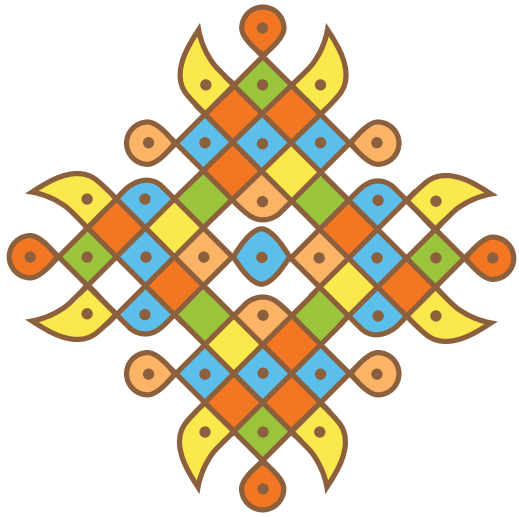
9

SYMMETRY



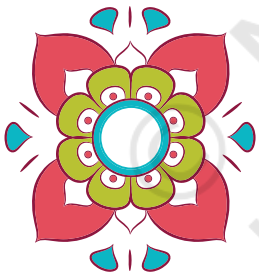
Look around you—you may find many objects that catch your attention. Some such things are shown below:



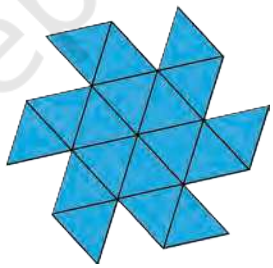
Flower



Butterfly



Rangoli



Pinwheel

There is something beautiful about the pictures above.

The flower looks the same from many different angles. What about the butterfly? No doubt, the colours are very attractive. But what else about the butterfly appeals to you?

In these pictures, it appears that some parts of the figure are repeated and these repetitions seem to occur in a definite pattern. Can you see what repeats in the beautiful *rangoli* figure? In the

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rangoli, the red petals come back onto themselves when the flower is rotated by 90° around the centre and so do the other parts of the *rangoli*.

What about the pinwheel? Can you spot which pattern is repeating?
Hint: Look at the hexagon first.

Now, can you say what figure repeats along each side of the hexagon? What is the shape of the figure that is stuck to each side? Do you recognise it? How do these shapes move as you move along the boundary of the hexagon? What about the other pictures—what is it about those structures that appeals to you and what are the patterns in those structures that repeat?



Clouds

On the other hand, look at this picture of clouds. There is no such repetitive pattern.

We can say that the first four figures are symmetrical and the last one is not symmetrical. A symmetry refers to a part or parts of a figure that are repeated in some definite pattern.



Taj Mahal



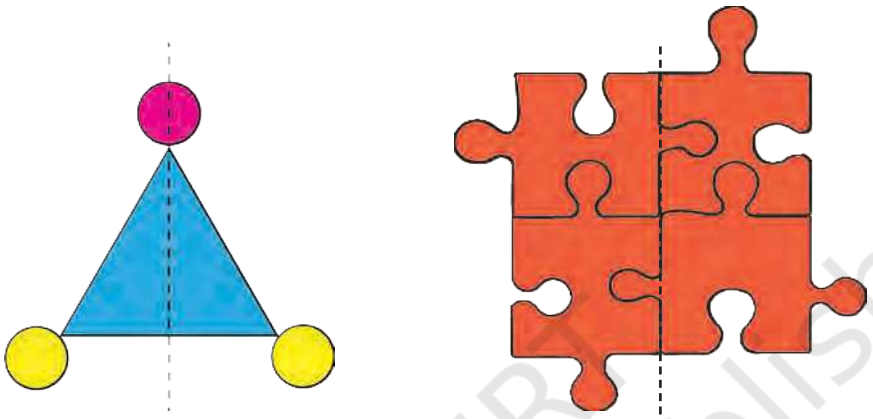
Gopuram

What are the symmetries that you see in these beautiful structures?

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9.1 Line of Symmetry

Figure (a) shows the picture of a blue triangle with a dotted line. What if you fold the triangle along the dotted line? Yes, one half of the triangle covers the other half completely. These are called **mirror halves**!



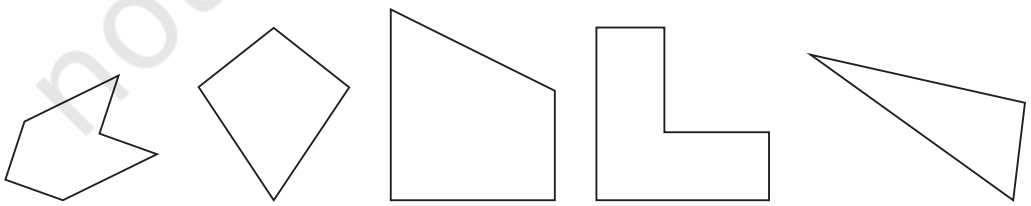
(a) (b)

What about Figure (b) with the four puzzle pieces and a dotted line passing through the middle? Are they mirror halves? No, when we fold along the line, the left half does not exactly fit over the right half.

A line that cuts a figure into two parts that exactly overlap when folded along that line is called a **line of symmetry** of the figure.

Figure it Out

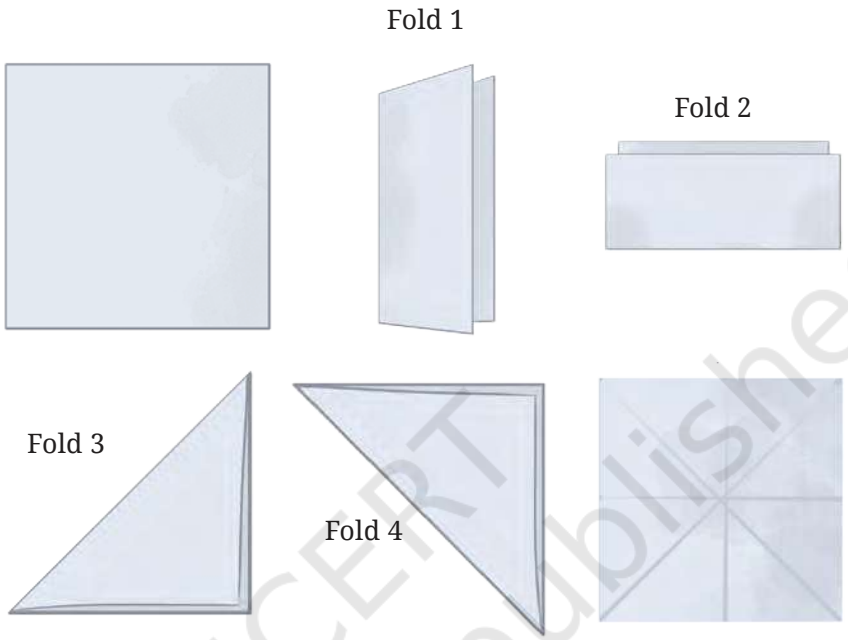
1. Do you see any line of symmetry in the figures at the start of the chapter? What about in the picture of the cloud?
2. For each of the following figures, identify the line(s) of symmetry if it exists.



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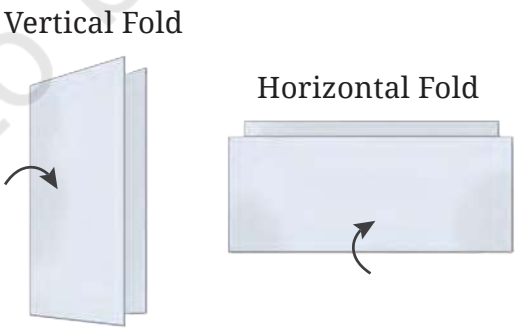
Figures with more than one line of symmetry

Does a square have only one line of symmetry?
Take a square piece of paper. By folding, find all its lines of symmetry.



Here are the different folds giving different lines of symmetry.

- Fold the paper into half vertically.
- Fold it again into half horizontally (i.e., you have folded it twice). Now open out the folds.



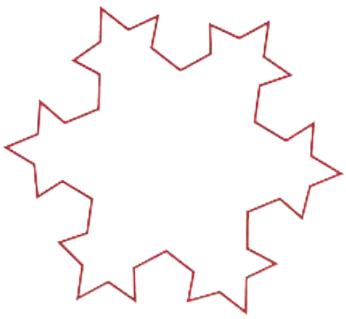
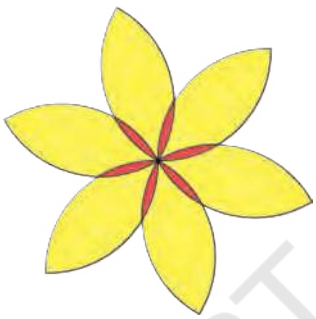
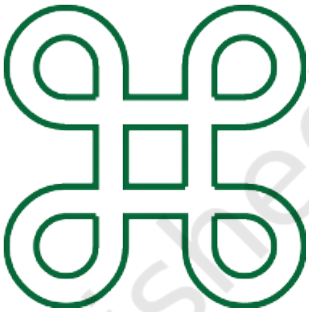
Again fold the square into half (for a third time now), but this time along a diagonal, as shown in the figure. Again, open it.

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Fold it into half (for the fourth time), but this time along the other diagonal, as shown in the figure. Open out the fold.

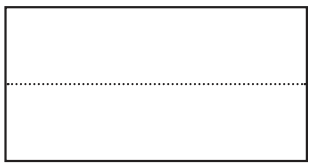
Is there any other way to fold the square so that the two halves overlap? How many lines of symmetry does the square shape have?

Thus, figures can have multiple lines of symmetry. The figures below also have multiple lines of symmetry. Can you find them all?

We saw that the diagonal of a square is also a line of symmetry. Let us take a rectangle that is not a square. Is its diagonal a line of symmetry?

First, see the rectangle and answer this question. Then, take a rectangular piece of paper and check if the two parts overlap by folding it along its diagonal. What do you observe?



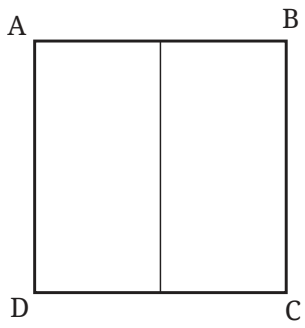
Reflection

So far we have been saying that when we fold a figure along a line of symmetry, the two parts overlap completely. We could also say that the part of the figure on one side of the line of symmetry is reflected by the line to the other side; similarly, the part of the figure on the other side of the line of symmetry is reflected to the first side! Let us understand this by labeling some points on the figure.

The figure shows a square with its corners labeled A, B, C and D. Let us first consider the vertical line of symmetry. When we reflect

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the square along this line, the points B, C on the right get reflected to the left side and occupy the positions occupied earlier by A, D. What happens to the points A, D? A occupies the position occupied by B and D that of C!



What if we reflect along the diagonal from A to C? Where do points A, B, C and D go? What if we reflect along the horizontal line of symmetry?

A figure that has a line or lines of symmetry is thus, also said to have **reflection symmetry**.

Generating shapes having lines of symmetry

So far we have seen symmetrical figures and asymmetrical figures. How does one generate such symmetrical figures? Let us explore this.

Ink Blot Devils

You enjoyed doing this earlier in Class 5. Take a piece of paper. Fold it in half. Open the paper and spill a few drops of ink (or paint) on one half.

Now press the halves together and then open the paper again.

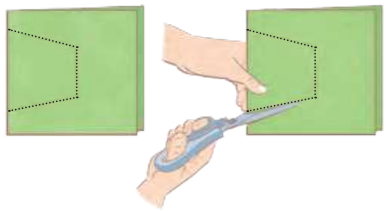
- What do you see?
- Is the resulting figure symmetric?
- If yes, where is the line of symmetry?
- Is there any other line along which it can be folded to produce two identical parts?
- Try making more such patterns.

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Paper Folding and Cutting

Here is another way of making symmetric shapes!

In these two figures, a sheet of paper is folded and a cut is made along the dotted line shown. Draw a sketch of how the paper will look when unfolded.



Do you see a line of symmetry in this figure? What is it?

Make different symmetric shapes by folding and cutting.

There are more ways of folding and cutting pieces of paper to get symmetric shapes!

Use thin rectangular coloured paper. Fold it several times and create some intricate patterns by cutting the paper, like the one shown here. Identify the lines of symmetry in the repeating design. Use such decorative paper cut-outs for festive occasions.

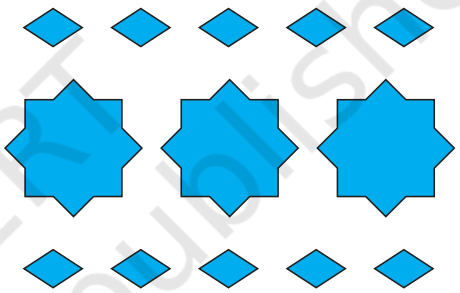
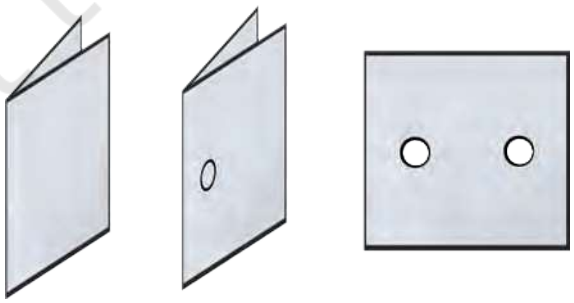


Figure it Out

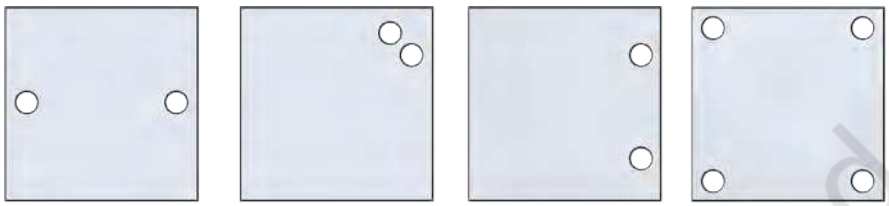
Punching Game

The fold is a line of symmetry. Punch holes at different locations of a folded square sheet of paper using a punching machine and create different symmetric patterns.



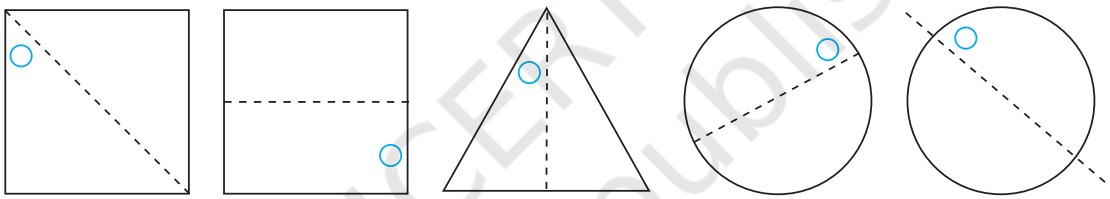
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1. In each of the following figures, a hole was punched in a folded square sheet of paper and then the paper was unfolded. Identify the line along which the paper was folded.
   Figure (d) was created by punching a single hole. How was the paper folded?



a. b. c. d.

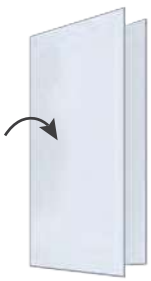
2. Given the line(s) of symmetry, find the other hole(s):



a. b. c. d. e.

3. Here are some questions on paper cutting.
   Consider a vertical fold. We represent it this way:

Vertical Fold



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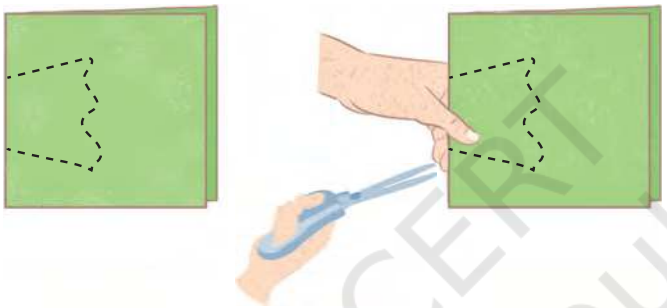
Similarly, a horizontal fold is represented as follows:

Horizontal Fold

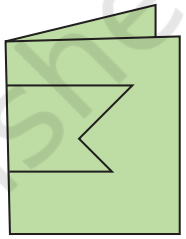


4. After each of the following cuts, predict the shape of the hole when the paper is opened. After you have made your prediction, make the cutouts and verify your answer.

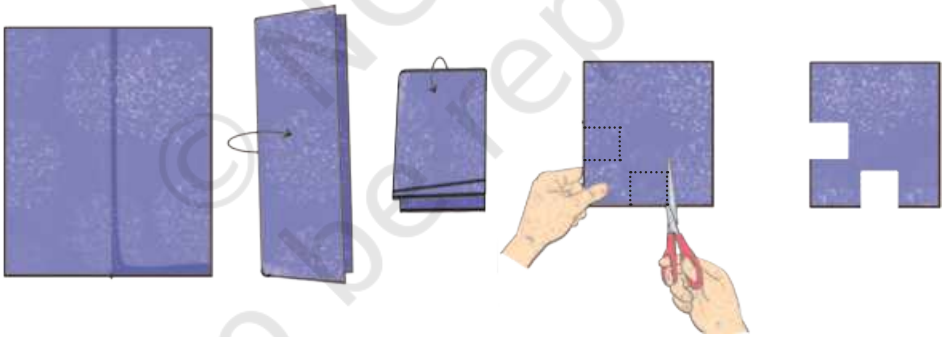
a.



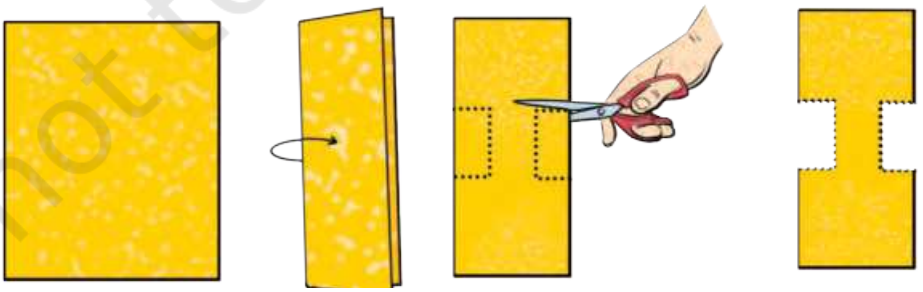
b.



c.



d.



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5 Suppose you have to get each of these shapes with some folds and a single straight cut. How will you do it?

a. The hole in the centre is a square.



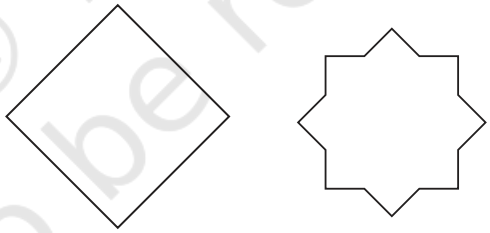
b. The hole in the centre is a square.



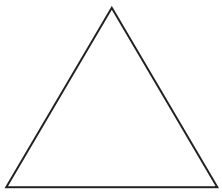
Note: For the above two questions, check if the 4-sided figures in the centre satisfy both the properties of a square.

6. How many lines of symmetry do these shapes have?

a.

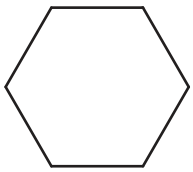


b. A triangle with equal sides and equal angles.

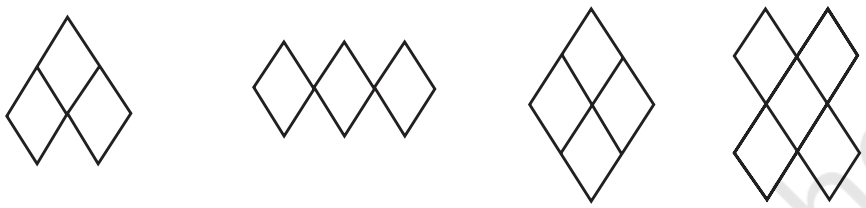


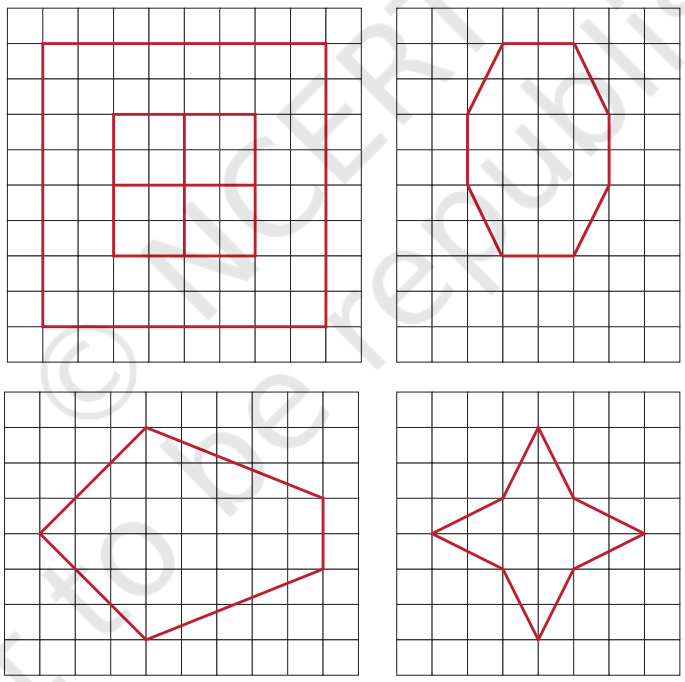
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c. A hexagon with equal sides and equal angles.



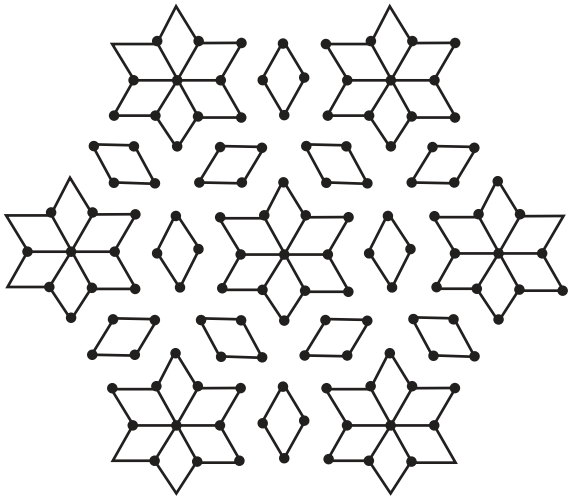
7. Trace each figure and draw the lines of symmetry, if any:





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8. Find the lines of symmetry for the *kolam* below.

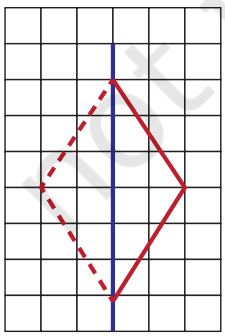


9. Draw the following.
   a. A triangle with exactly one line of symmetry.
   b. A triangle with exactly three lines of symmetry.
   c. A triangle with no line of symmetry.

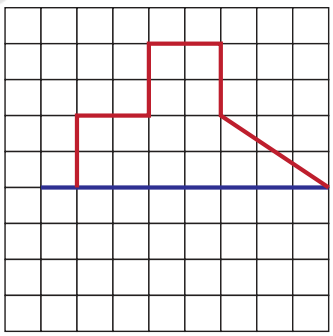
   Is it possible to draw a triangle with exactly two lines of symmetry?

10. Draw the following. In each case, the figure should contain at least one curved boundary.
    a. A figure with exactly one line of symmetry.
    b. A figure with exactly two lines of symmetry.
    c. A figure with exactly four lines of symmetry.

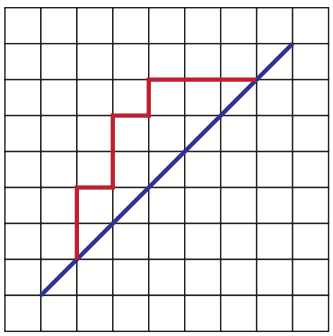
11. Copy the following on squared paper. Complete them so that the blue line is a line of symmetry. Problem (a) has been done for you.



(a)

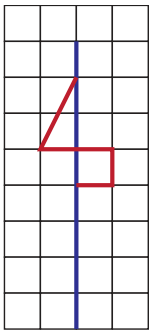


(b)

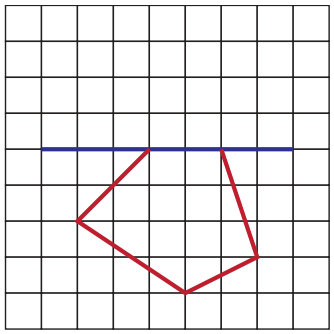


(c)

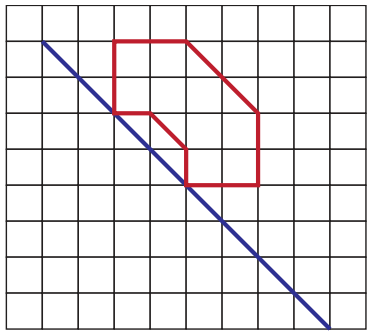
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(d)



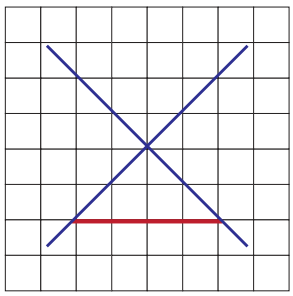
(e)



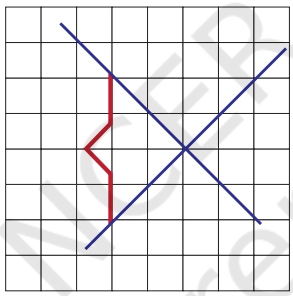
(f)

Hint: For (c) and (f), see if rotating the book helps!

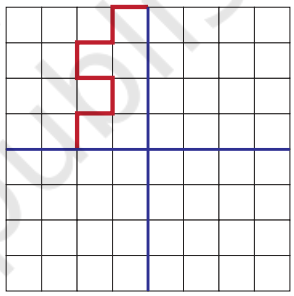
12. Copy the following drawing on squared paper. Complete each one of them so that the resulting figure has the two blue lines as lines of symmetry.



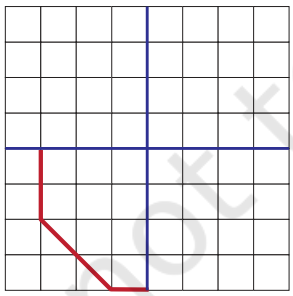
(a)



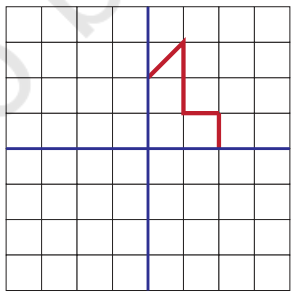
(b)



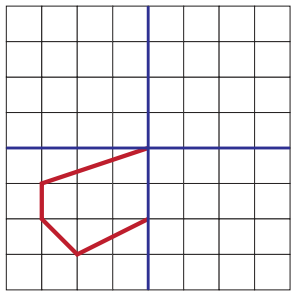
(c)



(d)



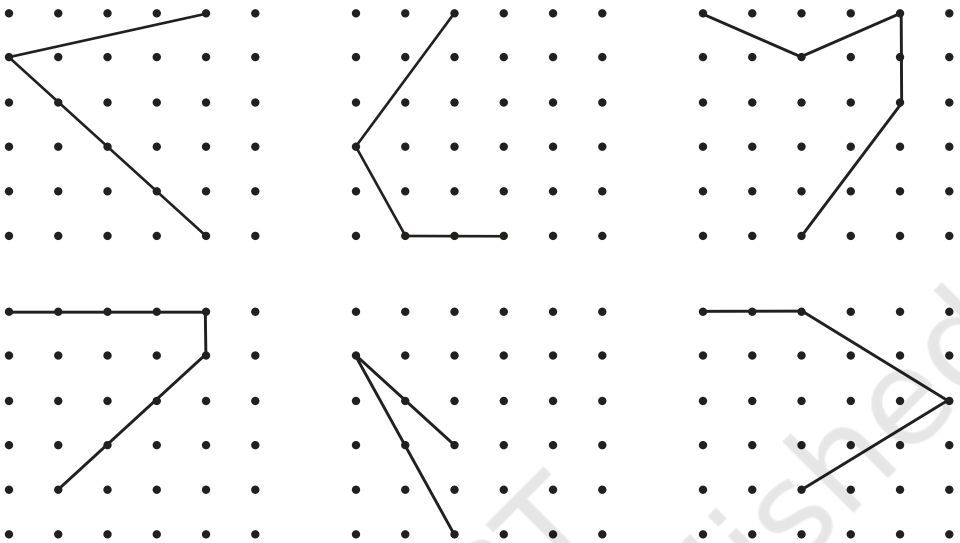
(e)



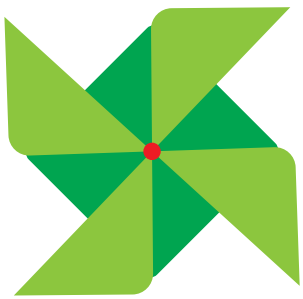
(f)

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13. Copy the following on a dot grid. For each figure draw two more lines to make a shape that has a line of symmetry.



9.2 Rotational Symmetry



The paper windmill in the picture looks symmetrical but there is no line of symmetry! However, if you fold it, the two halves will not exactly overlap. On the other hand, if you rotate it by 90° about the red point at the centre, the windmill looks exactly the same.

We say that the windmill has **rotational symmetry**.

When talking of rotational symmetry, there is always a fixed point about which the object is rotated. This fixed point is called the **centre of rotation**.

Will the windmill above look exactly the same when rotated through an angle of less than 90°?

No!

An angle through which a figure can be rotated to look exactly the same is called an **angle of rotational symmetry**, or just an **angle of symmetry**, for short.

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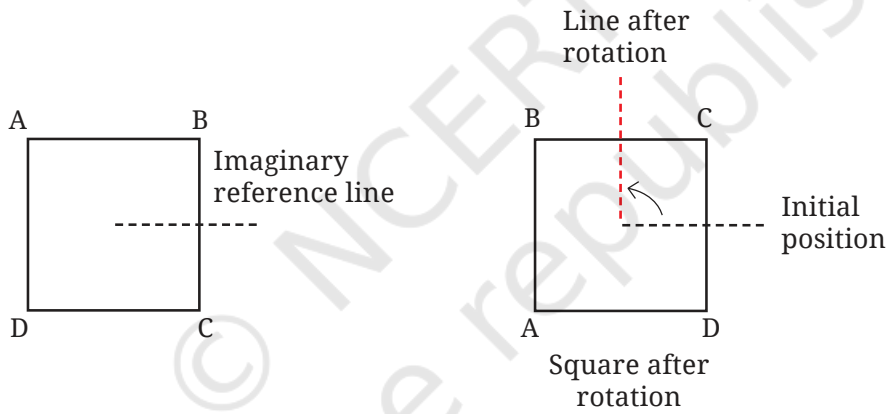
For the windmill, the angles of symmetry are 90° (quarter turn), 180° (half turn), 270° (three-quarter turn) and 360° (full turn). Observe that when any figure is rotated by 360°, it comes back to its original position, so 360° is always an angle of symmetry.

Thus, we see that the windmill has 4 angles of symmetry.

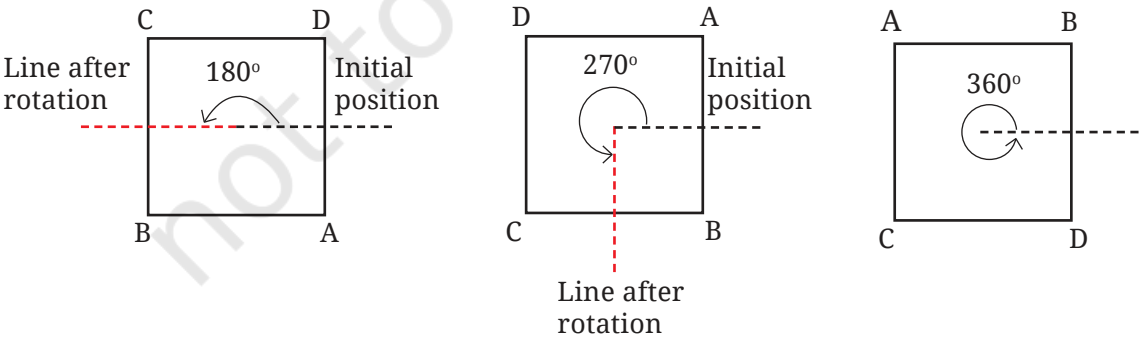
Do you know of any other shape that has exactly four angles of symmetry?

How many angles of symmetry does a square have? How much rotation does it require to get the initial square?

We get back a square overlapping with itself after 90° of rotation. This takes point A to the position of point B, point B to the position of point C, point C to the position of point D, and point D back to the position of point A. Do you know where to mark the centre of rotation?

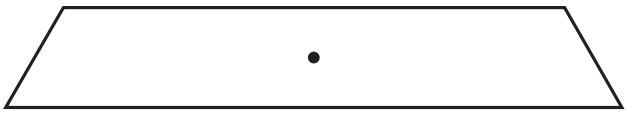


What are the other angles of symmetry?

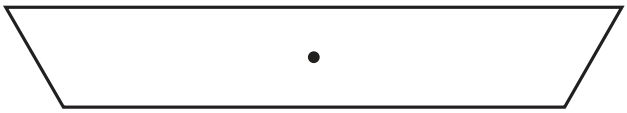


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Example: Find the angles of symmetry of the following strip.



Solution: Let us rotate the strip in a clockwise direction about its centre.



A rotation of 180° results in the figure above. Does this overlap with the original figure.

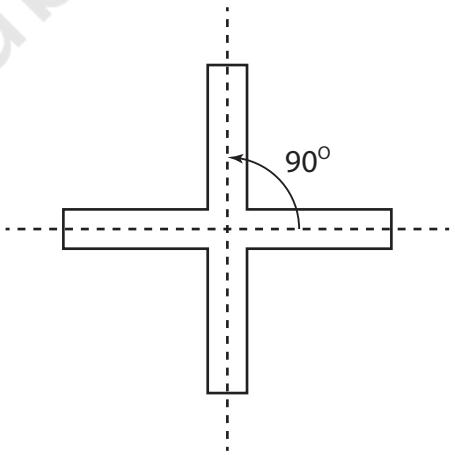
No. Why?

Another rotation through 180° from this position gives the original shape.

This figure comes back to its original shape only after **one** complete rotation through 360°. So, we say that this figure **does not have rotational symmetry**.

Rotational Symmetry of Figures with Radial Arms

Consider this figure, a picture with 4 radial arms. How many angles of symmetry does it have? What are they? Note that the angle between adjacent central dotted lines is 90°.



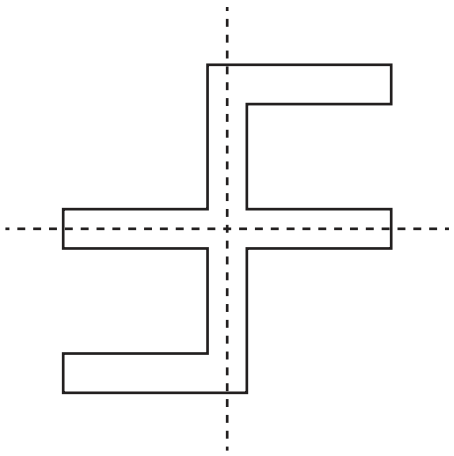
Can you change the angles between the radial arms so that the figure still has 4 angles of symmetry? Try drawing it.

To check if the figure drawn indeed has 4 angles of symmetry, you could draw the figure on two different pieces of paper. Cut out the radial arms from one of the papers. Keep the figure on the paper fixed and rotate the cutout to check for rotational symmetry.

How will you modify the figure above so that it has only two angles of symmetry?

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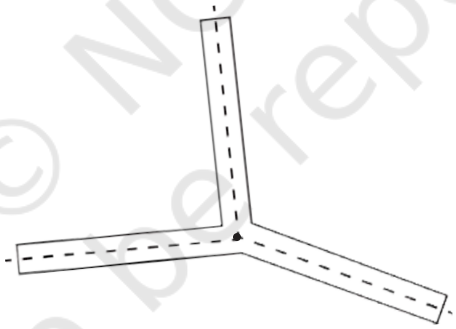
Here is one way:



We have seen figures having 4 and 2 angles of symmetry. Can we get a figure having exactly 3 angles of symmetry? Can you use radial arms for this?

Let us try with 3 radial arms as in the figure below. How many angles of symmetry does it have and what are they?

Here is a figure with three radial arms.



Trace and cut out a copy of this figure. By rotating the cutout over this figure determine its angles of rotation.

We see that only a full turn or a rotation of 360° will bring the figure back into itself. So this figure does not have rotational symmetry as 360 degrees is its only angle of symmetry.

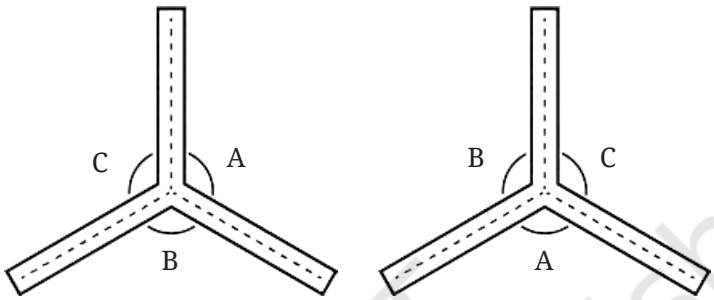
However, can anything in the figure be changed to make it have 3 angles of symmetry?

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Can it be done by changing the angles between the dotted lines?

If a figure with three radial arms should have rotational symmetry, then a rotated version of it should overlap with the original. Here are rough diagrams of both of them.

If these two figures must overlap, what can you tell about the angles?



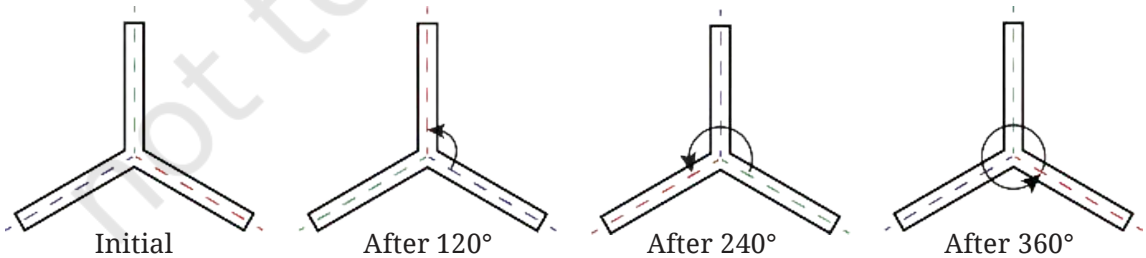
Observe that ∠A must overlap ∠B, ∠B must overlap ∠C and ∠C must overlap ∠A.

So, ∠A = ∠B = ∠C. What must this angle be?

We know that a full turn has 360 degrees. This is equally distributed amongst these three angles. So each angle must be $\frac{360°}{3}$ = 120°.

So, the radial arms figure with 3 arms shows rotational symmetry when the angle between the adjacent dotted lines is 120°. Use paper cutouts to verify this observation.

Now how many angles of rotation does the figure have and what are they?



Note: The colours have been added to show the rotations.

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Let us explore more figures.

Can you draw a figure with radial arms that has a) exactly 5 angles of symmetry, b) 6 angles of symmetry? Also find the angles of symmetry in each case.

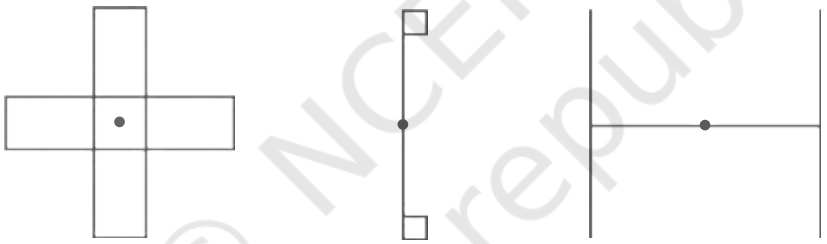
Hint: Use 5 radial arms for the first case. What should the angle between two adjacent radial arms be?

Consider a figure with radial arms having exactly 7 angles of symmetry. What will be its smallest angle of symmetry? Is the number of degrees a whole number in this case? If not, express it as a mixed fraction.

Let us find the angles of symmetry for other kinds of figures.

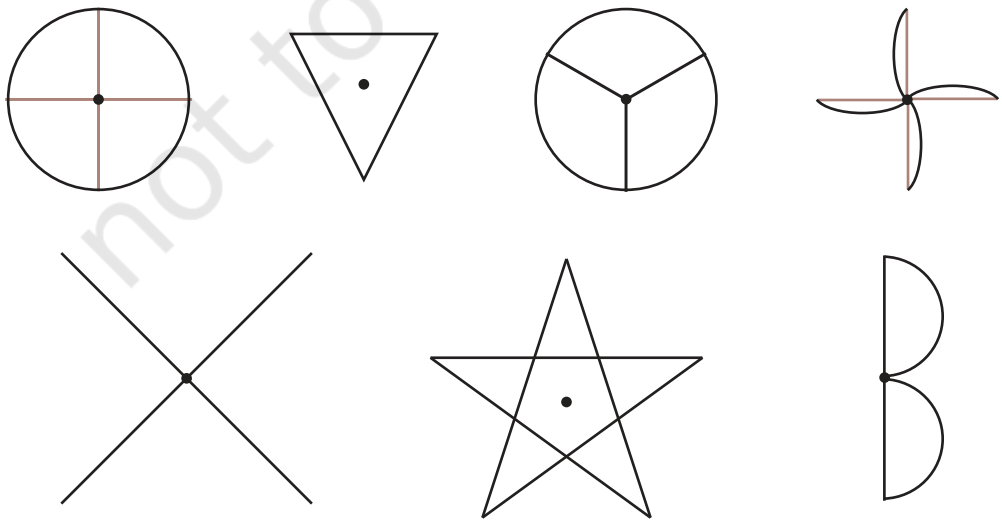
Figure it Out

1. Find the angles of symmetry for the given figures about the point marked •.



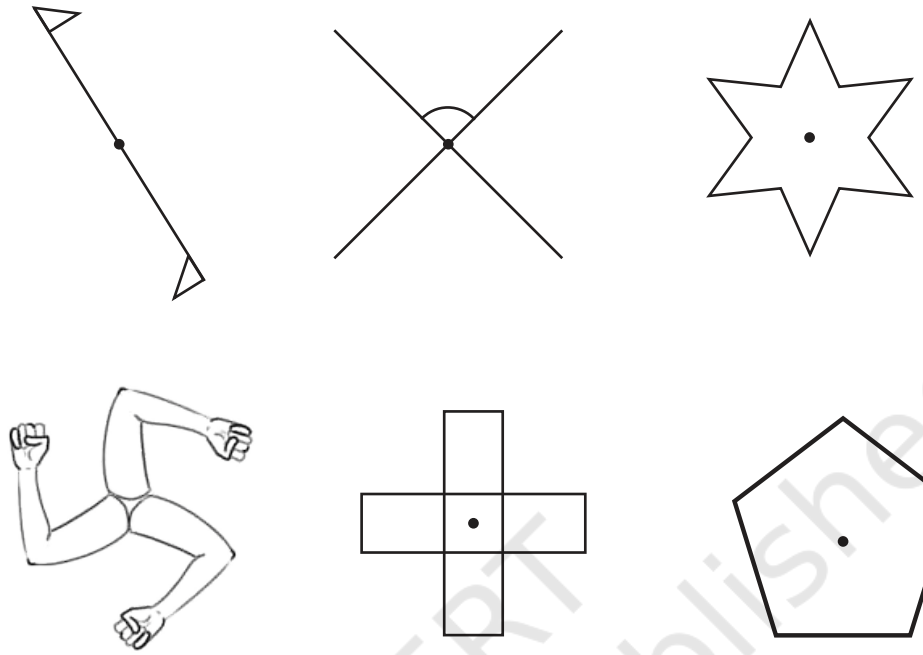
(a) (b) (c)

2. Which of the following figures have more than one angle of symmetry?



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3. Give the order of rotational symmetry for each figure:



Let us list down the angles of symmetry for all the cases above.

- Angles of symmetry when there are exactly 2 of them: 180°, 360°.
- Angles of symmetry when there are exactly 3 of them: 120°, 240°, 360°.
- Angles of symmetry when there are exactly 4 of them: 90°, 180°, 270°, 360°.

Do you observe something common about the angles of symmetries in these cases? The first set of numbers are all multiples of 180. The second are all multiples of 120. The third are all multiples of 90.

In each case, the angles are the multiples of the smallest angle. You may wonder and ask if this will always happen. What do you think?

True or False

- Every figure will have 360 degrees as an angle of symmetry.

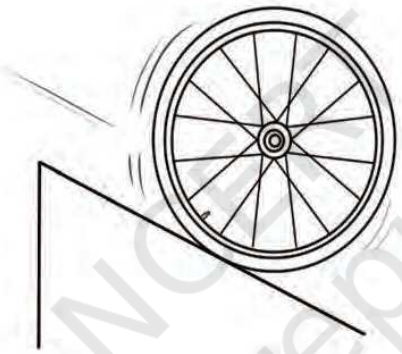
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- If the smallest angle of symmetry of a figure is a natural number in degrees, then it is **a factor** of 360.

Is there a smallest angle of symmetry for all figures? It turns out that this is the case for most figures, except for the most symmetric shapes like the circle, whose symmetries we now discuss.

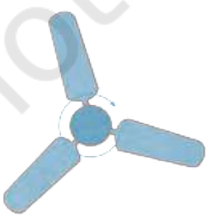
Symmetries of a circle

The circle is a fascinating figure. What happens when you rotate a circle clockwise about its centre? It coincides with itself. It does not matter what angle you rotate it by! So, for a circle, every angle is an angle of symmetry.

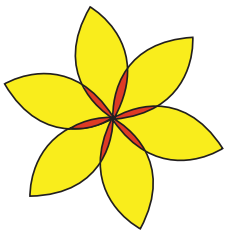


Now take a point on the rim of the circle and join it to the centre. Extend the segment to a diameter of the circle. Is that diameter a line of reflection symmetry? It is. Every diameter is a line of symmetry!

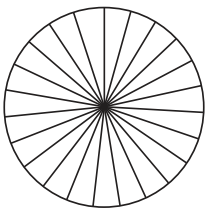
Like wheels, we can find other objects around us having rotational symmetry. Find them. Some of them are shown below:



Fan



Flower

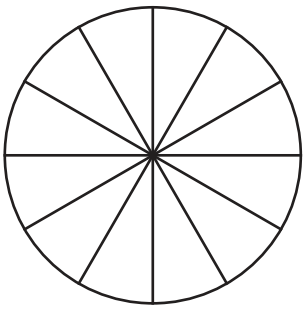


Wheel

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Figure it Out

1. Colour the sectors of the circle below so that the figure has i) 3 angles of symmetry, ii) 4 angles of symmetry, iii) what are the possible numbers of angles of symmetry you can obtain by colouring the sectors in different ways?



2. Draw two figures other than a circle and a square that have both reflection symmetry and rotational symmetry.
3. Draw, wherever possible, a rough sketch of:
   a. A triangle with at least two lines of symmetry and at least two angles of symmetry.
   b. A triangle with only one line of symmetry but not having rotational symmetry.
   c. A quadrilateral with rotational symmetry but no reflection symmetry.
   d. A quadrilateral with reflection symmetry but not having rotational symmetry.

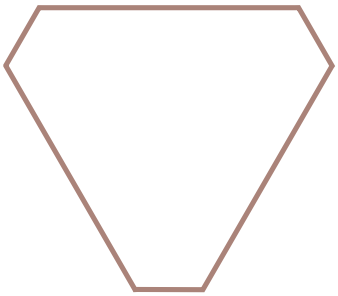
Try This

4. In a figure, 60° is the smallest angle of symmetry. What are the other angles of symmetry of this figure?
5. In a figure, 60° is an angle of symmetry. The figure has two angles of symmetry less than 60°. What is its smallest angle of symmetry?
6. Can we have a figure with rotational symmetry whose smallest angle of symmetry is:
   a. 45°?
   b. 17°?

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7. This is a picture of the new Parliament Building in Delhi.





   a. Does the outer boundary of the picture have reflection symmetry? If so, draw the lines of symmetries. How many are they?
   b. Does it have rotational symmetry around its centre? If so, find the angles of rotational symmetry.

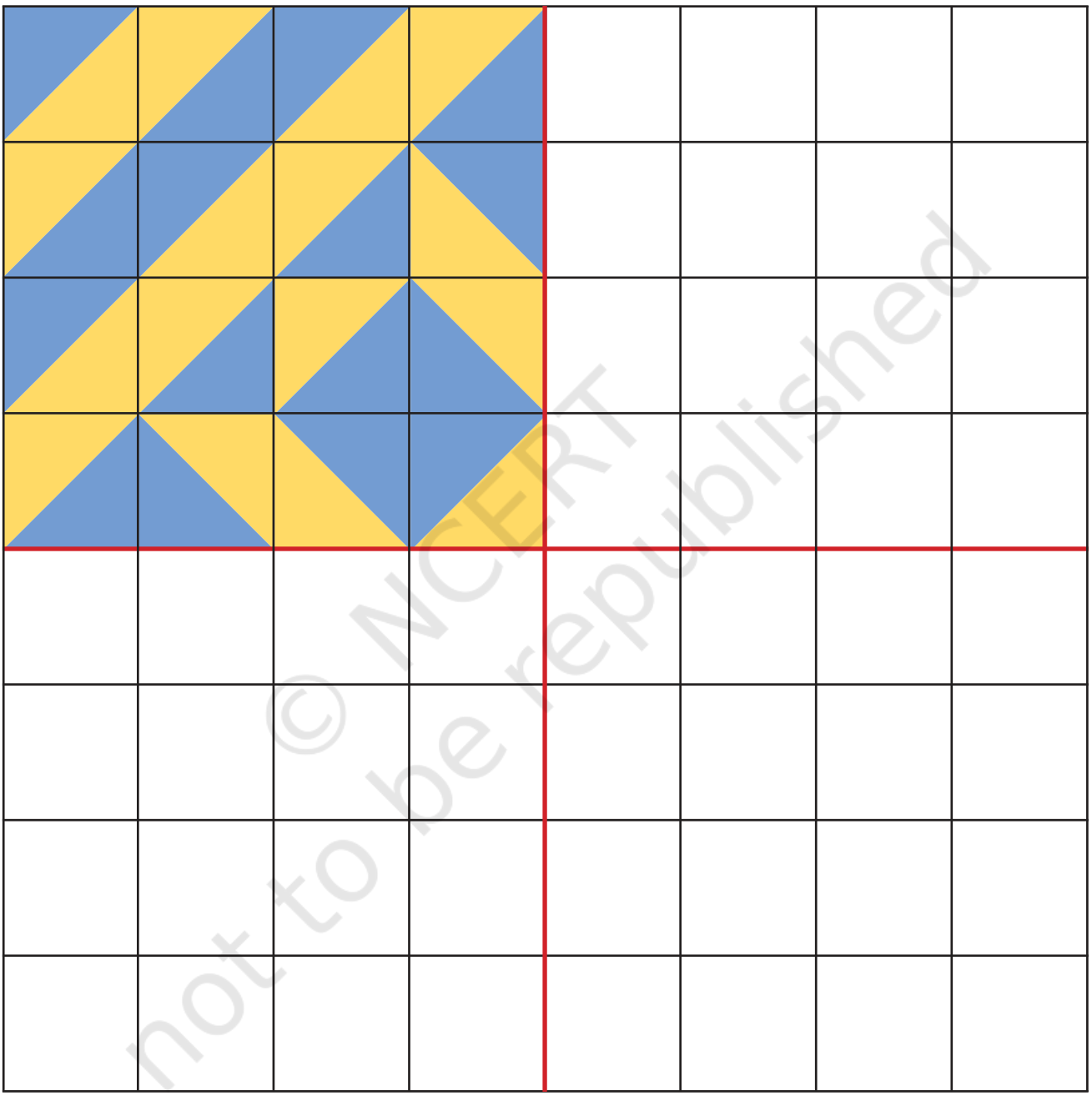
8. How many lines of symmetry do the shapes in the first shape sequence in Chapter 1, Table 3, the Regular Polygons, have? What number sequence do you get?
9. How many angles of symmetry do the shapes in the first shape sequence in Chapter 1, Table 3, the Regular Polygons, have? What number sequence do you get?
10. How many lines of symmetry do the shapes in the last shape sequence in Chapter 1, Table 3, the Koch Snowflake sequence, have? How many angles of symmetry?
11. How many lines of symmetry and angles of symmetry does *Ashoka Chakra* have?



Playing with Tiles

a. Use the colour tiles  given at the end of the book to complete the following figure so that it has exactly 2 lines of symmetry.
b. Use 16 such tiles to make figures that have exactly:
   1 line of symmetry
   2 lines of symmetry
c. Use these tiles in making creative symmetric designs.

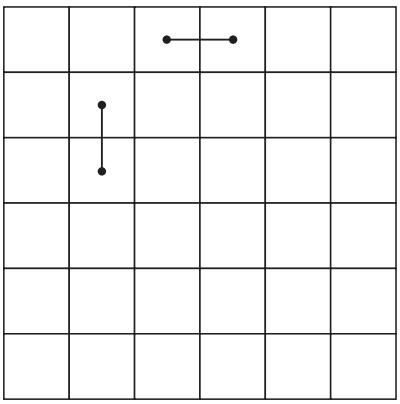
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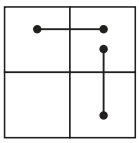
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Game

Draw a 6 by 6 grid. Two players take turns covering two adjacent squares by drawing a line. The line can be placed either way: horizontally or vertically. The lines cannot overlap. The game goes on till a player is not able to place any more lines. The player who is not able to place a line loses.



Not allowed



With what strategy can one play to win this game?

SUMMARY

- When a figure is made up of parts that repeat in a definite pattern, we say that the figure has **symmetry**. We say that such a figure is **symmetrical**.
- A line that cuts a plane figure into two parts that exactly overlap when folded along that line is called a **line of symmetry** or **axis of symmetry** of the figure.
- A figure may have multiple lines of symmetry.
- Sometimes a figure looks exactly the same when it is rotated by an angle about a fixed point. Such an angle is called an **angle of symmetry** of the figure. A figure that has an angle of symmetry strictly between 0 and 360 degrees is said to have **rotational symmetry**. The point of the figure about which the rotation occurs is called the **centre of rotation**.
- A figure may have multiple angles of symmetry.
- Some figures may have a line of symmetry but no angle of symmetry, while others may have angles of symmetry but no lines of symmetry. Some figures may have both lines of symmetry as well as angles of symmetry.

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CHAPTER 9 — SOLUTIONS

Symmetry

Section 9.1

Page No. 219

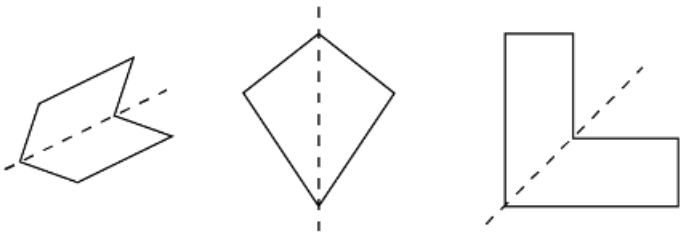
Figure it Out

Q1. Do you see any line of symmetry in the figures at the start of the chapter? What about in the picture of the cloud?

Ans. Yes, there are 6, 4 and 1 lines of symmetry in the figures of flower, rangoli and butterfly respectively. There is no line of symmetry in the figures of pinwheel and cloud.

Q2. For each of the following figures, identify the line(s) of symmetry if it exists.

Ans.



Section 9.1

Page no. 221

Q. Is there any other way to fold the square so that the two halves overlap? How many lines of symmetry does the square shape have?

Ans. No, there is no other way to fold the square.

The square shape has 4 lines of symmetry.

Q. We saw that the diagonal of a square is also a line of symmetry. Let us take a rectangle that is not a square. Is its diagonal a line of symmetry?

Ans. No, Rectangle's diagonal is not a line of symmetry.

Page no. 222

Q. What if we reflect along the diagonal from A to C? Where do points A, B, C and D go? What if we reflect along the horizontal line of symmetry?

Ans. If we reflect along the diagonal from A to C, D occupies the position occupied by B earlier. A and C remain at the same place.

If we reflect along the horizontal line of symmetry, D and C occupies the position earlier occupied by A and B respectively.

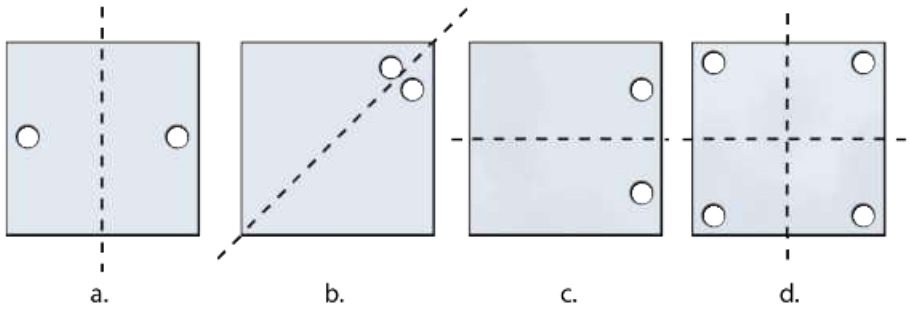
Section 9.1

Page no. 223

Q1 In each of the following figures, a hole was punched in a folded square sheet of paper and then the paper was unfolded. Identify the line along which the paper was folded.

Figure (d) was created by punching a single hole. How was the paper folded?

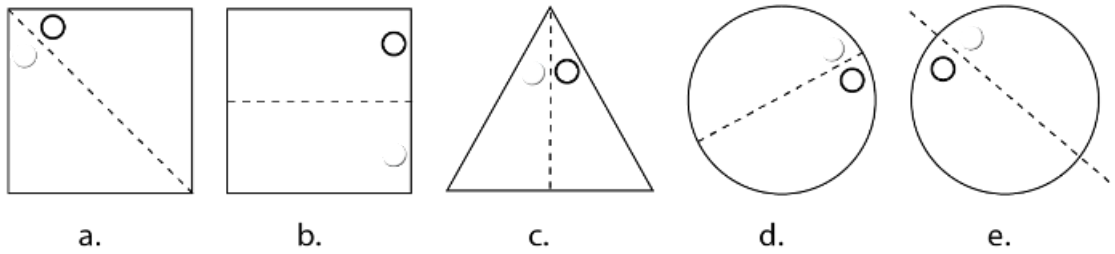
Ans.



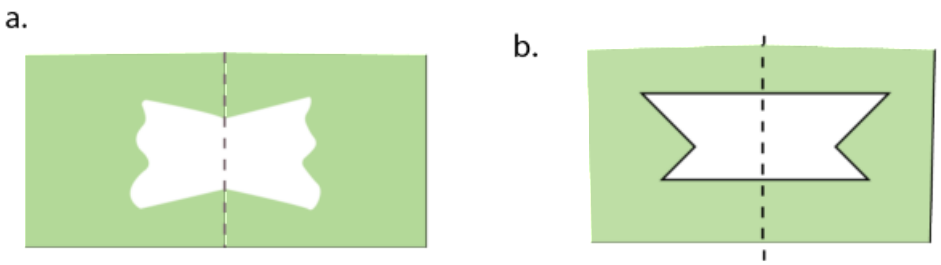
For figure (d), paper was folded vertically and then horizontally or vice versa.

Q2. Given the line(s) of symmetry, find the other hole(s):

Ans.



Q4. After each of the following cuts, predict the shape of the hole when the paper is opened. After you have made your prediction, make the cutouts and verify your answer.



c. 

d. 

Q5. Suppose you have to get each of these shapes with some folds and a single straight cut. How will you do it?

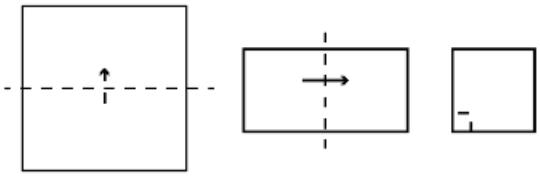
a. The hole in the centre is square.

b. The hole in the centre is a square.

Note : For the above two questions, check if the 4-sided figures in the centre satisfy both the properties of a square.

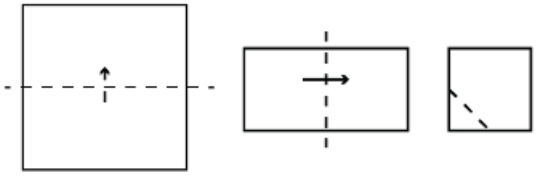
Ans. a. At first fold the paper horizontally then vertically.

Now cut a small square at the center (all sides closed corner).



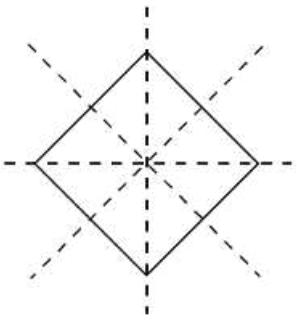
b. At first fold the paper horizontally then vertically.

Now at closed corner, cut along with a slanting line.

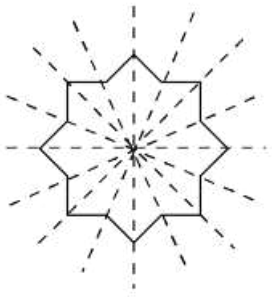


Q6. How many lines of symmetry do these shapes have?

Ans. a.

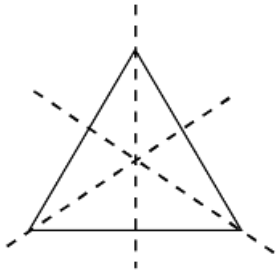


4 Lines of Symmetry



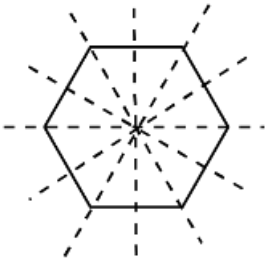
8 Lines of Symmetry

b. A triangle with equal sides and equal angles.



3 Lines of Symmetry

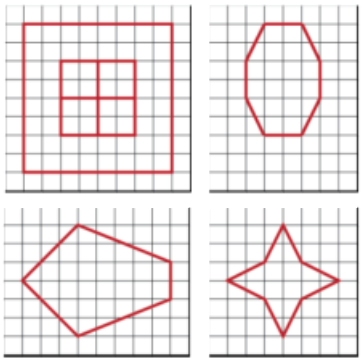
c. A hexagon with equal sides and equal angles.



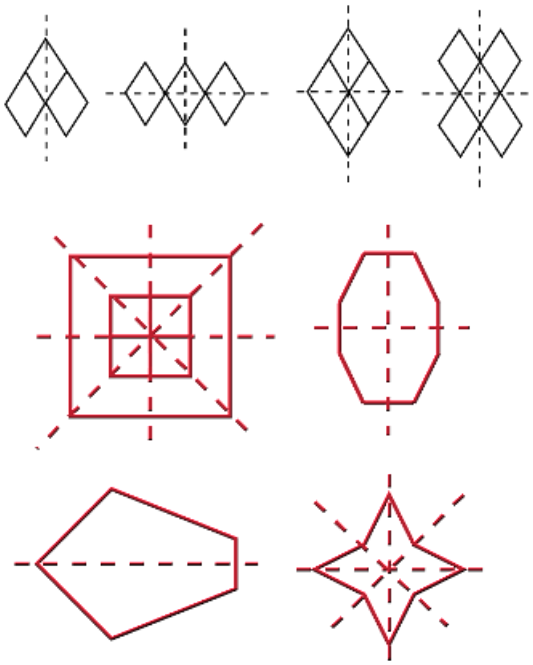
6 Lines of Symmetry

Q7. Trace each figure and draw the lines of symmetry, if any:



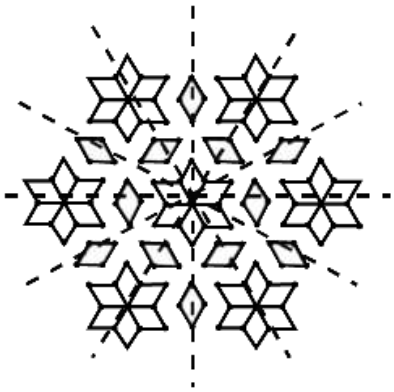


Ans.



Q8. Find the lines of symmetry for the *kolam* below.

Ans.



Q9. Draw the following.

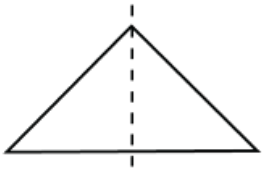
a. A triangle with exactly one line of symmetry

b. A triangle with exactly three lines of symmetry

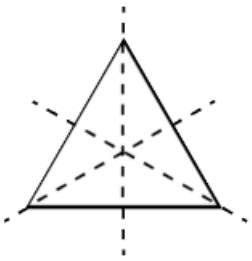
c. A triangle with no line of symmetry

Is it possible to draw a triangle with exactly two lines of symmetry?

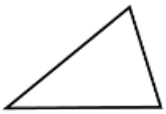
Ans. **a.**



b.



c.



No, it is not possible to draw a triangle with exactly two lines of symmetry.

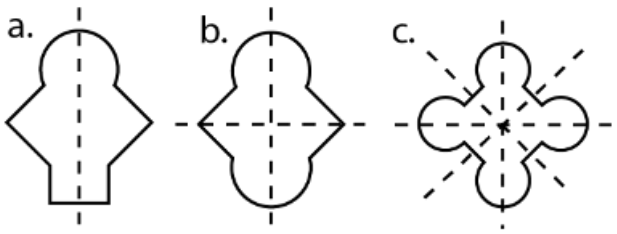
Q10. Draw the following. In each case, the figure should contain at least one curved boundary.

a. A figure with exactly one line of Symmetry

b. A figure with exactly two lines of symmetry

c. A figure with exactly four lines of symmetry

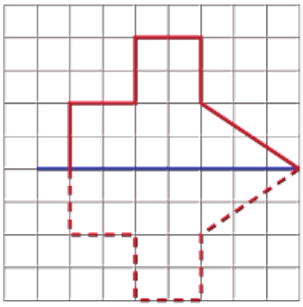
Ans.



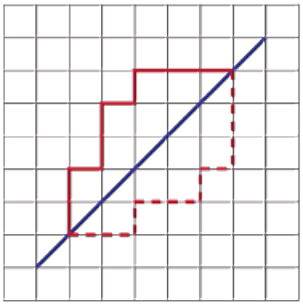
Q11. Copy the following on squared paper. Complete them so that the blue line is a line of symmetry. Problem (a) has been done for you.

Hint: For (c) and (f), see if rotating the book helps!

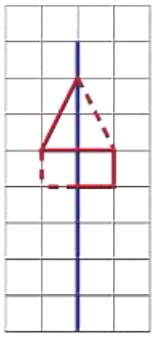
Ans.



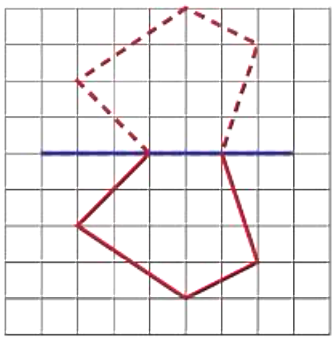
(b)



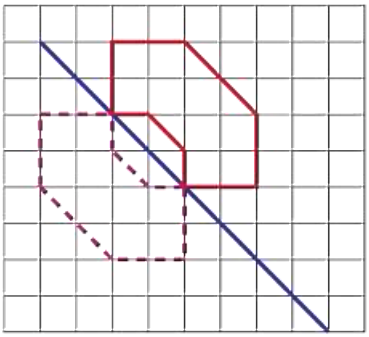
(c)



(d)



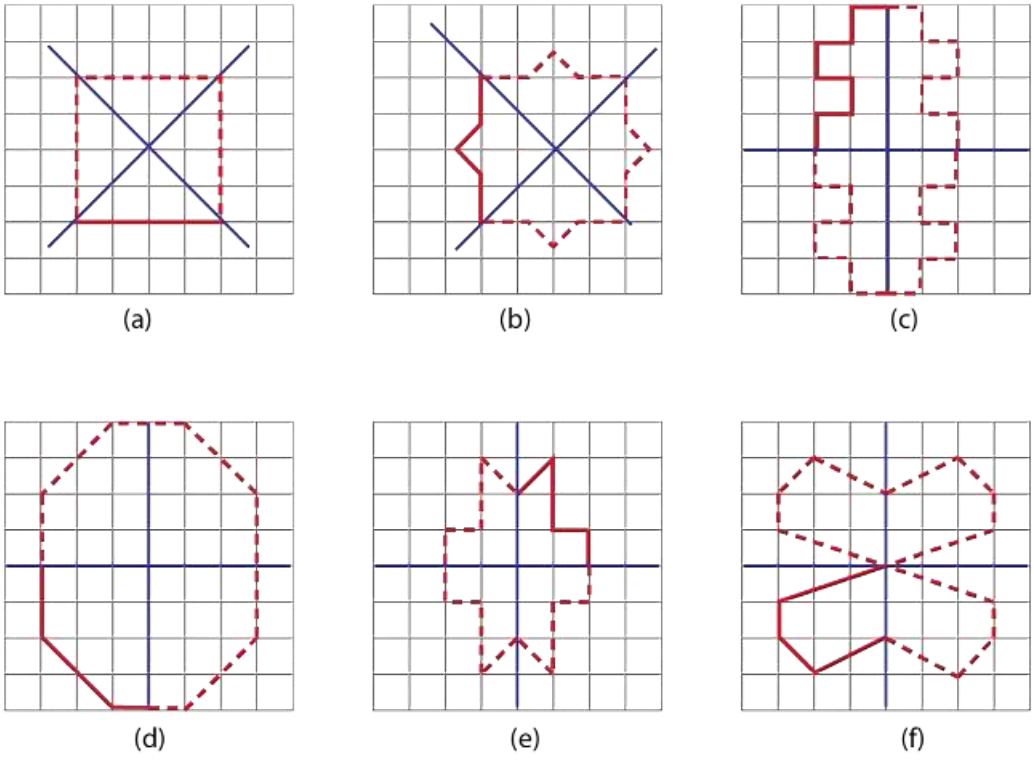
(e)



(f)

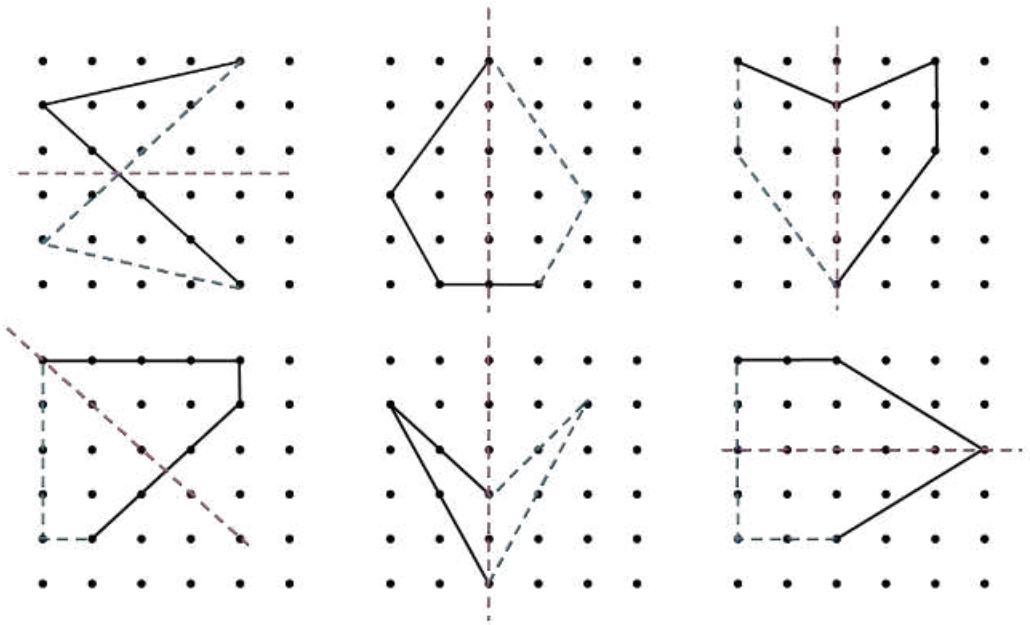
Q12. Copy the following drawing on squared paper. Complete each one of them so that the resulting figure has the two blue lines as lines of symmetry.

Ans.



Q13. Copy the following on a dot grid. For each figure draw two more lines to make a shape that has a line of symmetry.

Ans.

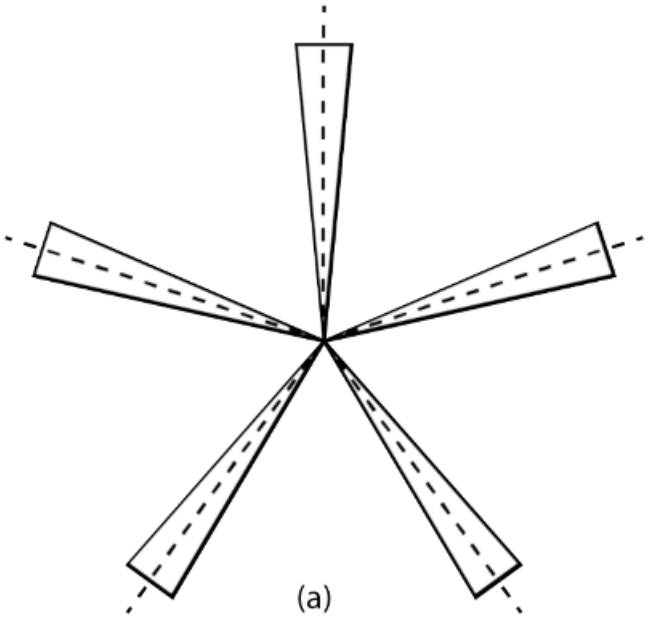


Section 9.2

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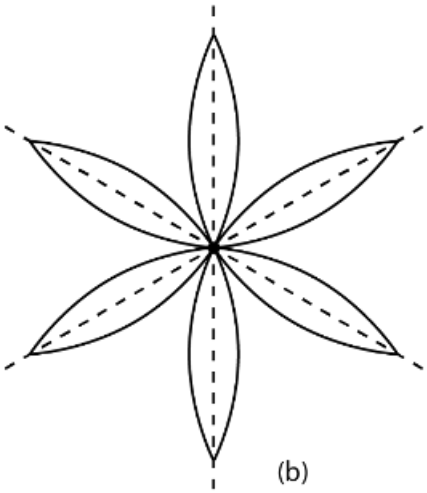
Can you draw a figure with radial arms that has a) exactly 5 angles of symmetry, b) 6 angles of symmetry? Also find the angles of symmetry in each case.

Hint: Use 5 radial arms for the first case. What should the angle between two adjacent radial arms be?



Ans.

Angles of symmetry = 72°, 144°, 216°, 288°, 360°.



Angles of symmetry = 60°, 120°, 180°, 240°, 300°, 360°.

Consider a figure with radial arms having exactly 7 angles of symmetry. What will be its smallest angle of symmetry? Is the number of degrees a whole number in this case? If not, express it as a mixed faction.

Let us find the angles of symmetry for other kinds of figures.

Ans. $360° \div 7 = 51\frac{3}{7}$

No, its smallest angle of symmetry is not a whole number.

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Figure it out

Q1. Find the angles of symmetry for the given figures about the point marked.

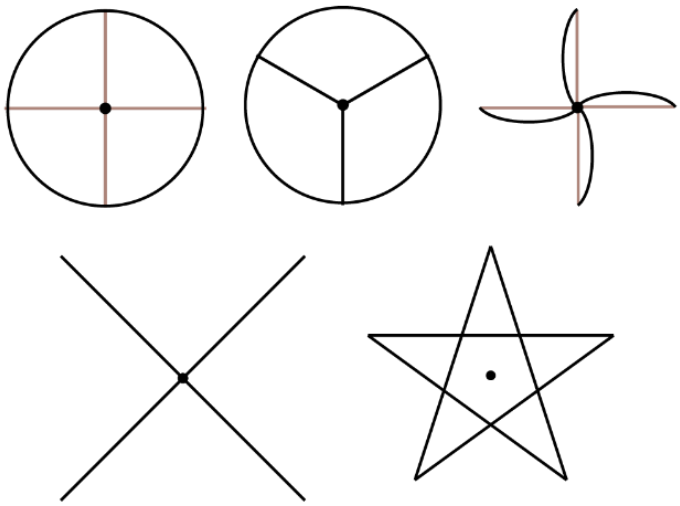
Ans. a. Angles of symmetry = 90°, 180°, 270°, 360°

b. Angle of symmetry = 360°

c. Angles of symmetry = 180°, 360°

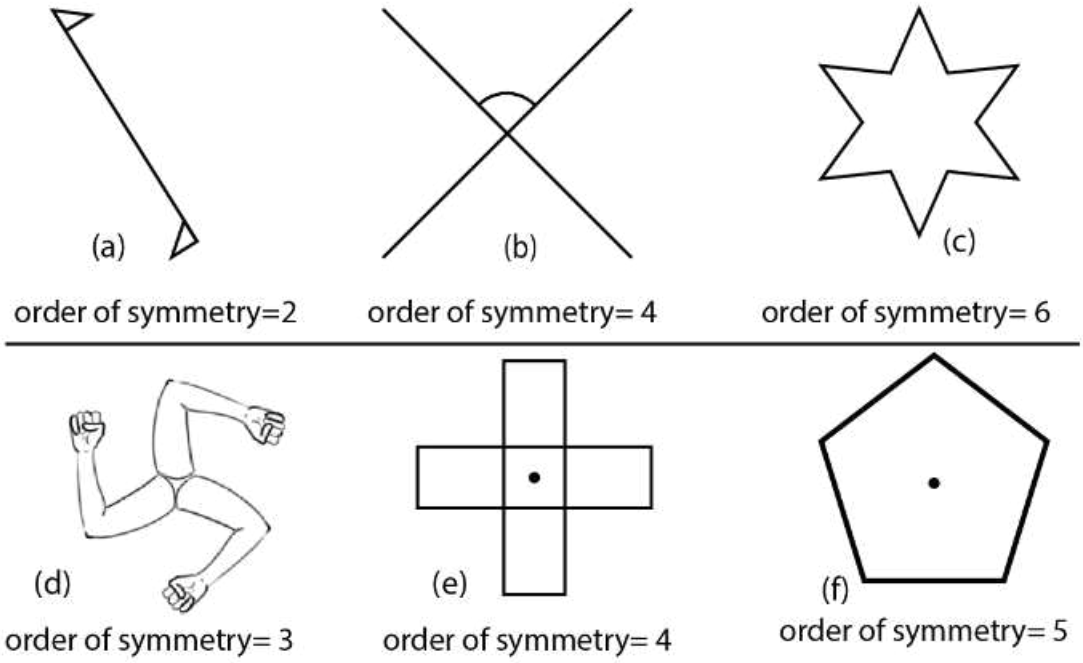
Q2. Which of the following figures have more than one angle of symmetry?

Ans. The following figures have more than one angle of symmetry–



Q3. Give the order of rotational symmetry for each figure:

Ans. Orders of rotational symmetry



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In each case, the angles are the multiples of the smallest angle. You may wonder and ask if this will always happen. What do you think?

Ans. Yes, the angles of symmetry are always the multiples of the smallest angle. For example, the second angle is twice the amount of rotation of the first rotation.

True or False

- **Every figure will have 360 degrees as an angle of symmetry.**
- **If the smallest angle of symmetry of a figure is a natural number in degrees, then it is a factor of 360.**

Ans. True

True

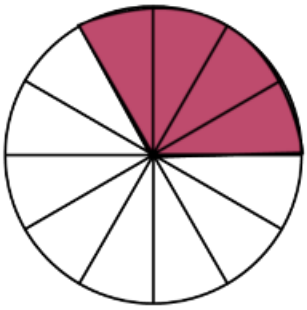
Section 9.2

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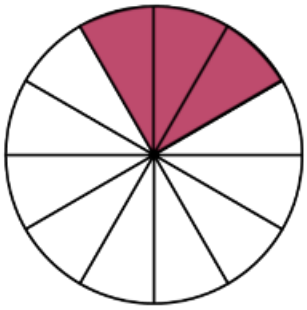
Figure it out

Q1. Color the sectors of the circle below so that the figure has i) 3 angles of symmetry, ii) 4 angles of symmetry, iii) what are the possible numbers of angles of symmetry you can obtain by coloring the sectors in different ways?

Ans.



(i) 3 Angles of Symmetry



(ii) Angles of Symmetry

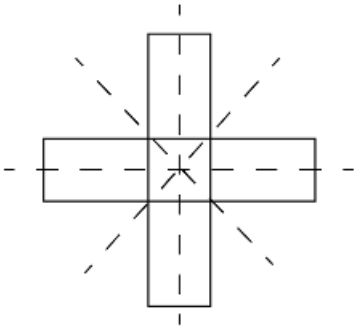
(i) 3 Angles of symmetry
(ii) 4 angles of symmetry
(iii) 12 angles of symmetry are possible to obtain.

Q2. Draw two figures other than a circle and a square that have both reflection symmetry and rotational symmetry.

Ans.

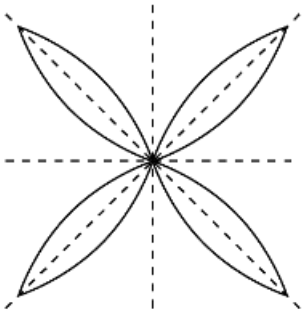
Numbers of line symmetry = 4

Order of rotational symmetry = 4



Numbers of line symmetry = 4

Order of rotational symmetry = 4



Q3. Draw, wherever possible, a rough sketch of

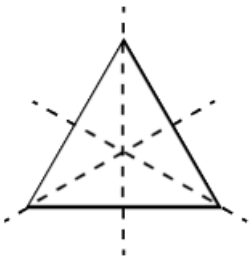
a. A triangle with at least two lines of symmetry and at least two angles of symmetry.

b. A triangle with only one line of symmetry but not having rotational symmetry.

c. A quadrilateral with rotational symmetry but no reflection symmetry.

d. A quadrilateral with reflection symmetry but not having rotational symmetry.

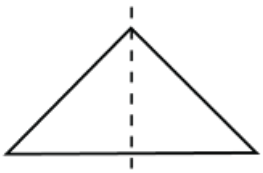
Ans. a.



3 lines of symmetry

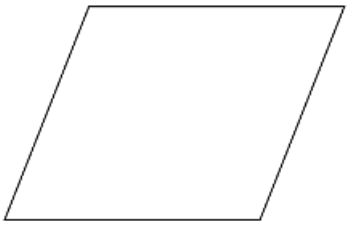
3 angles of symmetry

b.



1 line of symmetry

No rotational symmetry

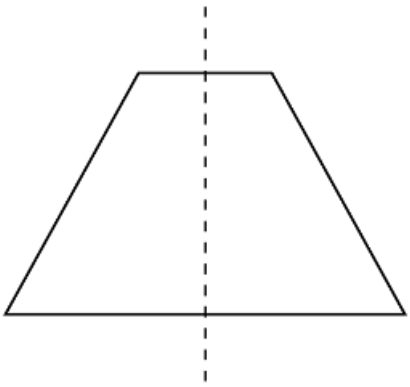


c.

No line of symmetry

2 angles (180°, 360°)

d. A quadrilateral with reflection symmetry but not having rotational symmetry



Q4. In a figure, 60° is the smallest angle of symmetry. What are the other angles of symmetry of the figure?

Ans. Other angles of symmetry = 120°, 180°, 240°, 300°, 360°.

Q5. In the figure, 60° in an angle of symmetry. The figure has two angles of symmetry less then 60°. What is its smallest angle of symmetry?

Ans. The smallest angle of symmetry = 20°

Q6. Can we have a figure with rotational symmetry whose smallest angle of symmetry is

a. 45°?

b. 17°?

Ans. a. yes, as 360° is a multiple of 45°.

b. No, as 360° is not a multiple of 17°.

Q7. This is a picture of the new Parliament Building in Delhi.

a. Does the outer boundary of the picture have reflection symmetry? If so, draw the lines of symmetries. How many are they?

b. Does it have rotational symmetry around its centre? If so, find the angles of rotational symmetry.

Ans. a. Yes, the outer boundary of the picture has 3 lines of symmetry.

b. Yes, the outer boundary has rotational symmetry. The angles of rotational symmetry are 120°, 240°, 360°.

Q8. How many lines of symmetry do the shapes in the first shape sequence in Chapter 1, Table 3, the Regular Polygons, have? What number sequence do you get?

Ans.

Regular Polygon	**No. of line symmetry**
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10

It is a counting number sequence.

Q10. How many lines of symmetry do the shapes in the last shape sequence in Chapter 1, Table 3, the Koch Snowflake sequence, have? How many angles of symmetry?

Ans. Number of lines of symmetry : 3, 6, 6,6,6

Angles of symmetry : 3, 6, 6,6,6

Q11. How many lines of symmetry and angles of symmetry does Ashoka Chakra have?

Ans. Number of lines of symmetry = 24

Number of angles of symmetry = 24