes, numbers 1–100 in rows of 10)? Which decades have the least number of primes? Which have the most number of primes?

SOLUTION

Count primes in each decade (row of 10):

1–10: 2, 3, 5, 7 → 4 primes
11–20: 11, 13, 17, 19 → 4 primes
21–30: 23, 29 → 2 primes
31–40: 31, 37 → 2 primes

41–50: 41, 43, 47 → 3 primes
51–60: 53, 59 → 2 primes
61–70: 61, 67 → 2 primes
71–80: 71, 73, 79 → 3 primes
81–90: 83, 89 → 2 primes
91–100: 97 → 1 prime

No, the count is not equal in every row.
Least primes: 91–100 (only 1 prime).
Most primes: 1–10 and 11–20 (4 primes each).

Answer: No; decades 1–10 and 11–20 have the most (4 primes each); decade 91–100 has the least (1 prime)

Q4 Which of the following numbers are prime: 23, 51, 37, 26?

SOLUTION

23: Check divisibility by 2, 3. 23 is odd; 2+3=5 not divisible by 3. Not divisible by any prime up to $\sqrt{23} \approx 4.8$. Prime. ✓

51: 5+1=6, divisible by 3. So 51 = 3 × 17. Composite.

37: Odd; 3+7=10 not divisible by 3; not divisible by 5. Check 7: 37÷7≈5.3. Not divisible by any prime up to $\sqrt{37} \approx 6.1$. Prime. ✓

26: Even (divisible by 2). 26 = 2 × 13. Composite.

Answer: 23 and 37 are prime

Q5 Write three pairs of prime numbers less than 20 whose sum is a multiple of 5.

SOLUTION

Primes less than 20: 2, 3, 5, 7, 11, 13, 17, 19.

Find pairs with sum divisible by 5:
2 + 3 = 5. ✓ (multiple of 5)
2 + 13 = 15. ✓
3 + 7 = 10. ✓
3 + 17 = 20. ✓
7 + 13 = 20. ✓

Three such pairs: (2, 3), (2, 13), (3, 7).

Answer: (2, 3), (2, 13), (3, 7) — and others like (3, 17) or (7, 13)

Q6 The numbers 13 and 31 are prime numbers. Both these numbers have the same digits 1 and 3. Find such pairs of prime numbers up to 100.

SOLUTION

Look for two-digit primes where reversing the digits gives another prime:

13 and 31: both prime. ✓ (already given as example)
17 and 71: both prime. ✓
37 and 73: both prime. ✓
79 and 97: both prime. ✓

Other candidates: 11 (reverse=11, same number); 12 (not prime); 14 (not prime).
So the pairs (other than the given example) are: (17, 71), (37, 73), (79, 97).

Answer: (17, 71), (37, 73), (79, 97)

Q7 Find seven consecutive composite numbers between 1 and 100.

SOLUTION

Look for a run of 7 consecutive composite numbers:

90 = 2×45, 91 = 7×13, 92 = 4×23, 93 = 3×31, 94 = 2×47, 95 = 5×19, 96 = 2×48.

All seven numbers 90, 91, 92, 93, 94, 95, 96 are composite. ✓
The next prime after 89 is 97, so there is a gap of 7 consecutive composites.

Answer: 90, 91, 92, 93, 94, 95, 96

Q8 Twin primes are pairs of primes having a difference of 2. For example, 3 and 5 are twin primes. So are 17 and 19. Find the other twin primes between 1 and 100.

SOLUTION

Primes up to 100: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Pairs with difference 2:
(3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73).

Answer: (3,5), (5,7), (11,13), (17,19), (29,31), (41,43), (59,61), (71,73)

Q9 Identify whether each statement is true or false. Explain.

a. There is no prime number whose units digit is 4.

b. A product of primes can also be prime.

c. Prime numbers do not have any factors.

d. All even numbers are composite numbers.

e. 2 is a prime and so is the next number, 3. For every other prime, the next number is composite.

SOLUTION

a. TRUE. A number ending in 4 is even (divisible by 2). The only even prime is 2 itself. So no prime ends in 4.

b. FALSE. The product of two or more primes has those primes as factors (in addition to 1 and itself), giving it more than two factors. So it is composite.

Example: 2 × 3 = 6 is composite.

c. FALSE. Every prime number has exactly two factors: 1 and itself.

d. FALSE. The number 2 is even but prime, not composite.

e. TRUE. Every prime other than 2 is odd. The number after any odd number is even. An even number greater than 2 is always composite (divisible by 2). Since the smallest such prime is 5 (and 5+1=6 is composite), this holds for all primes greater than 3.

Answer: a. True; b. False; c. False; d. False; e. True

Q10 Which of the following numbers is the product of exactly three distinct prime numbers: 45, 60, 91, 105, 330?

SOLUTION

Find the prime factorisation of each:

45 = 3 × 3 × 5 = 3^2 × 5. Only 2 distinct primes (3 and 5). ✗

60 = 2 × 2 × 3 × 5. Three distinct primes but 2 appears twice, so not a product of exactly 3 primes. ✗

91 = 7 × 13. Only 2 distinct primes. ✗

105 = 3 × 5 × 7. Exactly 3 distinct primes, each appearing once. ✓

330 = 2 × 3 × 5 × 11. Four distinct primes. ✗

Only 105 = 3 × 5 × 7 is the product of exactly three distinct prime numbers.

Answer: 105 (= 3 × 5 × 7)

Q11 How many three-digit prime numbers can you make using each of 2, 4 and 5 once?

SOLUTION

The possible three-digit numbers using 2, 4, 5 each once:
245, 254, 425, 452, 524, 542.

Check each for primality:
245 = 5 × 49 = 5 × 7². Composite.
254 = 2 × 127. Composite.
425 = 5 × 85 = 5 × 5 × 17. Composite.
452 = 4 × 113 = 2² × 113. Composite.
524 = 4 × 131 = 2² × 131. Composite.
542 = 2 × 271. Composite.

None of the six arrangements is prime.

Answer: None (0 three-digit prime numbers)

Q12 Observe that 3 is a prime number, and 2 × 3 + 1 = 7 is also a prime. Are there other primes for which doubling and adding 1 gives another prime? Find at least five such examples.

SOLUTION

We need primes p such that 2p + 1 is also prime (these are called Sophie Germain primes).

p = 2: 2×2+1 = 5. Prime. ✓
p = 3: 2×3+1 = 7. Prime. ✓ (given in question)
p = 5: 2×5+1 = 11. Prime. ✓
p = 11: 2×11+1 = 23. Prime. ✓
p = 23: 2×23+1 = 47. Prime. ✓
p = 29: 2×29+1 = 59. Prime. ✓
p = 41: 2×41+1 = 83. Prime. ✓

Five examples (apart from the given p=3): p=2, 5, 11, 23, 29.

Answer: Five examples: p=2 (2p+1=5), p=5 (11), p=11 (23), p=23 (47), p=29 (59)

Exercise 5.4 — Prime Factorisation

Q1 Find the prime factorisations of the following numbers: 64, 104, 105, 243, 320, 141, 1728, 729, 1024, 1331, 1000.

SOLUTION

64: $64 = 2\times32 = 2\times2\times16 = 2\times2\times2\times8 = 2\times2\times2\times2\times4 = 2\times2\times2\times2\times2\times2.$ → 2^6

104: $104 = 2\times52 = 2\times2\times26 = 2\times2\times2\times13.$ → $2^3 \times 13$

105: $105 = 3\times35 = 3\times5\times7.$ → $3 \times 5 \times 7$

243: $243 = 3\times81 = 3\times3\times27 = 3\times3\times3\times9 = 3\times3\times3\times3\times3.$ → 3^5

320: $320 = 2\times160 = 2\times2\times80 = 2\times2\times2\times40 = 2\times2\times2\times2\times20 = 2\times2\times2\times2\times2\times10 = 2\times2\times2\times2\times2\times2\times5.$ → $2^6 \times 5$

141: $141 = 3\times47$. (47 is prime: not divisible by 2,3,5,7) → 3×47

1728: $1728 = 2\times864 = 2\times2\times432 = 2\times2\times2\times216 = 2\times2\times2\times2\times108 = 2\times2\times2\times2\times2\times54 = 2\times2\times2\times2\times2\times2\times27 = 2^6 \times 3^3$.

729: $729 = 3\times243 = 3\times3\times81 = 3\times3\times3\times27 = 3\times3\times3\times3\times9 = 3\times3\times3\times3\times3\times3.$ → 3^6

1024: $1024 = 2^{10}$. ($2^{10} = 1024$). → 2^{10}

1331: $1331 = 11\times121 = 11\times11\times11.$ → 11^3

1000: $1000 = 8\times125 = 2^3 \times 5^3.$ → $2^3 \times 5^3$

Answer: $64=2^6$; $104=2^3\times13$; $105=3\times5\times7$; $243=3^5$; $320=2^6\times5$; $141=3\times47$; $1728=2^6\times3^3$; $729=3^6$; $1024=2^{10}$; $1331=11^3$; $1000=2^3\times5^3$

Q2 The prime factorisation of a number has one 2, two 3s, and one 11. What is the number?

SOLUTION

The number = $2^1 \times 3^2 \times 11^1$
= $2 \times 9 \times 11$
= 18×11
= 198

Answer: 198

Q3 Find three prime numbers, all less than 30, whose product is 1955.

SOLUTION

1955 ends in 5, so divisible by 5.
1955 ÷ 5 = 391.

Now factorise 391: check primes up to √391 ≈ 19.8.
391 ÷ 7 = 55.8… ✗. 391 ÷ 11 = 35.5… ✗. 391 ÷ 13 = 30.07… ✗. 391 ÷ 17 = 23. ✓

So 391 = 17 × 23.

Check: 5 × 17 × 23 = 5 × 391 = 1955. ✓
All three primes (5, 17, 23) are less than 30. ✓

Answer: 5, 17, and 23

Q4 Find the prime factorisation of these numbers without multiplying first:
a. 56 × 25
b. 108 × 75
c. 1000 × 81

SOLUTION

a. 56 × 25:
56 = 2 × 2 × 2 × 7
25 = 5 × 5
56 × 25 = 2 × 2 × 2 × 5 × 5 × 7
Verify: $2^3 \times 5^2 \times 7 = 8 \times 25 \times 7 = 1400$. ✓

b. 108 × 75:
108 = 2 × 2 × 3 × 3 × 3
75 = 3 × 5 × 5
108 × 75 = 2 × 2 × 3 × 3 × 3 × 3 × 5 × 5
Verify: 4 × 81 × 25 = 4 × 2025 = 8100. ✓

c. 1000 × 81:
1000 = 2 × 2 × 2 × 5 × 5 × 5
81 = 3 × 3 × 3 × 3
1000 × 81 = 2 × 2 × 2 × 3 × 3 × 3 × 3 × 5 × 5 × 5
Verify: 8 × 81 × 125 = 8 × 10125 = 81000. ✓

Answer: a. 2×2×2×5×5×7; b. 2×2×3×3×3×3×5×5; c. 2×2×2×3×3×3×3×5×5×5

Q5 What is the smallest number whose prime factorisation has:

a. three different prime numbers?

b. four different prime numbers?

SOLUTION

To get the smallest number, use the smallest primes (2, 3, 5, 7, …) each appearing exactly once.

a. Three different primes: 2, 3, 5.
Smallest number = 2 × 3 × 5 = 30.

b. Four different primes: 2, 3, 5, 7.
Smallest number = 2 × 3 × 5 × 7 = 210.

Answer: a. 30; b. 210

Exercise 5.5 — Prime Factorisation

Q1 Are the following pairs of numbers co-prime? Guess first and then use prime factorisation to verify your answer.

a. 30 and 45

b. 57 and 85

c. 121 and 1331

d. 343 and 216

SOLUTION

a. 30 and 45:

30 = 2 × 3 × 5

45 = 3 × 3 × 5

Common prime factors: 3 and 5. NOT co-prime.

b. 57 and 85:

57 = 3 × 19

85 = 5 × 17

No common prime factors. CO-PRIME. ✓

c. 121 and 1331:

121 = 11 × 11

1331 = 11 × 11 × 11

Common prime factor: 11. NOT co-prime.

d. 343 and 216:

343 = 7 × 7 × 7

216 = 2 × 2 × 2 × 3 × 3 × 3

No common prime factors. CO-PRIME. ✓

Answer: a. Not co-prime (common factors 3, 5); b. Co-prime; c. Not co-prime (common factor 11); d. Co-prime

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Q2 Is the first number divisible by the second? Use prime factorisation.

a. 225 and 27

b. 96 and 24

c. 343 and 17

d. 999 and 99

SOLUTION

a. 225 and 27:
225 = 3 × 3 × 5 × 5
27 = 3 × 3 × 3
27 needs three 3s but 225 has only two 3s. NOT divisible.

b. 96 and 24:
96 = 2 × 2 × 2 × 2 × 2 × 3
24 = 2 × 2 × 2 × 3
24's prime factorisation ($2^3 \times 3$) is included in 96's ($2^5 \times 3$). DIVISIBLE. ✓
96 ÷ 24 = 4. ✓

c. 343 and 17:
343 = 7 × 7 × 7
17 = 17 (prime)
17 does not appear in 343's factorisation. NOT divisible.

d. 999 and 99:
999 = 3 × 3 × 3 × 37
99 = 3 × 3 × 11
99 needs 11 as a factor, but 999 has no factor of 11. NOT divisible.

Answer: a. No; b. Yes (96 = 24 × 4); c. No; d. No

Q3 The first number has prime factorisation 2 × 3 × 7 and the second number has prime factorisation 3 × 7 × 11. Are they co-prime? Does one of them divide the other?

SOLUTION

First number = 2 × 3 × 7 = 42.
Second number = 3 × 7 × 11 = 231.

Co-prime? Common prime factors = {3, 7}. They share 3 and 7, so they are NOT co-prime.

Does one divide the other?

For 42 to divide 231: 231 must contain 2 × 3 × 7. But 2 is not in 231's factorisation. NO.

For 231 to divide 42: 42 must contain 3 × 7 × 11. But 11 is not in 42's factorisation. NO.

Neither number divides the other.

Answer: Not co-prime (both share factors 3 and 7); neither divides the other

Q4 Guna says, 'Any two prime numbers are co-prime'. Is he right?

SOLUTION

Yes, Guna is right.

Two distinct prime numbers p and q each have only two factors: 1 and themselves.
Since p and q are different primes, p is not a factor of q and q is not a factor of p.
Their only common factor is 1.
Therefore, any two distinct prime numbers are always co-prime.

Examples: (2, 3), (3, 11), (5, 13), (7, 17).

Answer: Yes, any two distinct prime numbers are always co-prime

Exercise 5.6 — Divisibility Tests

Q1 2024 is a leap year (as February has 29 days). Leap years occur in the years that are multiples of 4, except for those years that are evenly divisible by 100 but not 400.

a. From the year you were born till now, which years were leap years? (Personal answer — varies by student)

b. From the year 2024 till 2099, how many leap years are there?

SOLUTION

a. This depends on the student's birth year. For example, if born in 2012:
Leap years from 2012 to 2024: 2012, 2016, 2020, 2024.

b. Leap years from 2024 to 2099:
In this range there is no century year (2100 is outside the range).
So every year divisible by 4 is a leap year.
Years: 2024, 2028, 2032, …, 2096.
Count = (2096 − 2024) ÷ 4 + 1 = 72 ÷ 4 + 1 = 18 + 1 = 19.

Answer: a. Varies by birth year; b. 19 leap years (2024, 2028, …, 2096)

Q2 Find the largest and smallest 4-digit numbers that are divisible by 4 and are also palindromes.

SOLUTION

A 4-digit palindrome has the form abba (e.g., 2112).
Divisibility by 4: only the last two digits matter. The last two digits of abba are 'ba', i.e., the number 10b + a.
We need (10b + a) divisible by 4.

List all 4-digit palindromes abba (a from 1 to 9, b from 0 to 9) where (10b+a) is divisible by 4:
a=2, b=1: 2112, last two digits 12, 12÷4=3. ✓ (smallest candidate)
a=8, b=8: 8888, last two digits 88, 88÷4=22. ✓ (largest candidate)

Smallest: 2112 (last two digits = 12, divisible by 4). ✓
Largest: 8888 (last two digits = 88, divisible by 4). ✓

Note: 1001 and 9999 end in 01 and 99 respectively, neither divisible by 4.

Answer: Smallest: 2112; Largest: 8888

Q3 Explore and find out if each statement is always true, sometimes true or never true. You can give examples to support your reasoning.
a. Sum of two even numbers gives a multiple of 4.
b. Sum of two odd numbers gives a multiple of 4.

SOLUTION

a. Sum of two even numbers gives a multiple of 4?
2 + 6 = 8 = 4 × 2. ✓ (multiple of 4)
2 + 4 = 6 (not a multiple of 4). ✗
SOMETIMES TRUE.

b. Sum of two odd numbers gives a multiple of 4?
1 + 3 = 4 = 4 × 1. ✓
1 + 5 = 6 (not a multiple of 4). ✗
SOMETIMES TRUE.

Answer: a. Sometimes true; b. Sometimes true

Q4 Find the remainders obtained when each of the following numbers are divided by (a) 10, (b) 5, (c) 2.
Numbers: 78, 99, 173, 572, 980, 1111, 2345

SOLUTION

Remainder when divided by 10 = units digit.
Remainder when divided by 5 = units digit mod 5 (0 or 1–4).
Remainder when divided by 2 = 0 if even, 1 if odd.

78: ÷10 = rem 8; ÷5 = rem 3; ÷2 = rem 0
99: ÷10 = rem 9; ÷5 = rem 4; ÷2 = rem 1
173: ÷10 = rem 3; ÷5 = rem 3; ÷2 = rem 1
572: ÷10 = rem 2; ÷5 = rem 2; ÷2 = rem 0
980: ÷10 = rem 0; ÷5 = rem 0; ÷2 = rem 0
1111: ÷10 = rem 1; ÷5 = rem 1; ÷2 = rem 1
2345: ÷10 = rem 5; ÷5 = rem 0; ÷2 = rem 1

Answer: 78: (8,3,0); 99: (9,4,1); 173: (3,3,1); 572: (2,2,0); 980: (0,0,0); 1111: (1,1,1); 2345: (5,0,1)

Q5 The teacher asked if 14560 is divisible by all of 2, 4, 5, 8 and 10. Guna checked for divisibility of 14560 by only two of these numbers and then declared that it was also divisible by all of them. What could those two numbers be?

SOLUTION

Observe the relationships among the numbers 2, 4, 5, 8, 10:

$8 = 2^3$, so divisibility by 8 implies divisibility by 4 and 2.

$10 = 2 \times 5$, so divisibility by both 8 and 5 implies divisibility by 2, 4, 5, 8, 10.

If 14560 is divisible by 8 AND by 5:

Divisible by 8 → divisible by 4 and 2.

Divisible by 5 and 2 → divisible by 10.

So all five numbers (2, 4, 5, 8, 10) are covered.

Verify: 14560 ÷ 8 = 1820. ✓ 14560 ÷ 5 = 2912. ✓

Answer: 8 and 5 (checking divisibility by 8 covers 2 and 4; with 5 it also covers 10)

Q6 Which of the following numbers are divisible by all of 2, 4, 5, 8 and 10: 572, 2352, 5600, 6000, 77622160?

SOLUTION

A number divisible by all of 2, 4, 5, 8, 10 must be divisible by LCM(2,4,5,8,10) = 40.

Check divisibility by 40 (equivalently, by 8 and 5):

572: 572 ÷ 5 → units digit 2, not 0 or 5. NOT divisible by 5. ✗

2352: 2352 ÷ 5 → units digit 2. NOT divisible by 5. ✗

5600: 5600 ÷ 40 = 140. ✓

6000: 6000 ÷ 40 = 150. ✓

77622160: last 3 digits = 160, 160 ÷ 8 = 20 ✓; ends in 0, div by 5 ✓. 77622160 ÷ 40 = 1940554. ✓

Answer: 5600, 6000, and 77622160

Q7 Write two numbers whose product is 10000. The two numbers should not have 0 as the units digit.

SOLUTION

$10000 = 10^4 = 2^4 \times 5^4$.

To avoid 0 as the units digit, split the 2s and 5s so neither factor ends in 0.

Factor A = $2^4 = 16$ (units digit 6). ✓

Factor B = $5^4 = 625$ (units digit 5). ✓

Check: $16 \times 625 = 10000$. ✓

Neither 16 nor 625 ends in 0. ✓

Answer: $16 \times 625 = 10000$ (neither ends in 0)