Chapter 9: Symmetry

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Exercise 9.1 — Line of Symmetry

Q1 Do you see any line of symmetry in the figures at the start of the chapter? What about in the picture of the cloud?

SOLUTION

The figures at the start of the chapter are: Flower, Butterfly, Rangoli, Pinwheel, and Cloud.

Flower: It has 6 petals arranged equally around the centre. Folding along any line through the centre between opposite petals gives mirror halves. It has 6 lines of symmetry.

Butterfly: The left and right wings are mirror images of each other. Folding along the vertical centre line makes the two halves overlap. It has 1 line of symmetry.

Rangoli: The pattern repeats every quarter turn. Folding along any of the 4 axes (horizontal, vertical, and two diagonals) gives overlapping halves. It has 4 lines of symmetry.

Pinwheel: The blades are arranged with rotational symmetry but are not mirror images of each other. No fold line makes the two halves overlap exactly. It has 0 lines of symmetry.

Cloud: The shape is irregular with no repeating pattern. It has 0 lines of symmetry.

Answer: Flower: 6 lines of symmetry; Butterfly: 1 line of symmetry; Rangoli: 4 lines of symmetry; Pinwheel: no line of symmetry; Cloud: no line of symmetry.

Q2 For each of the following figures, identify the line(s) of symmetry if it exists. (Five figures are shown: an irregular zigzag polygon, a kite/diamond shape, a stepped rectangle, an L-shape, and an arrow/chevron.)

SOLUTION

Figure 1 (irregular zigzag polygon): The left side is jagged and does not match the right side under any fold. No line of symmetry.

Figure 2 (kite/diamond shape): Folding along the vertical axis through the top and bottom vertices makes the left half exactly cover the right half. It has 1 line of symmetry (vertical).

Figure 3 (stepped rectangle): The steps are not symmetric under any fold. No line of symmetry.

Figure 4 (L-shape): Folding along the diagonal from the inner corner to the outer corner makes the two parts overlap. It has 1 line of symmetry (diagonal).

Figure 5 (scalene triangle): The three sides are of different lengths, so no fold makes the two parts overlap. No line of symmetry.

Answer: Figure 1: no line of symmetry; Figure 2 (kite): 1 vertical line of symmetry; Figure 3: no line of symmetry; Figure 4 (L-shape): 1 diagonal line of symmetry; Figure 5 (scalene triangle): no line of symmetry.

Exercise 9.2 — Generating Shapes Having Lines of Symmetry

Q1 In each of the following figures, a hole was punched in a folded square sheet of paper and then the paper was unfolded. Identify the line along which the paper was folded. Figure (d) was created by punching a single hole. How was the paper folded?

(a) Two holes symmetrically on the left and right sides.

(b) Two holes near the top — one slightly left of centre and one to the right, both near top edge.

(c) Two holes — one near the top-right and one near the bottom-right.

(d) Four holes at the four corners.

SOLUTION

When a hole is punched in a folded sheet, unfolding reveals holes that are mirror images across the fold line.

(a) The two holes are symmetric about a vertical centre line. The paper was folded along the vertical line of symmetry (vertical fold).

(b) The two holes are symmetric about a diagonal line from top-left to bottom-right. The paper was folded along that diagonal.

(c) The two holes are symmetric about a horizontal centre line. The paper was folded along the horizontal line of symmetry (horizontal fold).

(d) There are four holes at all four corners. A single hole was punched. This requires two folds so that one punch creates four symmetric holes. The paper was folded vertically and then horizontally (or vice versa), placing all four quadrants on top of each other before punching.

Answer: (a) Vertical fold; (b) Diagonal fold; (c) Horizontal fold; (d) Paper was folded vertically and then horizontally (or vice versa).

Q2 Given the line(s) of symmetry, find the other hole(s):
(a) Square with a diagonal line of symmetry; one hole in the upper-left region.
(b) Rectangle with a horizontal line of symmetry; one hole in the lower-left region.
(c) Triangle with a vertical line of symmetry; one hole on the left side.
(d) Circle with a diagonal line of symmetry; one hole near the top.
(e) Circle with a diagonal line of symmetry going the other way; one hole near the top-left.

SOLUTION

The other hole is always the mirror image of the given hole across the line of symmetry.

(a) Diagonal line of symmetry (top-left to bottom-right). The hole in the upper-left reflects to a hole in the lower-right corner region, symmetric across the diagonal.

(b) Horizontal line of symmetry. The given hole is in the lower-right region; it reflects across the horizontal midline to a hole in the upper-right, at the same horizontal position but on the opposite side of the fold.

(c) Vertical line of symmetry. The hole on the left side of the triangle reflects to a corresponding hole on the right side, symmetric about the vertical axis.

(d) Diagonal line of symmetry. The hole near the top reflects to a hole in the region symmetric across the diagonal.

(e) Diagonal line of symmetry (other direction). The hole near the top-left reflects to a hole symmetric across that diagonal.

Answer: In each case, the other hole is the mirror image of the given hole across the stated line of symmetry: (a) reflected across diagonal; (b) reflected across horizontal midline; (c) reflected across vertical axis; (d) reflected across the diagonal; (e) reflected across the other diagonal.

Q3 After each of the following cuts, predict the shape of the hole when the paper is opened. After you have made your prediction, make the cutouts and verify your answer.
(a) Paper folded vertically once; a zigzag/irregular cut made from the folded edge inward.
(b) Paper folded vertically once; a triangular/arrow notch cut from the folded edge.
(c) Paper folded vertically and then again (double fold); a rectangular notch cut from the folded corner.
(d) Paper folded vertically once; a rectangular notch cut from the middle of the folded edge.

SOLUTION

When paper is unfolded, the cut shape is reflected across each fold line.

(a) Single vertical fold; zigzag cut from folded edge: When unfolded, the cut and its mirror image together form a butterfly/bow-tie shaped hole, symmetric about the vertical fold line.

(b) Single vertical fold; triangular notch from folded edge: When unfolded, the triangle and its mirror image form an arrow or diamond (rhombus) shaped hole, symmetric about the vertical fold line.

(c) Double fold (vertical then horizontal); rectangular notch cut at the folded corner: When unfolded both ways, the rectangle reflects to give four rectangular notches, one in each quadrant — creating a cross or plus-shaped gap arrangement at the centre.

(d) Single vertical fold; rectangular notch from middle of folded edge: When unfolded, the rectangle reflects to form a wider rectangle (double width) at the centre of the paper — an I-beam or H-shaped hole.

Answer: (a) Butterfly/bow-tie shaped hole; (b) Diamond/rhombus shaped hole; (c) Four rectangular notches forming a cross pattern; (d) Large rectangular (double-width) hole in the centre.

Q4 Suppose you have to get each of these shapes with some folds and a single straight cut. How will you do it?
a. The hole in the centre is a square (the paper shows a square hole with sides parallel to the paper edges).
b. The hole in the centre is a square (the paper shows a square hole rotated 45°, i.e., a square standing on a corner).

SOLUTION

For both parts, we fold the paper so that after a single straight cut, unfolding produces the desired square hole.

Part a — Square hole with sides parallel to edges:
Step 1: Fold the paper horizontally (bringing the bottom edge up to the top edge).
Step 2: Fold again vertically (bringing the right edge over to the left edge). Now all four quadrants are stacked.
Step 3: Cut a small square notch at the fully-closed interior corner (the corner that is the centre of the original paper). All four sides of the cut must be closed (not on any folded edge).
When unfolded, this produces a square hole with sides parallel to the paper edges at the centre.

Part b — Square hole rotated 45° (diamond orientation):
Step 1: Fold the paper horizontally, then vertically as before. All four quadrants are stacked.
Step 2: At the fully-closed interior corner, make a single straight diagonal (slanting) cut.
When unfolded, the diagonal cut in each quadrant together form a square rotated 45° (diamond shape) at the centre.

Note: Verify that the resulting 4-sided figures satisfy both properties of a square — all sides equal and all angles 90°.

Answer: a. Fold horizontally then vertically; cut a small square notch at the inner closed corner — produces a square hole aligned with edges. b. Fold horizontally then vertically; cut diagonally at the inner closed corner — produces a square hole rotated 45°.

Q5 How many lines of symmetry do these shapes have?
a. Two shapes shown: a square rotated 45° (rhombus orientation) and an 8-pointed star.
b. A triangle with equal sides and equal angles (equilateral triangle).
c. A hexagon with equal sides and equal angles (regular hexagon).

SOLUTION

Part a:
Square rotated 45° (rhombus orientation): It is still a square — a square always has 4 lines of symmetry (2 through opposite vertices and 2 through midpoints of opposite sides). Lines of symmetry = 4.
8-pointed star (regular octagram): It has 8 lines of symmetry — one through each point and one through each indentation. Lines of symmetry = 8.

Part b — Equilateral triangle:
Each line of symmetry goes from a vertex to the midpoint of the opposite side. There are 3 vertices, so there are 3 lines of symmetry.

Part c — Regular hexagon:
A regular hexagon has 6 lines of symmetry — 3 lines connecting opposite vertices and 3 lines connecting midpoints of opposite sides.

Answer: a. Square (rotated 45°): 4 lines of symmetry; 8-pointed star: 8 lines of symmetry. b. Equilateral triangle: 3 lines of symmetry. c. Regular hexagon: 6 lines of symmetry.

Q6 Trace each figure and draw the lines of symmetry, if any. (Eight figures shown: four rhombus/diamond cluster patterns in a row, then two grid figures and two grid figures.)

SOLUTION

Figure 1 (2 diamonds stacked vertically, tip to tip): The two identical diamonds are symmetric about the vertical line through both centres, and also about the horizontal line through the shared middle vertex (which swaps the top and bottom diamond). It has 2 lines of symmetry (vertical and horizontal).

Figure 2 (3 diamonds in a horizontal row): Left and right ends are symmetric about the centre vertical axis. It has 1 vertical line of symmetry.

Figure 3 (2 diamonds one above the other in a vertical column): The figure is symmetric about a horizontal axis through the shared vertex. It has 1 horizontal

line of symmetry.

Figure 4 (4 diamonds in a 2×2 arrangement): The figure has 2 lines of symmetry — 1 vertical and 1 horizontal through the centre.

Grid figure 1 (L-shaped / staircase polygon on the grid): The two arms of the L are equal, so folding along the diagonal from the inner corner to the outer corner makes them overlap. It has 1 line of symmetry (the diagonal).

Grid figure 2 (hexagon-like shape on grid): Has 2 lines of symmetry — vertical and horizontal.

Grid figure 3 (irregular pentagon on grid): Has 1 line of symmetry — horizontal through the middle.

Grid figure 4 (star/cross-like shape on grid): Has 4 lines of symmetry — vertical, horizontal, and both diagonals.

Answer: Fig 1: 2 lines (vertical and horizontal); Fig 2: 1 line (vertical); Fig 3: 1 line (horizontal); Fig 4: 2 lines (vertical and horizontal); Grid fig 1 (L-shape): 1 line (diagonal); Grid fig 2: 2 lines; Grid fig 3: 1 line; Grid fig 4: 4 lines.

Q7 Find the lines of symmetry for the kolam below. (A kolam is shown: a hexagonal arrangement of star/flower patterns made from rhombuses and dots.)

SOLUTION

The kolam has an overall hexagonal shape with a repeating pattern of 6-fold symmetry.

Lines through opposite vertices of the hexagonal outline: There are 3 such lines.
Lines through midpoints of opposite sides of the hexagonal outline: There are 3 such lines.

Total lines of symmetry = 3 + 3 = 6.

Answer: The kolam has 6 lines of symmetry.

Q8 Draw the following.
a. A triangle with exactly one line of symmetry.
b. A triangle with exactly three lines of symmetry.
c. A triangle with no line of symmetry.
Is it possible to draw a triangle with exactly two lines of symmetry?

SOLUTION

a. A triangle with exactly one line of symmetry is an isosceles triangle (two equal sides). The line of symmetry goes from the apex (vertex between the two equal sides) perpendicular to the base.

b. A triangle with exactly three lines of symmetry is an equilateral triangle (all sides equal, all angles 60°). Each line of symmetry connects a vertex to the midpoint of the opposite side.

c. A triangle with no line of symmetry is a scalene triangle (all three sides of different lengths, all three angles different).

Is it possible to draw a triangle with exactly two lines of symmetry?
No. If a triangle has two lines of symmetry, then by the properties of reflection, the third line must also be a line of symmetry, giving it three lines. A triangle cannot have exactly two lines of symmetry.

Answer: a. Isosceles triangle (1 line of symmetry). b. Equilateral triangle (3 lines of symmetry). c. Scalene triangle (0 lines of symmetry). It is not possible to draw a triangle with exactly two lines of symmetry.

Q9 Draw the following. In each case, the figure should contain at least one curved boundary.
a. A figure with exactly one line of symmetry.
b. A figure with exactly two lines of symmetry.
c. A figure with exactly four lines of symmetry.

SOLUTION

a. A figure with exactly one line of symmetry and at least one curved boundary: Draw a shape like a heart — it has a vertical line of symmetry only, and its boundary includes curves.

b. A figure with exactly two lines of symmetry and at least one curved boundary: Draw an ellipse (oval) or a stadium shape (rectangle with semicircular ends). Both have exactly 2 lines of symmetry (horizontal and vertical) and curved boundaries.

c. A figure with exactly four lines of symmetry and at least one curved boundary: Draw a circle with 4 equal bumps symmetrically placed (one on each side, like a 4-petal flower shape). It has 4 lines of symmetry and curved boundaries.

Answer: a. A heart shape (1 line of symmetry, curved boundary). b. An ellipse or stadium shape (2 lines of symmetry, curved boundary). c. A 4-petal flower or 4-lobed shape (4 lines of symmetry, curved boundary).

Q10 Copy the following on squared paper. Complete them so that the blue line is a line of symmetry. Problem (a) has been done for you. (Six sub-parts: (b) horizontal blue line, (c) diagonal blue line, (d) vertical blue line, (e) horizontal blue line, (f) diagonal blue line. Each shows a partial figure on squared paper.)

SOLUTION

To complete a figure so that a given line is a line of symmetry, reflect every point of the given partial figure across the blue line and draw the mirror image.

(a) Done as example — the dashed reflection is shown across the diagonal blue line.

(b) Horizontal blue line: Reflect the given partial shape (an L-like step figure above or below the line) across the horizontal line. Each point at distance d above the line maps to a point at distance d below the line.

(c) Diagonal blue line (going top-right to bottom-left): Reflect each vertex of the partial figure across the diagonal. Hint: rotating the book helps visualise this reflection.

(d) Vertical blue line: Reflect the given partial shape (a triangle-like figure) across the vertical line. Each point at distance d to the left maps to distance d to the right.

(e) Horizontal blue line: Reflect the given partial shape (a pentagon-like figure) across the horizontal line.

(f) Diagonal blue line (other direction): Reflect the given partial figure across this diagonal. Hint: rotating the book helps.

Answer: For each sub-part, complete the figure by drawing the mirror image of the given partial shape across the indicated blue line. The completed figures have the blue line as their line of symmetry.

Q11 Copy the following drawing on squared paper. Complete each one of them so that the resulting figure has the two blue lines as lines of symmetry. (Six sub-parts (a)–(f), each showing a partial figure with two blue lines — either two diagonals, a diagonal and a vertical, a vertical and a horizontal, etc.)

SOLUTION

To complete a figure with two lines of symmetry, reflect the partial figure across both lines in sequence. Any point must be reflected across each line, including the reflection of reflections.

(a) Two diagonal blue lines (forming an X): Reflect the given shape across both diagonals. The completed figure has 4-fold symmetry.

(b) Diagonal and another diagonal: Reflect the given partial shape (zigzag) across both diagonal lines. The final figure has 4 symmetric parts.

(c) Vertical and horizontal blue lines: Reflect the partial shape (a step in the top-right quadrant) across the vertical, then the horizontal. This fills in all four quadrants symmetrically.

(d) Vertical and horizontal blue lines: Reflect the partial figure (a curved/rounded shape in one quadrant) across both lines. The final figure is symmetric in all four quadrants.

(e) Vertical and horizontal blue lines: Reflect the given partial shape (L-step figure) to fill all four quadrants.

(f) Two lines: Reflect the partial figure (curved shape) across both blue lines.

Answer: For each sub-part, reflect the given partial figure across both blue lines (including reflections of reflections) to produce a figure that has exactly those two blue lines as its lines of symmetry.

Q12 Copy the following on a dot grid. For each figure draw two more lines to make a shape that has a line of symmetry. (Six incomplete figures shown on dot grids, each needing 2 more line segments to form a shape with at least one line of symmetry.)

SOLUTION

For each figure, add two line segments so that the resulting closed shape has a line of symmetry.

The strategy: identify what fold line would make the existing lines into mirror

halves, then draw the two missing segments as the mirror image of existing segments, or draw them to create a symmetric closure.

Figure 1 (partial arrow/chevron shape): Add two lines to form a symmetric arrowhead or kite shape with a vertical line of symmetry.

Figure 2 (partial open polygon): Add two lines to close the shape symmetrically, giving a horizontal or vertical line of symmetry.

Figure 3 (partial stepped shape): Add two lines to mirror the existing steps, completing a shape with a vertical line of symmetry.

Figure 4 (partial open bracket shape): Add two lines to form a closed figure with a line of symmetry.

Figure 5 (partial V or wedge shape): Add two lines to complete a triangle or symmetric polygon with a vertical line of symmetry.

Figure 6 (partial diagonal lines): Add two lines to complete a shape with a line of symmetry.

Answer: For each dot-grid figure, draw 2 more line segments to close the shape so that the resulting figure has at least one line of symmetry. The exact solution depends on the specific partial figure shown.

Exercise 9.3 — Rotational Symmetry

Q1 Find the angles of symmetry for the given figures about the point marked •.
(a) A plus/cross shape made of squares — a square in the centre with four square arms, centre marked •.
(b) An arrow or angular chevron shape pointing left, centre marked •.
(c) A horizontal rectangle (long and thin), centre marked •.

SOLUTION

(a) Plus/cross shape:
The cross has 4 equal arms at 90° to each other. Rotating by 90° brings each arm onto the next arm — the figure looks the same. Rotating by 180°, 270°, and 360° also gives the same figure.
Angles of symmetry: 90°, 180°, 270°, 360°.

(b) Chevron/arrow shape:
This shape points in a specific direction. Rotating by 90° or 180° does not bring it back to itself. Only a full 360° rotation returns it to the original position.
Angle of symmetry: 360° only.

(c) Horizontal rectangle:
Rotating the rectangle by 180° brings it exactly back onto itself (left end goes to right, right goes to left, top to bottom). Rotating by 360° also gives the original.
Angles of symmetry: 180°, 360°.

Answer: (a) 90°, 180°, 270°, 360°; (b) 360° only; (c) 180°, 360°.

Q2 Which of the following figures have more than one angle of symmetry? (Seven figures shown: a circle with cross inside and centre dot, a downward-pointing triangle with centre dot, a circle divided into 3 equal parts, a 4-blade fan/pinwheel shape, an X-cross made of two lines, a 5-pointed star with centre dot, and a D-shape/half-circle.)

SOLUTION

A figure has more than one angle of symmetry if it looks the same after rotating by some angle less than 360°.

Circle with cross (4 divisions): Rotating by 90° gives the same figure. Angles: 90°, 180°, 270°, 360°. More than one angle of symmetry: YES.

Downward-pointing triangle with dot: Rotating by 120° gives the same figure

(equilateral triangle). Angles: 120°, 240°, 360°. More than one angle of symmetry: YES.

Circle divided into 3 equal parts: Rotating by 120° gives the same figure. Angles: 120°, 240°, 360°. More than one angle of symmetry: YES.

4-blade fan/pinwheel: Rotating by 90° gives the same figure. Angles: 90°, 180°, 270°, 360°. More than one angle of symmetry: YES.

X-cross (two intersecting lines): Rotating by 90° gives the same figure. Angles: 90°, 180°, 270°, 360°. More than one angle of symmetry: YES.

5-pointed star: Rotating by 72° gives the same figure. Angles: 72°, 144°, 216°, 288°, 360°. More than one angle of symmetry: YES.

D-shape/half-circle: Only a full 360° rotation brings it back. More than one angle of symmetry: NO.

Answer: The following figures have more than one angle of symmetry: circle with cross, downward triangle, circle divided into 3 parts, 4-blade fan, X-cross, and 5-pointed star. The D-shape/half-circle does not.

Q3 Give the order of rotational symmetry for each figure:
(a) A line segment with an arrowhead at each end (a double-headed arrow).
(b) An X-shape (two straight lines crossing at the centre, like an oblique X).
(c) A 6-pointed star.
(d) A 3-legged running figure (like the Isle of Man triskelion).
(e) A plus/cross shape made of rectangles, with a centre dot.
(f) A regular pentagon with a centre dot.

SOLUTION

The order of rotational symmetry is the number of angles of symmetry (including 360°) for which the figure looks the same.

(a) Double-headed arrow (arrowhead at each end): Rotating by 180° swaps the two arrowheads and the figure looks exactly the same. Order = 2.

(b) X-shape (two straight lines crossing at the centre, like an oblique X): Rotating by 180° maps each line onto itself and brings the figure back to its original appearance. A rotation smaller than 180° does not. Order = 2.

(c) 6-pointed star: Rotating by 60° (= 360°/6) gives the same star. Order = 6.

(d) 3-legged figure (triskelion): Rotating by 120° (= 360°/3) gives the same figure.

Order = 3.

(e) Plus/cross shape with centre dot: Rotating by 90° gives the same cross. Order = 4.

(f) Regular pentagon with dot: Rotating by 72° (= 360°/5) gives the same pentagon. Order = 5.

Answer: (a) Order 2; (b) Order 2; (c) Order 6; (d) Order 3; (e) Order 4; (f) Order 5.

Exercise 9.4 — Symmetries of a Circle

Q1 Colour the sectors of the circle below so that the figure has i) 3 angles of symmetry, ii) 4 angles of symmetry, iii) what are the possible numbers of angles of symmetry you can obtain by colouring the sectors in different ways? (The circle is divided into 12 equal sectors.)

SOLUTION

The circle is divided into 12 equal sectors. Each sector subtends 30° at the centre.

(i) 3 angles of symmetry (120°, 240°, 360°):
The colouring pattern must repeat every 120° = every 4 sectors.
Colour sectors 1–4 with one pattern, 5–8 identically, 9–12 identically.
For example, colour 4 consecutive sectors in a repeating block of the same pattern. A simple way: colour sectors 1, 5, 9 (every 4th sector) one colour, and leave the rest white — this gives 3-fold rotational symmetry.

(ii) 4 angles of symmetry (90°, 180°, 270°, 360°):
The colouring pattern must repeat every 90° = every 3 sectors.
Colour sectors 1, 4, 7, 10 the same colour (every 3rd sector). This gives 4-fold symmetry.

(iii) Possible numbers of angles of symmetry:
The 12 sectors can give rotational symmetry of order n only if n divides 12.
Divisors of 12: 1, 2, 3, 4, 6, 12.
So the possible numbers of angles of symmetry obtainable are: 1 (trivial, only 360°), 2, 3, 4, 6, or 12.

Answer: i) Colour in a repeating pattern every 4 sectors to get 3 angles of symmetry (120°, 240°, 360°). ii) Colour in a repeating pattern every 3 sectors to get 4 angles of symmetry (90°, 180°, 270°, 360°). iii) The possible numbers of angles of symmetry are 1, 2, 3, 4, 6, or 12 (the divisors of 12).

Q2 Draw two figures other than a circle and a square that have both reflection symmetry and rotational symmetry.

SOLUTION

Figure 1 — Regular cross (plus shape):
A plus/cross shape made of a vertical rectangle and a horizontal rectangle of equal dimensions, centred at the same point.
Lines of symmetry: 4 (vertical, horizontal, and the two diagonals).
Order of rotational symmetry: 4 (angles 90°, 180°, 270°, 360°).

Figure 2 — Equilateral triangle:
A triangle with all three sides equal and all angles 60°.
Lines of symmetry: 3 (one from each vertex to the midpoint of the opposite side).
Order of rotational symmetry: 3 (angles 120°, 240°, 360°).

Both figures have both reflection symmetry and rotational symmetry.

Answer: Example 1: A regular cross/plus shape — 4 lines of symmetry, order 4 rotational symmetry. Example 2: An equilateral triangle — 3 lines of symmetry, order 3 rotational symmetry.

Q3 Draw, wherever possible, a rough sketch of:
a. A triangle with at least two lines of symmetry and at least two angles of symmetry.
b. A triangle with only one line of symmetry but not having rotational symmetry.
c. A quadrilateral with rotational symmetry but no reflection symmetry.
d. A quadrilateral with reflection symmetry but not having rotational symmetry.

SOLUTION

a. Triangle with at least two lines of symmetry and at least two angles of symmetry:
An equilateral triangle has 3 lines of symmetry and angles of symmetry 120°, 240°, 360° (3 angles, so at least two). This is the only triangle that works.

b. Triangle with only one line of symmetry and no rotational symmetry:
An isosceles triangle (but not equilateral) has exactly one line of symmetry (through the apex and midpoint of base) and no rotational symmetry (it only returns to itself after 360°, so does not qualify as having rotational symmetry).

c. Quadrilateral with rotational symmetry but no reflection symmetry:
A parallelogram (that is not a rectangle or rhombus) has rotational symmetry of

order 2 (180° rotation) but no line of symmetry. Draw a slanted parallelogram.

d. Quadrilateral with reflection symmetry but no rotational symmetry:
An isosceles trapezium has one line of symmetry (vertical axis) but no rotational symmetry (180° rotation does not map it to itself).

Answer: a. Equilateral triangle (3 lines of symmetry, angles 120°, 240°, 360°). b. Isosceles triangle (1 line of symmetry, no rotational symmetry). c. Non-rectangular parallelogram (no lines of symmetry, 180° rotational symmetry). d. Isosceles trapezium (1 line of symmetry, no rotational symmetry).

Q4 In a figure, 60° is the smallest angle of symmetry. What are the other angles of symmetry of this figure?

SOLUTION

When 60° is the smallest angle of symmetry, all other angles of symmetry are multiples of 60°.

Multiples of 60° up to and including 360°:
60° × 2 = 120°
60° × 3 = 180°
60° × 4 = 240°
60° × 5 = 300°
60° × 6 = 360°

So the other angles of symmetry are 120°, 180°, 240°, 300°, and 360°.

Answer: The other angles of symmetry are 120°, 180°, 240°, 300°, and 360°.

Q5 In a figure, 60° is an angle of symmetry. The figure has two angles of symmetry less than 60°. What is its smallest angle of symmetry?

SOLUTION

All angles of symmetry are multiples of the smallest angle of symmetry.

Let the smallest angle of symmetry be s degrees.
Since 60° is an angle of symmetry, 60° must be a multiple of s: 60 = k × s for some positive integer k.

There are two angles of symmetry less than 60°. These would be s and 2s (since 3s = 60°).

So k = 3, meaning s = 60° ÷ 3 = 20°.

Check: Angles of symmetry would be 20°, 40°, 60°, 80°, ... — but 20° and 40° are the two angles less than 60°. This fits.

The smallest angle of symmetry = 20°.

Answer: The smallest angle of symmetry is 20°.

Q6 Can we have a figure with rotational symmetry whose smallest angle of symmetry is:
a. 45°?
b. 17°?

SOLUTION

A figure can have rotational symmetry with smallest angle s if and only if s divides 360° exactly (i.e., 360 ÷ s is a whole number), because after repeating the rotation s degrees, we must return exactly to the start at 360°.

a. Smallest angle = 45°:
360° ÷ 45° = 8. This is a whole number.
Yes, a figure with smallest angle of symmetry 45° is possible (e.g., a regular 8-pointed star). The angles of symmetry would be 45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°.

b. Smallest angle = 17°:
360° ÷ 17° = 21.176... This is not a whole number.
No, such a figure is not possible, because the rotations would not return to exactly 360°.

Answer: a. Yes, 45° is possible since 360 ÷ 45 = 8 (a whole number). b. No, 17° is not possible since 360 ÷ 17 is not a whole number.

Q7 This is a picture of the new Parliament Building in Delhi. (The outer boundary is shown as a regular triangle with concave or flat sides — essentially a triangle with 3 lines of symmetry.)
a. Does the outer boundary of the picture have reflection symmetry? If so, draw the lines of symmetries. How many are they?
b. Does it have rotational symmetry around its centre? If so, find the angles of rotational symmetry.

SOLUTION

The outer boundary of the new Parliament Building is triangular in shape (a triangle with equal sides — essentially an equilateral triangle form).

a. Reflection symmetry:
An equilateral triangle has 3 lines of symmetry — one from each vertex perpendicular to the opposite side (i.e., from each vertex to the midpoint of the opposite side).
Yes, the outer boundary has reflection symmetry, with 3 lines of symmetry.

b. Rotational symmetry:
An equilateral triangle looks the same after rotating by 120° (one-third of 360°) about its centre.
Angles of symmetry: 120°, 240°, 360°.
Yes, it has rotational symmetry. The angles of rotational symmetry are 120°, 240°, and 360°.

Answer: a. Yes, the outer boundary has reflection symmetry with 3 lines of symmetry. b. Yes, it has rotational symmetry; the angles of rotational symmetry are 120°, 240°, and 360°.

Q8 How many lines of symmetry do the shapes in the first shape sequence in Chapter 1, Table 3, the Regular Polygons, have? What number sequence do you get?

SOLUTION

The Regular Polygon sequence starts from the equilateral triangle (3 sides) and increases: triangle, square, pentagon, hexagon, heptagon, octagon, ...

A regular n-gon has exactly n lines of symmetry.

Triangle (3 sides): 3 lines of symmetry
Square (4 sides): 4 lines of symmetry

Pentagon (5 sides): 5 lines of symmetry
Hexagon (6 sides): 6 lines of symmetry
Heptagon (7 sides): 7 lines of symmetry
Octagon (8 sides): 8 lines of symmetry
Nonagon (9 sides): 9 lines of symmetry
Decagon (10 sides): 10 lines of symmetry

The number sequence is: 3, 4, 5, 6, 7, 8, 9, 10, ... — this is the sequence of counting numbers starting from 3.

Answer: A regular n-gon has n lines of symmetry. The sequence is 3, 4, 5, 6, 7, 8, 9, 10, ... — the counting numbers starting from 3.

Q9 How many angles of symmetry do the shapes in the first shape sequence in Chapter 1, Table 3, the Regular Polygons, have? What number sequence do you get?

SOLUTION

A regular n-gon has exactly n angles of symmetry (the multiples of 360°/n up to and including 360°).

Triangle (3 sides): 3 angles of symmetry (120°, 240°, 360°)
Square (4 sides): 4 angles of symmetry (90°, 180°, 270°, 360°)
Pentagon (5 sides): 5 angles of symmetry
Hexagon (6 sides): 6 angles of symmetry
Heptagon (7 sides): 7 angles of symmetry
Octagon (8 sides): 8 angles of symmetry
Nonagon (9 sides): 9 angles of symmetry
Decagon (10 sides): 10 angles of symmetry

The number sequence is: 3, 4, 5, 6, 7, 8, 9, 10, ... — the same sequence of counting numbers starting from 3.

Answer: A regular n-gon has n angles of symmetry. The sequence is 3, 4, 5, 6, 7, 8, 9, 10, ... — counting numbers starting from 3.

Q10 How many lines of symmetry do the shapes in the last shape sequence in Chapter 1, Table 3, the Koch Snowflake sequence, have? How many angles of symmetry?

SOLUTION

The Koch Snowflake sequence starts from an equilateral triangle and builds by replacing each side with a Koch curve.

Stage 0 (equilateral triangle): 3 lines of symmetry; 3 angles of symmetry.

Stage 1 and beyond (Koch Snowflake iterations): The resulting snowflake retains the 3-fold symmetry of the original triangle, but each of the 3 lines of symmetry is still a line of symmetry. However, as the fractal develops, additional symmetries arise — at each stage the snowflake has 6 lines of symmetry (3 through vertices and 3 through midpoints) and 6 angles of symmetry.

From the official solution: the sequence of (lines of symmetry, angles of symmetry) for successive stages is:
Stage 0: 3, 3
Stage 1: 6, 6
Stage 2: 6, 6
Stage 3: 6, 6
Stage 4: 6, 6

Number of lines of symmetry: 3, 6, 6, 6, 6, ...
Number of angles of symmetry: 3, 6, 6, 6, 6, ...

Answer: Lines of symmetry sequence: 3, 6, 6, 6, 6, ... Angles of symmetry sequence: 3, 6, 6, 6, 6, ...

Q11 How many lines of symmetry and angles of symmetry does Ashoka Chakra have?

SOLUTION

The Ashoka Chakra is the 24-spoked wheel appearing on the Indian national flag.

Lines of symmetry: Each spoke is a diameter of the wheel. Folding along any spoke creates two halves that overlap. Since there are 24 spokes, and each diameter is a line of symmetry, but each line of symmetry passes through 2 opposite spokes.
Number of distinct lines of symmetry = 24 ÷ 2 × 2 = 24.

The Ashoka Chakra has 24 lines of symmetry.

Angles of symmetry: The wheel looks the same after rotating by 360°/24 = 15°.
So the angles of symmetry are all multiples of 15° from 15° to 360°.
Number of angles of symmetry = 360° ÷ 15° = 24.
The Ashoka Chakra has 24 angles of symmetry.

Answer: The Ashoka Chakra has 24 lines of symmetry and 24 angles of symmetry.