Chapter 8: Playing with Constructions

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Exercise 8.1 — Artwork

Q1 What radius should be taken in the compass to get this half circle (for the Wavy Wave with central line AB = 8 cm)? What should be the length of AX?

SOLUTION

The Wavy Wave is drawn along a central line AB = 8 cm.
The first wave is a half circle sitting on segment AX.

For the wave to be a half circle, the diameter of the half circle equals AX.
The two waves together span the full length AB = 8 cm, so each wave spans 4 cm.

Therefore: AX = 4 cm.
Radius = AX ÷ 2 = 4 ÷ 2 = 2 cm.

Answer: Radius = 2 cm; AX = 4 cm

Exercise 8.2 — Squares and Rectangles

Q1 Which of the following is NOT a name for the square with corners S (top-left), P (top-right), Q (bottom-right), R (bottom-left)?

1. PQSR
2. SPQR
3. RSPQ
4. QRSP

SOLUTION

A valid name for a square (or rectangle) lists its corners in the order they appear when travelling around the figure — clockwise or anti-clockwise — starting from any corner.

The corners in clockwise order are: S → P → Q → R → S.

Check each option:

• SPQR: S→P→Q→R — consecutive clockwise travel ✓

• RSPQ: R→S→P→Q — consecutive clockwise travel ✓

• QRSP: Q→R→S→P — consecutive clockwise travel ✓

• PQSR: P→Q→S→R — this jumps from Q to S, skipping R, then back to R. Not a consecutive travel ✗

So PQSR is NOT a valid name for the square.

Answer: PQSR (option 1) is not a valid name for the square

Q2 Identify if there are any squares in this collection (figures A, B, C, D drawn on a dot grid). Use measurements if needed.

SOLUTION

Examine each figure on the dot grid by checking the two properties of a square:
S1) All sides are equal, and
S2) All angles are 90°.

Figure A: Its corners lie on the dots such that all four sides span the same diagonal distance on the grid, and the angles between adjacent sides are 90°. Both properties hold — A is a square.

Figures B, C, D: Measuring (or reasoning from the dot positions) shows that either the sides are not all equal or the angles are not all 90°. None of them is a square.

Note: On a dot grid, you can verify equal sides and right angles by counting dot-distances and checking that adjacent sides are perpendicular (one side goes m right + n up, the adjacent side goes n right − m up, for any m, n).

Answer: Only figure A is a square

Exercise 8.3 — Constructing Squares and Rectangles

Q1 Draw a rectangle with sides of length 4 cm and 6 cm. After drawing, check if it satisfies both the rectangle properties.

SOLUTION

Steps to construct rectangle ABCD with AB = 6 cm and BC = 4 cm:
Step 1: Draw segment AB = 6 cm.
Step 2: At A, construct a perpendicular to AB. Mark C on this perpendicular such that AC = 4 cm.
Step 3: At B, construct a perpendicular to AB. Mark D on this perpendicular such that BD = 4 cm.
Step 4: Join CD.

Verification:
R1) Opposite sides are equal: AB = CD = 6 cm and AC = BD = 4 cm. ✓
R2) All angles ∠A = ∠B = ∠C = ∠D = 90°. ✓

Answer: Rectangle with sides 4 cm and 6 cm satisfies both properties: opposite sides equal (6 cm = 6 cm, 4 cm = 4 cm) and all angles = 90°

Q2 Draw a rectangle of sides 2 cm and 10 cm. After drawing, check if it satisfies both the rectangle properties.

SOLUTION

Steps to construct rectangle PQRS with PQ = 10 cm and QR = 2 cm:
Step 1: Draw segment PQ = 10 cm.
Step 2: At P, construct a perpendicular. Mark S on it such that PS = 2 cm.
Step 3: At Q, construct a perpendicular. Mark R on it such that QR = 2 cm.
Step 4: Join SR.

Verification:
R1) Opposite sides are equal: PQ = SR = 10 cm and PS = QR = 2 cm. ✓
R2) All angles ∠P = ∠Q = ∠R = ∠S = 90°. ✓

Answer: Rectangle with sides 2 cm and 10 cm satisfies both properties: opposite sides equal (10 cm = 10 cm, 2 cm = 2 cm) and all angles = 90°

Q3 Is it possible to construct a 4-sided figure in which all the angles are equal to 90° but opposite sides are not equal?

SOLUTION

No, it is not possible.

Reasoning: In any quadrilateral ABCD, the sum of all interior angles = 360°.
If all four angles are 90°, then ∠A + ∠B = 180°, which means AB ∥ CD.
Similarly, ∠A + ∠D = 180°, which means BC ∥ AD.

A quadrilateral with both pairs of opposite sides parallel is a parallelogram.
In a parallelogram, opposite sides are always equal.
A parallelogram with all angles equal to 90° is a rectangle.

Therefore, any 4-sided figure with all angles equal to 90° must have equal opposite sides.

Answer: No, it is not possible. Any 4-sided figure with all angles equal to 90° must be a rectangle, and a rectangle always has equal opposite sides.

Exercise 8.4 — An Exploration in Rectangles

Q1 In rectangle ABCD with AB = 7 cm and BC = 4 cm, X is on side AD and Y is on side BC. Fill in the table: When X is 5 mm away from A and Y is 3 cm away from B, XY = ___ cm ___ mm. When X is 1 cm away from A and Y is 1 cm away from B, XY = ___ cm ___ mm. When X is 2 cm away from A and Y is 4 cm away from B, XY = ___ cm ___ mm.

SOLUTION

Set up coordinates: A = (0, 4), B = (7, 4), C = (7, 0), D = (0, 0).
AD is the left side (vertical), BC is the right side (vertical).

Case 1: X is 5 mm = 0.5 cm from A along AD, so X = (0, 3.5).
Y is 3 cm from B along BC, so Y = (7, 1).
XY = $\sqrt{[(7-0)^2 + (1-3.5)^2]} = \sqrt{[49 + 6.25]} = \sqrt{55.25} \approx 7.43$ cm ≈ 7 cm 4 mm.

Case 2: X is 1 cm from A, so X = (0, 3). Y is 1 cm from B, so Y = (7, 3).
XY = $\sqrt{[(7)^2 + (3-3)^2]} = \sqrt{49} = 7$ cm = 7 cm 0 mm.

Case 3: X is 2 cm from A, so X = (0, 2). Y is 4 cm from B = C, so Y = (7, 0).
XY = $\sqrt{[(7)^2 + (0-2)^2]} = \sqrt{[49 + 4]} = \sqrt{53} \approx 7.28$ cm ≈ 7 cm 3 mm.

Answer: Case 1: XY ≈ 7 cm 4 mm; Case 2: XY = 7 cm 0 mm; Case 3: XY ≈ 7 cm 3 mm

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Q2 When X and Y are placed at the same distance from A and B respectively (5 mm each; 1 cm each; 1 cm 5 mm each), find the length of XY in each case. What do you observe about (i) how XY compares to AB, and (ii) the shape of figure ABYX?

SOLUTION

Using coordinates: A=(0,4), B=(7,4), C=(7,0), D=(0,0).

Case 1: X = 5 mm = 0.5 cm from A → X = (0, 3.5); Y = 5 mm from B → Y = (7, 3.5).
XY = $\sqrt{[(7)^2 + (3.5-3.5)^2]} = \sqrt{49} = 7$ cm.

Case 2: X = 1 cm from A → X = (0, 3); Y = 1 cm from B → Y = (7, 3).
XY = $\sqrt{[49 + 0]} = 7$ cm.

Case 3: X = 1 cm 5 mm from A → X = (0, 2.5); Y = 1 cm 5 mm from B → Y = (7, 2.5).

XY = √[49 + 0] = 7 cm.

Observations:
(i) XY = AB = 7 cm in every case.
(ii) ABYX is a rectangle (since AB ‖ XY and AX ‖ BY, with all angles 90°).

Answer: XY = AB = 7 cm in all cases; the 4-sided figure ABYX is a rectangle

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Q3 How does the farthest distance between X and Y compare with the length of AC and BD (the diagonals of rectangle ABCD)?

SOLUTION

The diagonal of rectangle ABCD: AC = BD = √(AB² + BC²) = √(7² + 4²) = √(49 + 16) = √65 ≈ 8.06 cm.

The farthest distance between X and Y is achieved when X is at one end of AD and Y is at the opposite end of BC.
For example: X = D = (0, 0) and Y = B = (7, 4).
XY = √(7² + 4²) = √65 ≈ 8.06 cm = DB.

Or: X = A = (0, 4) and Y = C = (7, 0).
XY = √(7² + 4²) = √65 ≈ 8.06 cm = AC.

So the farthest distance between X and Y equals the length of the diagonal AC (= BD).

Answer: The farthest distance between X and Y equals the diagonal length AC = BD = √65 ≈ 8.06 cm

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Q4 Construct a rectangle that can be divided into 3 identical squares. Give the lengths of the sides.

SOLUTION

Let the side of each identical square be s.
Placing 3 identical squares side by side: the rectangle has length = 3s and breadth = s.

For example, choose s = 4 cm.
Then the rectangle has sides 4 cm and 12 cm.

Construction steps:
Step 1: Draw segment AB = 12 cm.

Step 2: At A, draw a perpendicular and mark D such that AD = 4 cm.
Step 3: At B, draw a perpendicular and mark C such that BC = 4 cm.
Step 4: Join DC.
Step 5: Using the compass set to 4 cm, mark points E and F on AB dividing it into three 4 cm segments. Draw perpendiculars at E and F to get the two internal dividing lines.

Answer: The rectangle has its length equal to 3 times its breadth (e.g., 12 cm × 4 cm, divided into three 4 cm × 4 cm squares)

Q5 Give the lengths of the sides of a rectangle that cannot be divided into (a) two identical squares; (b) three identical squares.

SOLUTION

A rectangle can be divided into 2 identical squares only if its length = 2 × breadth.
A rectangle can be divided into 3 identical squares only if its length = 3 × breadth.

(a) A rectangle that CANNOT be divided into two identical squares:
Choose sides where length ≠ 2 × breadth.
Example: 4 cm × 6 cm (since 6 ≠ 2 × 4 = 8). ✓

(b) A rectangle that CANNOT be divided into three identical squares:
Choose sides where length ≠ 3 × breadth.
Example: 7 cm × 2 cm (since 7 ≠ 3 × 2 = 6). ✓

Answer: (a) E.g., 4 cm × 6 cm cannot be divided into 2 identical squares; (b) e.g., 7 cm × 2 cm cannot be divided into 3 identical squares

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Q6 A Square within a Rectangle: Construct a rectangle of sides 8 cm and 4 cm. Construct a square inside it such that the centre of the square is the same as the centre of the rectangle.

SOLUTION

The shorter side of the rectangle is 4 cm, so the largest square that fits has side = 4 cm.

To centre the square inside the 8 cm × 4 cm rectangle:
The square occupies 4 cm of the 8 cm length, leaving 8 − 4 = 4 cm unused.
Divided equally: 2 cm on each side.

Construction steps:

Step 1: Draw rectangle ABCD with AB = 8 cm and BC = 4 cm.

Step 2: Find the midpoint M of AB (at 4 cm from A). Find the midpoint N of DC.

Step 3: Mark point P on AB at 2 cm from A. Mark point Q on AB at 6 cm from A.

Step 4: Draw perpendiculars to AB at P and Q, meeting DC at R and S respectively.

Step 5: PQSR is the required 4 cm × 4 cm square centred in the rectangle.

The distance from each corner of the square to the nearer short side of the rectangle = 2 cm.

Answer: The inner square has side 4 cm, with its sides parallel to those of the rectangle and positioned 2 cm from each short end

Exercise 8.5 — Exploring Diagonals of Rectangles and Squares

Q1 How should the rectangle be constructed so that the diagonal divides the opposite angles into equal parts?

SOLUTION

In a rectangle, each angle is 90°. A diagonal divides the angle at each opposite corner into two parts.

For the diagonal to divide each opposite angle into equal parts, each half must be 45°.

In a rectangle with sides a and b, the diagonal makes an angle of arctan(b/a) with one side.
For this angle to be 45°, we need arctan(b/a) = 45°, which means b/a = 1, i.e., a = b.

A rectangle with all sides equal is a square.

Therefore, the rectangle must be constructed as a square (all four sides equal).

Answer: The rectangle must be a square — only when all sides are equal does each diagonal divide the opposite angles into equal parts (each 45°)

Q2 Construct a rectangle in which one of the diagonals divides the opposite angles into 50° and 40°.

SOLUTION

The diagonal divides the right angle (90°) at a corner into 50° and 40°. Check: 50° + 40° = 90°. ✓

Construction steps:
Step 1: Draw segment AB of any convenient length.
Step 2: At B, draw a perpendicular to AB.
Step 3: At A, draw a line making 50° with AB (using a protractor). This line is the diagonal AC. Let it meet the perpendicular at C.
Step 4: At A, draw a perpendicular to AB. At C, draw a perpendicular to BC. They meet at D.
Step 5: ABCD is the required rectangle. The diagonal AC divides ∠A into 50° (= ∠BAC) and 40° (= ∠DAC), and divides ∠C into 40° and 50°.

Answer: Construct by drawing AB, erecting perpendicular at B, drawing the diagonal at 50° from A to meet the perpendicular at C, then completing the rectangle; the diagonal divides the 90° angles into 50° and 40°

Q3 Construct a rectangle in which one of the diagonals divides the opposite angles into 45° and 45°. What do you observe about the sides?

SOLUTION

The diagonal divides the right angle at each corner into 45° and 45°. Check: 45° + 45° = 90°. ✓

Construction steps:
Step 1: Draw segment AB.
Step 2: At B, draw a perpendicular to AB.
Step 3: At A, draw a line at 45° to AB (the diagonal). It meets the perpendicular at C.
Step 4: Complete the rectangle to get ABCD.

Observation: When the diagonal makes 45° with each side, the adjacent sides are equal (AB = BC).
This is because tan(45°) = BC/AB = 1, so BC = AB.
A rectangle with all sides equal is a square.

Therefore, the constructed rectangle is actually a square.

Answer: When the diagonal divides each angle into 45° and 45°, the rectangle is a square (all sides are equal)

Q4 Construct a rectangle one of whose sides is 4 cm and the diagonal is of length 8 cm.

SOLUTION

Let side CD = 4 cm and diagonal DB = 8 cm (in rectangle ABCD).

Step 1: Draw CD = 4 cm.
Step 2: At C, draw a perpendicular to CD. Let this perpendicular line be l.
Step 3: With D as centre and radius 8 cm, draw an arc. The arc cuts line l at B.
Step 4: CB = $\sqrt{(DB^2 - DC^2)} = \sqrt{(64 - 16)} = \sqrt{48} \approx 6.93$ cm.
Step 5: Complete the rectangle: draw perpendiculars at D and B. They meet at A.

Verification: DC = AB = 4 cm, CB = DA = $\sqrt{48} \approx 6.93$ cm, diagonal = 8 cm. ✓

Answer: The rectangle has sides 4 cm and √48 ≈ 6.93 cm, with diagonal = 8 cm; construct by drawing 4 cm base, erecting perpendicular, then using a compass arc of radius 8 cm from one end to locate the opposite corner

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Q5 Construct a rectangle one of whose sides is 3 cm and the diagonal is of length 7 cm.

SOLUTION

Let side CD = 3 cm and diagonal DB = 7 cm.

Step 1: Draw CD = 3 cm.
Step 2: At C, draw a perpendicular to CD. Call this line l.
Step 3: With D as centre and radius 7 cm, draw an arc. The arc cuts line l at B.
Step 4: CB = √(DB² − DC²) = √(49 − 9) = √40 ≈ 6.32 cm.
Step 5: Complete the rectangle: draw perpendiculars at D and B to get point A.

Verification: DC = AB = 3 cm, CB = DA = √40 ≈ 6.32 cm, diagonal = 7 cm. ✓

Answer: The rectangle has sides 3 cm and √40 ≈ 6.32 cm, with diagonal = 7 cm

Exercise 8.6 — Points Equidistant from Two Given Points

Q1 Construct a bigger house in which all the sides are of length 7 cm.

SOLUTION

The house figure has a rectangular base with a triangular/arc roof, where all boundary segments are equal in length.

Construction steps (mirroring the 5 cm house, scaled to 7 cm):
Step 1: Draw the base DE = 7 cm.
Step 2: At D, draw a perpendicular upward. Mark B on it such that BD = 7 cm.
Step 3: At E, draw a perpendicular upward. Mark C on it such that CE = 7 cm.
Step 4: Join BC = 7 cm (top of the rectangular portion).
Step 5: To find apex A (equidistant from B and C, at distance 7 cm from each):
— With B as centre and radius 7 cm, draw an arc above BC.
— With C as centre and radius 7 cm, draw an arc above BC.
— The intersection point is A.
Step 6: With A as centre and radius 7 cm, draw the arc from B to C (the roof arc).
Step 7: Join AB and AC to complete the house outline.

Answer: Construct the house with all sides 7 cm: rectangular base BDEC (each side 7 cm) with apex A located 7 cm from both B and C using compass arcs, then draw arc of radius 7 cm from A

Q2 Is there a 4-sided figure in which all the sides are equal in length but is not a square? If such a figure exists, can you construct it?

SOLUTION

Yes, such a figure exists. It is called a rhombus.

A rhombus has all four sides equal in length, but its angles are not necessarily 90°.
A square is a special rhombus where all angles are also 90°.

Construction of a rhombus (that is not a square):
Step 1: Draw segment AB = 5 cm (choose any side length).
Step 2: At A, draw a line at an angle other than 90° to AB (say 60°). Mark D on it such that AD = 5 cm.
Step 3: With B as centre and radius 5 cm, draw an arc.
Step 4: With D as centre and radius 5 cm, draw another arc. The arcs intersect at

C.

Step 5: Join BC and DC. ABCD is the required rhombus.

All sides AB = BC = CD = DA = 5 cm, but ∠A = 60° ≠ 90°, so it is not a square.

Answer: Yes — a rhombus has all sides equal but is not a square (its angles are not 90°). It can be constructed by drawing a quadrilateral with all four sides equal but with non-right angles.